Computation of the diffracted field of a toothed occulter by the semi-infinite rectangle method

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To observe the solar corona, stray light in the coronagraph, arising primarily from an external occulter and diaphragm illuminated directly by the Sun, should be strongly suppressed. A toothed occulter and diaphragm can be used to suppress stray light because they diffract much less light in the central area than a circular disk. This study develops a method of computing the light diffracted by a toothed occulter and diaphragm, obtaining the optimum shape using this method. To prove the method's feasibility, the diffracted fields of circular and rectangular disks are computed and compared with those calculated by a conventional method. © 2013 Optical Society of America OCIS codes: (050.0050) Diffraction and gratings; (260.0260) Physical optics.

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1. INTRODUCTION

Since the first coronagraph flew to space in 1963 [1], many solar observing satellites have been equipped with coronagraphs, including Skylab [2], Solar Maximum Mission (SMM) [3], Solar and Heliospheric Observatory (SOHO) [4], and Solar TErrestrial RElations Observatory (STEREO) [5]. The elimination of stray light is the key problem in coronagraph manufacturing because the corona is much less bright than the Sun, typically on the order of 10^{-5} to $10^{-13}B_{\odot}$ [4], where B_{\odot} is the mean solar brightness. An external occulter and diaphragm are used in an externally occulted coronagraph. Figure 1 shows a schematic diagram of a typical externally occulted coronagraph [4,6]. It is composed of four stops, two occulters, three groups of lenses, and a Lyot spot. The external occulter D₁ is used to block the photospheric light from the entrance aperture A_1 . The objective lens O_1 images the corona at the position of the field stop, and at the same time images the external occulter D₁ onto the internal occulter D₂. The external diaphragm blocks the light from neighboring instruments and other sources outside the field of view. The field stop is not located exactly at the conjugate position of A_0 , but it can still block the light diffracted from A_0 , as shown in Fig. 1. The Lyot stop is located at the conjugate position of the entrance aperture A₁ and blocks the light diffracted from A_1 . The relay lens O_3 reimages the corona image onto a CCD camera positioned at the focal plane F.

The external occulter and diaphragm in the coronagraph, which are directly illuminated by photospheric light, are the primary contributors to the diffracted light. Although they are blocked by the field stop and internal occulter, respectively, the diffracted light can still illuminate the objective lens, producing scattered light strong enough to overwhelm the outer coronal signal. Therefore, suppressing this diffracted light becomes a key problem in coronagraph fabrication. Many occulters have been developed to limit stray light since Evans [7] invented the externally occulted coronagraph, such as the two-disk occulter [8], three-disk occulter [8,9], multiple-disk occulter [8], toothed disk occulter [9,10], multithreaded occulter [9], polished cone occulter [9], spider web mask [11], and petal occulter [12]. Apparently, different shapes, including those of teeth, spider webs, and petals, can be used for the diaphragm. Among these three shapes, the latter two are too hard to fabricate. Only the toothed shape combines efficient diffracted light reduction with ease of fabrication. Because the toothed diaphragm and the occulter are the same in theory, we discuss only the toothed occulter in the following.

Along with the shape design, many methods have been developed to compute the diffracted field, for example, the analytically approximate method [13], Fresnel zone plate method [14–17], rectangular diffraction method [18], Fourier transform method [19], and boundary diffraction method [20]. Among them, the analytically approximate method is not precise enough to calculate the optimum shape of the occulter. The Fourier transform and rectangular diffraction methods are based on Fraunhofer diffraction, but the diffraction in an occulter usually falls within the scope of Fresnel diffraction. Therefore, these two methods are not appropriate for calculating the diffraction in an occulter in most circumstances. The Fresnel zone and boundary diffraction methods are two generic and reliable ways of computing the diffracted field of a toothed occulter. However, obtaining the optimized shape of toothed occulters is complicated in both of them. This paper aims to determine the optimum shape of external occulters and diaphragms. To do this, we propose an algorithm for calculating the diffracted field of occulters on the basis of Fresnel integration theory [21,22]. We call this algorithm the semi-infinite rectangular method and prove that it is fast enough to optimize the shape of the toothed occulter.

The algorithm has the following three steps. First, consider the diffraction at every side of a polygon acting as a semiinfinite rectangular aperture, as illustrated in Fig. 2. Next,



Fig. 1. Schematic of the externally occulted coronagraph. A_0 , external diaphragm; D_1 , external occulter; A_1 , entrance aperture; O_1 , objective lens; D_2 , internal occulter; O_2 , field lens; O_3 , relay lens with Lyot spot; F, focal plane [4].



Fig. 2. Semi-infinite rectangle; the rectangle extends infinitely to the left.

calculate the diffracted field at each side of the polygon. Finally, take a coherent superposition of the diffracted field at all sides of the polygon. For instance, the diffraction by a square mask can be regarded as the superposition of the diffracted fields of four semi-infinite rectangles corresponding to the four sides of the square, as illustrated in Fig. <u>3</u>. In fact, the diffracted fields generated by the four corners are relatively weak, so they can be ignored. In Section <u>2.B</u>, it is proved that the ignored parts are much smaller than the four parts calculated. Theoretically, the strongest diffracted light in the receiving plane is along the direction perpendicular to the edge of the diffracted aperture. The direction perpendicular to the the edge of the corner part is always farther away from the center than that of the side part, so the diffracted field due to the corner part is always lower than that due to the side part.



Fig. 3. Square mask. Only the central square part is opaque; the rest, including the blue corners, transmits light. Parts 1–4 are the calculated parts of the semi-infinite rectangle. Infinite dimension always points outward. Parts 5–8 are ignored.

Similarly, the diffraction of a dodecagon mask can be regarded as the superposition of the diffraction of twelve semi-infinite rectangles (Fig. <u>4</u>). A circular disk can be considered as a polygon with N sides, where N is large enough that the polygon is close to a circle. Figure <u>5</u> shows a four-toothed mask and the semi-infinite rectangles that are calculated. Some of the semi-infinite rectangle areas overlap. However, the effect of these areas is small compared to the entire diffraction, so these overlaps are ignored.

To prove the feasibility of this algorithm, the diffracted fields of both square and circular masks were calculated using the proposed method and conventional methods for comparison. For a square mask, the diffracted field can be calculated directly by Fresnel integration. However, for a



Fig. 4. Dodecagon mask. Only the central dodecagon is opaque; the rest, including the white triangles, transmits light. Parts 1–12 are the calculated parts of the semi-infinite rectangle. Infinite dimension always points outward. Diffraction due to Part 14 can be obtained by symmetric transformation of that due to Part 13 with respect to the $\theta_0 = \theta_1$ axis. Diffraction due to Parts 13 and 14 with respect to the $\theta_0 = \theta_2$ axis. Diffraction due to Parts 13 and 14 with respect to the $\theta_0 = \theta_3$ axis. Diffraction due to Parts 13, 14, and 15 with respect to the $\theta_0 = \theta_3$ axis. Diffraction due to Part 17 can be obtained by symmetric transformation of that due to Part 16 with respect to the $\theta_0 = \theta_4$ axis.



Fig. 5. Four-toothed mask. Only the central four-toothed part is opaque; the rest, including the white parts, transmits light. Parts 5–12 are the calculated parts of the semi-infinite rectangle. Infinite dimension always points outward. Diffraction due to Parts 3 and 4 can be obtained by symmetric transformation of that due to Part 1.

circular mask, the diffracted field should be calculated by the Fresnel–Kirchhoff formula. This paper includes five sections and two appendices. In Section 2, the feasibility of the algorithm is demonstrated. In Section 3, the diffracted field of the toothed occulter is calculated. Section 4 presents the results and discussion, and the conclusions are drawn in Section 5. The diffracted field of a single semi-infinite rectangle is derived in Appendix <u>A</u>, and the feasibility of the method for other shapes is demonstrated in Appendix <u>B</u>.

2. FEASIBILITY DEMONSTRATION

A. Calculation of the Diffraction of a Semi-infinite Rectangle

Figure $\underline{6}$ shows a schematic diagram of the diffraction of a single semi-infinite rectangle. The incident light is considered to be a plane wave. From the Fresnel integration formula, the diffracted field in the screen is given by (see Appendix A) [21,22]

$$U(x_0, y_0) = \frac{U_0 e^{ikz}}{2j} \left\{ \left[C(\alpha_2) + \frac{1}{2} \right] + j \left[S(\alpha_2) + \frac{1}{2} \right] \right\} \\ \times \left\{ \left[C(\beta_2) - C(\beta_1) \right] + j \left[S(\beta_2) - S(\beta_1) \right] \right\}, \quad (1)$$

where U_0 is the amplitude of the incident plane wave in the mask plane, $\alpha_2 = [2/(\lambda z)]^{1/2}(-R-x_0)$, $\beta_1 = -[2/(\lambda z)]^{1/2}(w+y_0)$, $\beta_2 = [2/(\lambda z)]^{1/2}(w-y_0)$, x_0 and y_0 are the coordinates in the screen, and z, w, and R are illustrated in Fig. <u>6</u>. C(x) and S(x) represent the two basic integrations of the Fresnel integration formula [<u>20,21</u>] and are also shown in Appendix A.

B. Demonstration of the Feasibility of the Method for a Square Mask

Here, the diffraction of a square mask computed by the semiinfinite rectangle method is compared with that obtained by the conventional Fresnel integration formula. Figure <u>7</u> shows a schematic diagram of the diffracted set of a square mask. In the semi-infinite rectangle method, the calculation needs to superpose the diffracted field from each side of the square mask. As the square mask is symmetric, the coherent



Fig. 6. Schematic diagram of diffraction of the semi-infinite rectangle (the left-hand screen is opaque except for the semi-infinite rectangle).



Fig. 7. Schematic diagram of diffraction of the square mask.



Fig. 8. Diffracted field of the square mask calculated by the semiinfinite rectangle method.

superposition calculation can be simplified. To make a symmetrical transformation, it is convenient to change Eq. (1) to polar coordinates using $x_0 = r_0 \cos(\theta_0)$, $y_0 = r_0 \sin(\theta_0)$. According to Eq. (1), the diffracted field from region 1 in Fig. 3 is

$$U_{1}(r_{0},\theta_{0}) = \frac{U_{0}e^{ikz}}{2j} \left\{ \left[\frac{1}{2} - C(\alpha_{1}) \right] + j \left[\frac{1}{2} - S(\alpha_{1}) \right] \right\} \\ \times \left\{ \left[C(\beta_{2}) - C(\beta_{1}) \right] + j \left[S(\beta_{2}) - S(\beta_{1}) \right] \right\}, \quad (2)$$

where $\alpha_1 = [2/(\lambda z)]^{1/2} [w_x - r_0 \cos(\theta_0)]$, $\beta_1 = [2/(\lambda z)]^{1/2} [-w_y - r_0 \sin(\theta_0)]$, and $\beta_2 = [2/(\lambda z)]^{1/2} (w_y - r_0 \sin(\theta_0)]$. Further, $2w_x$ and $2w_y$ are the lengths of the sides of the square along the x and y axes, respectively, and r_0 and θ_0 are the polar coordinates on the receiving screen. Owing to the symmetry of the square mask, we can obtain the diffracted field from the square mask using



Fig. 9. Diffraction intensities of the square mask calculated by the semi-infinite rectangle method (dashed line) and the Fresnel integration formula (solid line). Intensities are calculated along the direction of $\theta_0 = \theta_1$ (Fig. 3), where the corner problem is largest.



Fig. 10. Ratio of diffraction intensity from the corner to that from the entire square. Calculation is along $\theta_0 = \theta_1$ (Fig. 3), where the corner problem is largest.



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Fig. 11. Diffracted intensities of a circular disk calculated by the semi-infinite rectangle method (N = 256) and the Fresnel-Kirchhoff formula.

$$Us(r_0, \theta_0) = U_1(r_0, \theta_0) + U_1(r_0, 2\theta_1 - \theta_0) + U_1(r_0, 2\theta_2 - \theta_0) + U_1(r_0, 2\theta_2 - 2\theta_1 + \theta_0).$$
(3)

Figure <u>8</u> shows the diffracted field on the receiving screen. By using the Fresnel integration formula, the diffracted field from the square mask can be given directly,

$$\begin{split} I(r_0,\theta_0) &= \frac{I_0}{4} \{ [C(\alpha_2) - C(\alpha_1)]^2 + [S(\alpha_2) - S(\alpha_1)]^2 \} \\ &\times \{ [C(\beta_2) - C(\beta_1)]^2 + [S(\beta_2) - S(\beta_1)]^2 \}, \end{split}$$

where $\alpha_1 = [2/(\lambda z)]^{1/2}(-w_x - r_0 \cos(\theta_0)], \quad \alpha_2 = [2/(\lambda z)]^{1/2}[w_x - r_0 \cos(\theta_0)], \quad \beta_1 = [2/(\lambda z)]^{1/2}[-w_y - r_0 \sin(\theta_0)], \quad \beta_2 = [2/(\lambda z)]^{1/2}$ $[w_y - r_0 \sin(\theta_0)], \quad 2w_x \text{ and } 2w_y \text{ are the lengths of the sides of the square along the x and y axes, respectively, and <math>r_0$ and θ_0 are the polar coordinates on the receiving screen.

The diffraction intensity along the radial direction calculated by the two methods is illustrated in Fig. 9. The calculations were made under the following conditions. The distance between the receiving screen and the mask plane, z, is 450 mm. The side lengths of the square, w_x and w_y , are 15 mm. The incident wavelength λ is 550 mm. The results calculated by the two methods agree very well.

Actually, it can be proved that the diffracted fields from the four corners are much smaller than that from the square. Using the Fresnel integration formula, the diffracted fields from region 5 can be calculated by the formula

$$U(r_0, \theta_0) = \frac{U_0 e^{ikz}}{2j} \left\{ \left[\frac{1}{2} - C(\alpha_1) \right] + j \left[\frac{1}{2} - S(\alpha_1) \right] \right\} \\ \times \left\{ \left[\frac{1}{2} - C(\beta_1) \right] + j \left[\frac{1}{2} - S(\beta_1) \right] \right\},$$
(5)

where $\alpha_1 = [2/(\lambda z)]^{1/2} [w_x - r_0 \cos(\theta_0)]$, $\beta_1 = [2/(\lambda z)]^{1/2} [w_y - r_0 \sin(\theta_0)]$. The ratio of the diffraction intensity due to region 5 in Fig. 3 to that from the entire square was calculated; the results are shown in Fig. 10. The diffraction intensity from the entire square is almost 10^4 times higher than that from region 5. Hence, the diffracted field from the corners can be ignored.

C. Demonstration of the Feasibility of the Method for a Circular Disk

The diffracted field from a circular disk was calculated by the proposed method, and the result was compared with that calculated by the Fresnel-Kirchhoff formula. In the semi-infinite rectangle method, a circular disk occulter can be regarded as a polygon of N sides, where N is very large. The results obtained by this method and by the Fresnel-Kirchhoff formula are compared in Fig. 11. The conditions of the calculation are as follows. The distance between the occulter and the receiving screen z is 450 mm. The radius of the circular disk r is 7.5 mm, the incident wavelength λ is 550 µm, and N = 256. Again, the results obtained by these two methods agree well. However, the calculation by the semi-infinite rectangle method required much less time (360.109169 s) than that by the Fresnel-Kirchhoff formula (29,601.084674 s), so the latter method is about 82 times slower than the proposed method.

The diffracted fields of other shapes are also compared and illustrated in Appendix B.



Fig. 12. Diagram of the toothed occulter or diaphragm [8].



Fig. 13. Schematic diagram of diffraction from the toothed disk.

3. CALCULATION OF THE DIFFRACTED FIELD OF A TOOTHED OCCULTER

Theoretically, the strongest diffracted light in the receiving plane is along the direction perpendicular to the edge of the diffracted aperture. The toothed occulter is designed such that all the lines perpendicular to the edge of the occulter do not fall in the center part. Thus, the diffracted light in the center is effectively suppressed (Fig. <u>12</u>) [9]. Figure <u>13</u>

shows a schematic diagram of the diffraction from a toothed disk.

Note that when the diffracted field of a toothed occulter is calculated, the sides of the calculated rectangle must be parallel to the coordinate axis. Therefore, a coordinate transformation is needed. The diffracted field of the semi-infinite rectangle is calculated under $x'_0 - y'_0$ coordinates as illustrated in Fig. <u>12</u>. It is then transformed to $x_0 - y_0$ coordinates. If the



Fig. 14. Toothed occulter with four teeth and its diffracted field.



Fig. 15. Toothed occulter with 16 teeth and its diffracted field.

angle between the two coordinate systems is δ , the relation between them is given by

$$\begin{pmatrix} x'_0 \\ y'_0 \end{pmatrix} = \begin{pmatrix} \cos \delta & \sin \delta \\ -\sin \delta & \cos \delta \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}.$$
 (6)

An equilateral toothed occulter can be described by three factors: the number of teeth N and the radii of the internal envelope r_i and the external envelope r_e (Fig. <u>12</u>). When N, r_i , and r_e are known, we obtain

$$\begin{aligned} \varphi &= 2\pi/N \\ L &= \sqrt{r_i^2 + r_e^2 - 2r_i r_e \cos \varphi} = 2w_x \\ \sin i_e &= r_i \sin \varphi/L \\ R &= r_e \sin i_e \\ T &= \sqrt{r_e^2 - R^2} - L \\ \delta &= \arccos(T/r_i) \end{aligned}$$
(7)

where φ , R, and T are illustrated in Fig. <u>12</u>.

According to the above equation, the diffracted field from the side is given by

$$\tilde{U}(r_0, \theta_0) = \frac{U_0 e^{ikz}}{2j} \{ [C(\alpha_2) - C(\alpha_1)] + j [S(\alpha_2) - S(\alpha_1)] \} \\ \times \{ [C(\beta_2) - C(\beta_1)] + j [S(\beta_2) - S(\beta_1)] \},$$
(8)

where $\alpha_1 = [2/(\lambda z)]^{1/2} (T - r_0 \cos \theta_0 \cos \delta - r_0 \sin \theta_0 \sin \delta),$ $\alpha_2 = [2/(\lambda z)]^{1/2} (T + L - r_0 \cos \theta_0 \cos \delta - r_0 \sin \theta_0 \sin \delta), \quad \beta_1 = -\infty, \text{ and } \beta_2 = [2/(\lambda z)]^{1/2} (-R + r_0 \cos \theta_0 \sin \delta - r_0 \sin \theta_0 \cos \delta).$

Finally, by a symmetric superposition, the diffracted field of the toothed occulter is obtained. The calculated diffracted fields of occulters with 4, 16, and 32 teeth are illustrated in Figs. <u>14–16</u>, respectively.

The diffracted fields of a toothed occulter and circular disk were calculated; the results are shown in Fig. <u>17</u>. The diffraction intensity of the toothed occulter is much lower than that of the circular disk. The average diffraction intensity of the circular disk is about 353 times that of the toothed disk. In fact, with a proper selection of the height and number of teeth, the scale can be improved to 10^3 .

4. DISCUSSION

To obtain the optimal shape of the toothed occulter, the diffracted field of the occulter with different numbers N and heights of teeth was calculated.



Fig. 16. Toothed occulter with 32 teeth and its diffracted field.



Fig. 17. Comparison of diffraction intensities of toothed occulter (lower curve) and circular disk (upper curve).



Fig. 18. Average diffraction intensity as a function of the number of teeth for different tooth heights (solid line, 0.5 mm; dashed line, 2 mm). The lines are averaged over the circular region of radius r = 2 mm in the central part of the screen.



Fig. 19. Diffraction intensity of toothed occulter with 32 teeth as a function of tooth height. Lines are averaged over a circular region of radius r = 2 mm in the central part of screen.

Figure 18 shows the average diffraction intensity as a function of the number of teeth N. The calculation was made under the following conditions. The distance between the occulter and the receiving screen, z, is 840 mm, and the radius of the internal envelope, r_i , is 40 mm. The diffraction intensity for teeth with a height of 2 mm declines first and then rises with increasing N beyond a critical point, whereas for a height of 0.5 mm, it declines with increasing N and then remains constant when N is larger than a critical value. The critical point for 0.5-mm-high teeth is larger than that for 2-mm-high teeth. The slope after the critical point is larger for the height of 2 mm. In fact, the value of the critical point and the slope are determined by the ratio of the height to the radius of the occulter. As the ratio decreases, the critical point value increases and the slope decreases. The diffraction intensity reportedly became flat for large N and the critical point was about 200 [15]. This is similar to our results for the 0.5-mm-high tooth, except that in our case the ratio of the height to the radius is even larger. We also calculated the diffraction intensity under the same conditions as in [15] with our proposed method, and the result agrees well with that of [15].

When N is smaller than the critical value, the diffraction intensity of the occulter for the tooth height of 2 mm is 10^3 orders of magnitude lower than that for the height of 0.5 mm, as shown in Fig. <u>18</u>. Figure <u>19</u> shows the diffraction intensity of the toothed occulter as a function of the tooth height with N = 32. The diffraction intensity initially decreases quickly with increasing tooth height and then tends to remain unchanged. In addition, the amount of vignetting increases with the tooth height. Therefore, the selection of the height is a trade-off between the diffraction intensity and the vignetting.

5. CONCLUSION

This paper proposes a method for computing the diffracted field of a toothed occulter or external diaphragm and proves that the method is feasible. The diffracted field of a toothed occulter decreases quickly as the number of teeth N increases when N is smaller than the critical value, and then it increases or becomes constant when N is larger than the critical value. The critical value increases as the ratio of the tooth height and occulter radius decreases. The number of teeth should be optimized considering that it becomes more difficult to fabricate the occulter as the number of teeth increases. In addition, the diffraction intensity decreases with increasing tooth height when the height is lower than a certain value and then remains unchanged when the height is higher than that value. If the number N and height of the teeth are properly chosen, the diffraction intensity can be reduced by a factor of 10^3 .

APPENDIX A: DERIVATION OF THE DIFFRACTED FIELD OF A SINGLE SEMI-INFINITE RECTANGLE

According to the theory of Fresnel integration, if the incident light is a plane wave, the complex form of the disturbance at $P(x_0, y_0)$ in Fig. <u>6</u> is [21]

$$U(x_0, y_0) = \frac{U_0}{\lambda z} \int_{y_1}^{y_2} \int_{x_1}^{x_2} e^{jkr} dx dy$$

$$\approx \frac{U_0}{\lambda z} e^{jkz} \int_{y_1}^{y_2} \int_{x_1}^{x_2} e^{jk\frac{(x-x_0)^2 + (y-y_0)^2}{2z}} dx dy, \quad (A1)$$

where U_0 is the amplitude of the incoming light in the mask plane, and $k = 2\pi/\lambda$. Equation (A1) does not consider the time factor of exp(-*iwt*). For convenience, we introduce two dimensionless variables α and β :

$$\alpha \equiv \left(\frac{2}{\lambda z}\right)^{\frac{1}{2}} (x - x_0), \qquad \beta \equiv \left(\frac{2}{\lambda z}\right)^{\frac{1}{2}} (y - y_0).$$
(A2)

By substituting Eq. (A2) into Eq. (A1), we obtain

$$U(x_0, y_0) = \frac{U_0}{2} e^{jkz} \int_{\alpha_1}^{\alpha_2} e^{j\frac{\pi a^2}{2}} d\alpha \int_{\beta_1}^{\beta_2} e^{j\frac{\pi \beta^2}{2}} d\beta.$$
(A3)

The Fresnel integrals are defined by

$$C(x) = \int_0^x \cos\left(\frac{\pi\xi^2}{2}\right) \mathrm{d}\xi, \quad S(x) = \int_0^x \sin\left(\frac{\pi\xi^2}{2}\right) \mathrm{d}\xi. \quad (A4)$$

Then Eq. (A3) becomes



Fig. 20. Diffraction intensity of the hexagonal mask calculated by the Fresnel–Kirchhoff formula (solid line) and the semi-infinite rectangle method (dashed line). Lines are plotted along one perpendicular bisector of the side line of the hexagon.



Fig. 21. Diffraction intensity of the decagonal mask calculated by the Fresnel–Kirchhoff formula (solid line) and the semi-infinite rectangle method (dashed line). Lines are plotted along one perpendicular bisector of the side line of the decagon.



Fig. 22. Diffracted fields of the four-toothed occulter by the Fresnel–Kirchhoff formula (dashed line) and the semi-infinite rectangle method (solid line).

$$U(x_0, y_0) = \frac{U_0}{2} e^{jkz} (C(\alpha) + jS(\alpha))|_{\alpha_1}^{\alpha_2} (C(\beta) + jS(\beta))|_{\beta_1}^{\beta_2}.$$
 (A5)

For the semi-infinite rectangle in Fig. <u>6</u>, we have $x_1 = -\infty$, $x_2 = -R$, $y_1 = -w$, and $y_2 = w$. Consequently, $\alpha_1 = -\infty$, $\alpha_2 = [2/(\lambda z)]^{1/2}(-R - x_0)$, $\beta_1 = [2/(\lambda z)]^{1/2}(-w - y_0)$, and $\beta_2 = [2/(\lambda z)]^{1/2}(w - y_0)$. Because $C(-\infty) = S(-\infty) = -1/2$, the diffracted field becomes

$$U(x_0, y_0) = \frac{U_0 e^{jkz}}{2j} \left\{ \left[C(\alpha_2) + \frac{1}{2} \right] + j \left[S(\alpha_2) + \frac{1}{2} \right] \right\} \\ \times \left\{ \left[C(\beta_2) - C(\beta_1) \right] + j \left[S(\beta_2) - S(\beta_1) \right] \right\}.$$
(A6)

APPENDIX B: DEMONSTRATION OF THE FEASIBILITY OF THE METHOD FOR OTHER SHAPES

The diffracted light intensities of hexagonal and decagonal masks were calculated by the Fresnel-Kirchhoff formula and the semi-infinite rectangle method; the results are shown in Figs. 20 and 21, respectively. The calculations were made under the following conditions: the distance between the center and the side of the mask, r, is 7.5 mm, and the distance between the occulter and the receiving screen, z, is 450 mm.

The diffracted light intensity from the four-toothed occulter was calculated by the Fresnel–Kirchhoff formula and semiinfinite rectangle method; the results are shown in Fig. 22.

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