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Method of Studying Surface Torsional Anchoring of Nematic Liquid Crystal

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In this paper we introduce a method to study surface torsional anchoring of both chiral-molecule-doped and pure nematic liquid crystal (NLC) without using any extra field. By measuring the transmission of the twisted or supertwisted NLC cells, the actual twist angles in NLC cells are obtained, which differ from the angle between two boundary rubbing directions. A theoretical discussion reveals the relationship of the surface torsional anchoring energy and the difference, and thus the surface torsional anchoring energy can be calculated.

KEYWORDS: nematic liquid crystal, surface anchoring, twist angle

1. Introduction

The mechanism of surface alignment of nematic liquid crystal molecules on a treated substrate is an interesting research field of liquid crystal physics.[1] Many methods to measure the polar surface anchoring energy have been proposed,[2-7] such as surface disclination, Freedericksz transition and high field. However there are few ways to study surface torsional anchoring. Sato *et al.* had investigated the relationship between rubbing strength and surface torsional anchoring of nematic liquid crystal using the Cano wedge cell,[8] and Sugiyama *et al.* had measured the torsional surface coupling strength using a high magnetic field.[9] These are very popular method. However, the first method is only applicable to chiral-molecule-doped nematic liquid crystals, and the second requires a complex system to apply extra-high magnetic field.

In this paper we introduce a new method of studying the torsional anchoring in NLC cells, which is applicable to either pure or chiral-molecule nematic liquid crystals. The transmission of NLC cells is related to the twist angle, which is the angle between the surface NLC directors on two substrates. After measuring the transmission under different conditions, we can calculate the twist angle and the difference between the actual twist angle and and the angle of two rubbing directions, which is the result of the balance of the surface torsional anchoring torque and the elastic bulk torque. Hence we can also obtain the surface torsional anchoring strength. By using this method we need only to measure the transmission of the NLC cell; no extra field is applied.

2. Analysis of Surface Torsional Anchoring

According to continum theory, the free energy per unit area is obtained as the sum of the bulk elastic energy F_b and the surface anchoring energy F_s as follows;[10]

$$F=F_b+F_s, \tag{1}$$

where

$$F_b=\frac{K_2}{2d}(\Phi_t-\Phi_0)^2$$

$$F_s=\frac{1}{2}A\sin^2\Delta\Phi,$$

where K_2 is the twist elastic constant, p is the LC's native pitch, d is the cell gap, A is the surface anchoring strength, and Φ_0 is the native twist angle of the NLC medium which can be expressed as

$$\Phi_0=2\pi d/p. \tag{2}$$

For pure NLCs, $p\to\infty$, $\Phi_0=0$. Φ_r is the angle between two rubbing directions, $\Delta\Phi$ is the deviation of the surface LC director from the rubbing direction, Φ_t is the actual twist angle related to Φ_r and $\Delta\Phi$ by

$$\Phi_t=\Phi_r-2\Delta\Phi. \tag{3}$$

By minimizing F or $\delta F/\delta\Delta\Phi=0$, the torque balance equation can be given as

$$\frac{2K_2}{d}(\Phi_t-\Phi_0)-A\sin(2\Delta\Phi)=0. \tag{4}$$

From this equation, the surface anchoring strength A is expressed as

$$\frac{Ad}{K_2}=\frac{2(\Phi_r-\Phi_0-2\Delta\Phi)}{\sin(2\Delta\Phi)}=\frac{2(\Phi_t-2\pi d/p)}{\sin(2\Delta\Phi)}. \tag{5}$$

In the eq. (5), the values of Φ_r, cell gap d, LC's twist elastic constant K_2 and LC's pitch p are easy to obtain. Clearly, from eq. (10), A can be calculated when the twist angle Φ_t or the deviation $\Delta\Phi$ is determined.

3. Measurement of Twist Angle and Deviation

The measuring system was set up using a polarizing microscope, as shown in Fig. 1. The optical components and NLC cell were arranged in the following sequence: the light source, the entrance polarizer, the interference filter, the filled NLC cell, the objective lens, the compensator, the exit polarizer, and the eye piece. The transmission axis of the entrance polarizer forms an angle ψ_0 with respect to the entrance LC director, and between two polarizers, there exists an angle ψ_p, shown in Fig. 2. Light is normally incident along the z axis. The interference filter is used to control the wavelength of incident light.

Based on Jones matrix representation, the optical transmission of the system can be described as:[10,11]

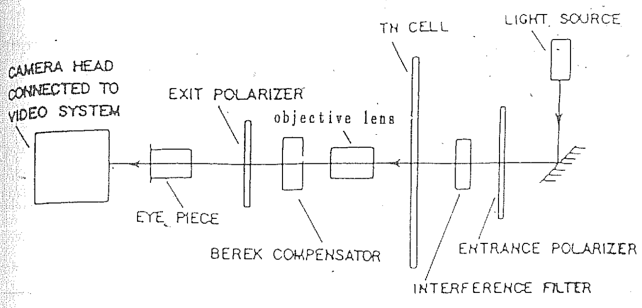

Fig. 1. Arrangement of measuring equipment.

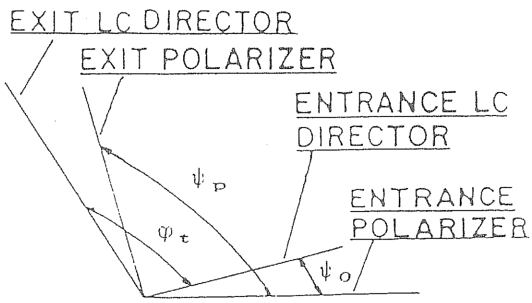

Fig. 2. Position of polarizers and LC cell.

$$T=\left|\left[\cos(\psi_p-\psi_0)\,e^{i\delta_x}\ \sin(\psi_p-\psi_0)\,e^{i\delta_y}\right]\times\begin{bmatrix}a & b\\ -b^* & a^*\end{bmatrix}\begin{bmatrix}\cos\psi_0\\ \sin\psi_0\end{bmatrix}\right|^2, \tag{6}$$

where ψ_p are the angle of the optical axes of two polarizers, ψ_0 is the angle between the entrance polarizer and entrance LC director, $\delta_x-\delta_y$ is the optical phase difference produced by the compensator, and

$$\begin{aligned}a=&\frac{1}{\sqrt{1+u^2}}\sin(\Phi_t\sqrt{1+u^2})\sin(\Phi_t)\\&+\cos(\Phi_t\sqrt{1+u^2})\cos(\Phi_t)\\&+i\frac{u^2}{\sqrt{1+u^2}}\cos(\Phi_t\sqrt{1+u^2})\cos(\Phi_t)\end{aligned}$$

$$\begin{aligned}b=&\frac{1}{\sqrt{1+u^2}}\sin(\Phi_t\sqrt{1+u^2})\cos(\Phi_t)\\&-\cos(\Phi_t\sqrt{1+u^2})\sin(\Phi_t)\\&+i\frac{u^2}{\sqrt{1+u^2}}\sin(\Phi_t\sqrt{1+u^2})\sin(\Phi_t),\end{aligned}$$

in which Φ_t is the twist angle and

$$u=\frac{\pi d}{\lambda\Phi_t}\left(\frac{n_e}{\sqrt{1+w\sin^2\theta}}-n_0\right),$$

and

$$w=\left(\frac{n_e}{n_0}\right)^2-1;\ \bar{\theta}=\frac{1}{d}\int_0^d\theta(z)\,dz.$$

Here, n_0 and n_e are the ordinary and extraordinary indices of the refraction of LC, λ is the wavelength of incident light, $\theta(z)$ is the tilt angle, θs is the pretilt angle, and $\theta(0)=\theta(d)=\theta s$. $\bar{\theta}$ is the average tilt angle throughout the NLC medium.

When θs of the measured NLC cell is small, we can assume that the director is tilted uniformly and twisted linearly throughout the entire cell, and $\bar{\theta}$ can be substituted simply by θs. But when θs is high, uniform tilt of the director profile is not a valid assumption; we must use the average tilt angle instead of the pretilt angle. Lien had detailed this problem in refs. 10 and 11. In his study, it was shown that the calulation results of using average tilt angle correspond well to those of using the 4×4 matrix method, which is applicable to both cases of low and high pretilt angle.

Transmission T can be minimized with respect to $(\delta_x-\delta_y)$. For $\psi_p=\pi/2$ and $\psi_0=\pi/4$, we can obtain[11]

$$\tan(\delta_x-\delta_y)=A/B, \tag{7}$$

where

$$\begin{aligned}A=&-u\sin(2\Phi_t\sqrt{1+u^2})\\B=&\sqrt{1+u^2}\cos(2\Phi_t)\cos(2\Phi_t\sqrt{1+u^2})\\&+\sin(2\Phi_t)\sin(2\Phi_t\sqrt{1+u^2}).\end{aligned}$$

The value of $(\delta_x-\delta_y)$ can be obtained from the compensator scale. It is easy to show that $-A/B$ is the phase difference between E_x and E_y produced by the NLC medium at its exit surface with $\psi_0=\pi/4$. The purpose of the compensator is to produce an opposite phase shift, A/B, such that the total phase difference of the system is zero.

Consider the case of $(\delta_x-\delta_y)=0$, transmission T can be minimized with respect to ψ_p and ψ_0:

$$\begin{aligned}\frac{\delta T}{\delta\psi_0}=0,\ \frac{\delta^2T}{\delta\psi_0^2}>0\\ \frac{\delta T}{\delta\psi_p}=0,\ \frac{\delta^2T}{\delta\psi_p^2}>0.\end{aligned} \tag{8}$$

From eqs. (6) and (8), the minimal transmission conditions are given as

$$\begin{aligned}&(1+u^2)\cos^2(\Phi_t\sqrt{1+u^2})\sin(2\Phi_t-2\psi_p)\\&-\sin^2(\Phi_t\sqrt{1+u^2})\sin(2\Phi_t-2\psi_p)\\&-\sqrt{1+u^2}\sin(2\Phi_t\sqrt{1+u^2})\cos(2\Phi_t-2\psi_p)=0.\end{aligned} \tag{9}$$

$(\delta_x-\delta_y)=0$ means the removal of the compensator from the system. ψ_p under minimal transmission can be measured easily by rotating the LC cell and polarizers.

After $(\delta_x-\delta_y)$ and ψ_p are determined, by solving eqs. (7) and (9), the actual twist angle Φ_t and variable u can be calculated. Furthermore, the thickness of the filled cell can also be calculated using eq. (5).

Using this method, twist angles of several NLC cells were measured and the results are shown in Table I. Here, for pure NLCs, $p\rightarrow\infty$, $\Phi_0=0$. From Table I, it is found that the actual twist angles of LC cells have different deviations from Φ_r, and from eq. (5), we can obtain different surface anchoring energy, which is also listed in Table I.

Because this method is mainly based on the measurement of optical transmisson of a NLC cell, a well-aligned NLC medium is required to obtain an accurate

Table I. Results of measurement. Here $p \rightarrow \infty$ means the LC material is pure nematic LC.

Sample	Spl.1	Spl.2	Spl.3	Spl.4	Spl.5
Pitch (μm)	15	15	15	125	∞
K_2 (10^{-12} N/m)	3.9	3.9	3.9	3.1	4.1
Gap (μm)	7.6	7.9	8.1	9.6	10.1
Φ_0 (°)	182	190	194	27.5	0
Φ_r (°)	240	240	225	90	90
Φ_t (°)	213	235	223	87	89
$2\Delta\Phi$	27	5	2.0	3.0	1.0
A (10^{-6} J/m^2)	1.22	8.91	14.0	12.8	71.7

value. The uniform transmission of the STN cell requires an extremely uniform thickness, therefore, the measurement of the anchoring strength of NLCs on the same surface using TN LC cells will be more accurate than that using STN LC cells.

4. Results and Discussion

The results listed in Table I show that the lower the surface torsional anchoring strength, the greater the deviation of the surface NLC director from the rubbing direction. This tendency is clearly shown in Fig. 3, the theoretical curves of $\Delta\Phi$ versus A of which are derived using eq. (5).

From the curve in Fig. 3 it can also be found that the thicker the cell gap, the smaller the deviation $\Delta\Phi$. This is easy to understand; when the cell gap is increased, the distortion ratio of NLC directors along the z axis, $(\Phi_r - \Phi_0)/d$, is reduced and the bulk elastic torsional torque will be decreased, so the surface anchoring strength becomes relatively stronger and consequently the deviation of the surface LC director decreases.

5. Conclusions

In this paper, we introduced a method for studying the surface torsional anchoring of NLCs. No extra field is used in this method, which is applicable to either pure or doped NLCs. The accuracy of this method is mainly dependent on the measuring precision of the optical transmission of NLC cells. Research on the surface anchoring of different NLC materials is now underway.

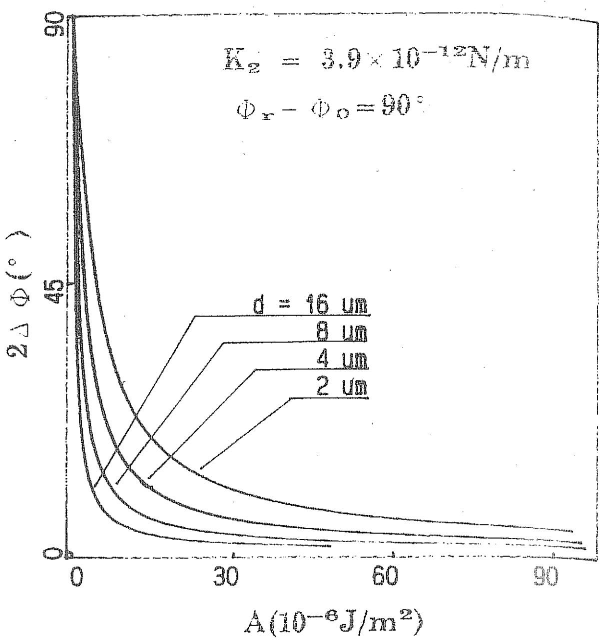

Fig. 3. Theoretical relationship between A and $\Delta\Phi$ with different values of cell gap.

In this method the deviation of the surface LC director at boundaries, from the rubbing direction can also be obtained. From our study we find that the value of the deviation greatly depends on the surface anchoring energy.

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