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Global μ-stability of impulsive reaction–diffusion neural networks with unbounded time-varying delays and bounded continuously distributed delays ☆

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ABSTRACT

This paper investigates the global μ-stability for impulsive cellular neural networks with reaction–diffusion terms and mixed delays, where mixed delays consist of unbounded time-varying delays and bounded continuously distributed delays. The model under consideration is more general than those investigated in most existing literature. By establishing a novel Lyapunov–Krasovskii functional and employing the delay differential inequality with impulsive initial conditions, boundary conditions and LMI, we obtain two concise and sufficient conditions ensuring global μ-stability of the impulsive reaction–diffusion cellular neural networks with mixed delays. The newly obtained criteria are concerned with the reaction–diffusion coefficients and the regional feature. However, they are independent of boundaries of variable time delays. Several corollaries are also presented. Finally, three concrete numerical examples are given to demonstrate the effectiveness and superiority of our main results.

1. Introduction

In recent few decades, cellular neural networks (CNNS), initially proposed by Chuan and Yang in 1988 [1], have attracted considerable attention due to their extensive applications in signal processing, classification of patterns, quadratic optimization, and other areas [1–3,10,12,20]. Such applications heavily depend on the dynamical behaviors of the CNNS. Therefore, dynamical behaviors of the CNNS, which involve discussions of stability properties, periodic oscillatory behaviors, bifurcation, chaos [3,4] and so on, have been investigated and meanwhile a great number of approaches for neural networks have also been proposed [6,7,10]. Especially, the global stability issue that is of great interest in many applications has long been a focused topic of theoretical as well as practical importance [12,20–22,40]. Thus, it has important meaning to do some research on cellular neural networks stability.

Time delays are usually unavoidable in signal transmission, engineering systems, biological systems and economic systems because of the finite processing speed of information, for example, the finite axonal propagation speed from soma to synapses, the diffusion of chemical across the synapses, the postsynaptic potential integration of membrane potential at the neuronal cell body and dendrites [5]. On the other hand, in the electronic implementation of artificial neural networks, time delays often inevitably occur and are usually time varying owing to the finite switching speed of amplifies [2]. Moreover, when designing a neural network or implementing it by VLSI in practice, neural networks usually have a spatial nature due to the presence of a multitude of parallel pathways with a variety of axon sizes and lengths, but the distribution of propagation is not instantaneous and cannot be modeled by discrete time delays. As a result, continuously distributed delays have been incorporated into neural networks [13,15]. It is well known to us that constant, time varying, or distributed time delays may lead to undesirable dynamical network behaviors such as bifurcation, divergence, instability or oscillation. Thus, the global stability of neural networks with time delays becomes an extremely important research topic and many researchers have already paid special attention to this area. Until now, some sufficient conditions have been reported to ensure the stability of neural networks with different types of time delays, see [8,9,11–15,19–22,38] and references therein.

On the other hand, it is noteworthy that a wide variety of evolutionary processes are characterized by abrupt changes at certain moments of time, which may be caused by a switching phenomenon or other sudden noises, existing the impulsive effects [25–27]. The instantaneous perturbations in turn affect

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dynamical behaviors of the neural network systems. Therefore, investigations of stability of neural networks with impulsive effects as well as delay effects have received much interest [18,25–32]. Fundamental theory of impulsive differential equations has been developed in [23,27,29,30]. In [40], several new sufficient conditions ensuring the global exponential stability of the equilibrium point for fuzzy cellular neural networks with delays and reaction–diffusion terms are obtained. For more details of the literature related to stability of impulsive neural networks, the reader is referred to [28–34,37,39,42] and the references cited therein.

In addition to delay and impulsive effects in artificial neural networks, diffusion effects are often encountered when electrons are moving in asymmetric electromagnetic fields. Therefore, the model of impulsive reaction–diffusion neural networks whose activations vary in space as well as in time will be more powerful for describing the evolutionary process of the practical systems. Recently, some results concerning the stability of impulsive neural networks involving time delays and diffusion effects have been reported [16,17,35,36,39–42]. For example, in [41], some new and concise algebraic criteria ensuring the global exponential stability of the equilibrium point for impulsive cellular neural networks with time-varying delays and reaction–diffusion terms are summarized by means of Hardy–Poincare inequality and Gronwall–Bellman-type impulsive integral inequality. Moreover, some authors have considered a class of neural networks with impulsive diffusion terms and mixed time delays. For example, the problem of mean square exponential stability for a class of impulsive stochastic fuzzy Cohen–Grossberg networks with mixed delays and reaction–diffusion terms is investigated in Ref. [39].

However, it should be noted that, in all the aforementioned literature, time delays are assumed to be bounded. Unfortunately, in many practical neural networks, this assumption is not realistic. As far as we know, several results have been available in some literature when time delays are unbounded [43–45]. For example, the global robust stability for uncertain stochastic neural networks with unbounded time-varying delays has been studied in [44], where a novel concept of global robust μ-stability in the mean square for neural networks is proposed. In [45], robust μ-stability criteria have been derived for a class of uncertain stochastic neural networks with unbounded time-varying delays. In addition, to the best of our knowledge, there are no results on the global μ-stability criteria for impulsive reaction–diffusion neural networks with both unbounded time-varying delays and bounded continuously distributed delays, which is very important in both theories and application but still remains as a challenging research issue.

Therefore, motivated by the above discussions, the objective of this paper is to study the global μ-stability for impulsive neural networks with reaction–diffusion terms and mixed delays which may be unbounded. The presented nervous model integrates impulses with reaction–diffusion terms and mixed delays. By establishing a novel Lyapunov–Krasovskii functional and employing the delay differential inequality with impulsive initial conditions, boundary conditions and LMI, we develop two sufficient conditions guaranteeing the global μ-stability which unifies the exponential stability, power stability, log-stability, log-log stability, etc., for cellular neural networks with unbounded time-varying delays.

The rest of this paper is organized as follows: in Section 2, models are formulated and preliminaries are given. In Section 3, two global μ-stability criteria for neural networks are derived and detailed proof is presented. In Section 4, several corollaries are also derived. In Section 5, two simple illustrative examples are provided to show the validity of the main results. Finally, conclusions are shortly drawn in Section 6.

Notation: The following notations will be used throughout this paper: For $A, B \in R^{m\times n}$ or $A, B \in R^{n\times n}$, $A \geq B$ ($A \leq B$) means that each pair of corresponding elements of A and B satisfies the inequality $\geq (\leq)$. $A \in R^{m\times n}$ is called a nonnegative matrix if $A \geq 0$ and $l \in R^n$ is called a positive vector if $l > 0$. $\lambda_{max}(P)$ and $\lambda_{min}(P)$ represent the maximum and minimum eigenvalues of a symmetric matrix P, respectively.
$PC(\Omega) = \{\varphi : (-\infty, t_0] \times \overline{\Omega} \to R^n \mid \varphi(s^-, x) = \varphi(s, x), s \in (-\infty, t_0]\}$ where $\overline{\Omega} = \Omega \cup \partial\Omega$, and $\varphi(s^+, x)$ exists for all $s \in (-\infty, t_0]$ but at a finite number of points. $PC[J \times \Omega, R^n] = \{u(t,x) : J \times \Omega \to R^n\}$ where $u(t,x)$ is continuous at $t \neq t_k$, $u(t_k^-, x) = u(t_k, x)$, and $J \subset R$ is an interval. $\| l \|_2 = \left(\int_\Omega |l|^2 dx\right)^{1/2}$, for any $l \in PC(\Omega)$, where $|\bullet|$ is Euclid norm of a vector $l \in R^n$ for any integer n. Let $S \triangleq S([-\infty, 0] \times R^m, R^n)$ be the Banach space of continuous functions which map $[-\infty, 0] \times R^m$ into R^n with the topology of uniform converge. $\Omega = \{x = (x_1, x_2, \ldots, x_m)^T \mid |x_i| < q_i, i = 1, 2, \ldots, m\}$ is an open bounded set with smooth boundary $\partial\Omega$ and mes $\Omega > 0$ denotes the measure of Ω in space R^m. $L^2(\Omega)$ is the space of real functions on Ω which are L^2 for the Lebesgue measure. It is a Banach space for the norm $\| u(t) \|_2 = \sqrt{\sum_{i=1}^n \| u_i(t) \|_2^2}$, where $u(t) = (u_1(t), \ldots, u_n(t))^T$, $\| u_i(t) \|_2 = \left(\int_\Omega |u_i(t,x)|^2 \, dx\right)^{1/2}$. For any $\psi(t,x) \in S([-\tau, 0] \times \Omega, R^n)$, we define $\| \psi \|_2 = \sqrt{\sum_{i=1}^n \| \psi_i \|_2^2}$, where $\psi(t,x) = (\psi_1(t,x), \ldots, \psi_n(t,x))^T$, $\| \psi_i \|_2 = \left(\int_\Omega |\psi_i(x)|_\tau^2 \, dx\right)^{1/2}$, $|\psi_i(x)|_\tau = \sup_{-\tau \leq s \leq 0} |\psi_i(s,x)|$.

2. Model description and preliminaries

We consider the following impulsive delayed reaction–diffusion cellular neural networks with boundary conditions:

$$\begin{aligned}\frac{\partial(u_i(t,x))}{\partial t} &= \sum_{s=1}^m \frac{\partial}{\partial x_s}\left(D_{is}\frac{\partial u_i(t,x)}{\partial x_s}\right) - a_i u_i(t,x) \\ &\quad + \sum_{j=1}^n b_{ij}\tilde{f}_j(u_j(t,x)) + \sum_{j=1}^n c_{ij}\tilde{f}_j(u_j(t-\tau_j(t),x)) \\ &\quad + \sum_{j=1}^n d_{ij}\int_0^{+\infty} k_{ij}(s)\tilde{f}_j(u_j(t-s,x))\,ds \\ &\quad + I_i(t), \quad t > 0, \ t \neq t_k, \ k \in N, \ x \in \Omega, i = 1, 2, \ldots, n, \end{aligned} \qquad (1)$$

$$\begin{aligned}\Delta u_i(t,x) &= d_i u_i(t^-, x) + \sum_{j=1}^n e_{ij}\tilde{h}_j(u_j(t^- - \tau_j(t), x)), \\ &\quad t = t_k, k \in N, x \in \Omega, i = 1, 2, \ldots, n,\end{aligned} \qquad (2)$$

$$\frac{\partial u_i}{\partial n} = \left(\frac{\partial u_i}{\partial x_1}, \frac{\partial u_i}{\partial x_2}, \ldots, \frac{\partial u_i}{\partial x_m}\right)^T = 0, \forall t \geq 0, x \in \partial\Omega, i = 1, 2, \ldots, n, \qquad (3)$$

$$u_i(s,x) = \phi_i(s,x), -\infty < s \leq 0, x \in \Omega, i = 1, 2, \ldots, n, \qquad (4)$$

where $\Delta u_i(t_k, x) = u_i(t_k^+, x) - u_i(t_k^-, x)$, $u_i(t_k^-, x) = \lim_{t\to t_k^-} u_i(t,x)$, $u_i(t_k^+, x) = \lim_{t\to t_k^+} u_i(t,x)$, $x \in \Omega$, $i = 1, 2, \ldots, n$, $n \geq 2$, $k \in N = \{1, 2, \ldots\}$ is the impulse at time instant t_k. The time sequence t_k satisfies $0 \leq t_0 < t_1 < t_2 < \cdots < t_k < t_{k+1} < \cdots$, and $\lim_{t\to+\infty} t_k = +\infty$; $x = (x_1, x_2, \ldots, x_m)^T \in \Omega \subset R^m$; n is the number of neurons in the network; $u = (u_1, u_2, \ldots, u_n)^T \in R^n$, $u_i(t,x)$ denotes the state of the ith neuron at time t and in space x; $u_i(t_k^+, x)$ and $u_i(t_k^-, x)$ denote the right-hand and left-hand limits at t_k, respectively. For all $k \in N$, it is always assumed that $u_i(t_k^-, x) = u_i(t_k, x)$. The smooth function $D_{is} = D_{is}(t,x,u) \geq 0$ corresponds to the transmission diffusion operator along the ith unit; the scalar $a_i > 0$ is the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs at time t and in space x; b_{ij}, c_{ij} and d_{ij} are known constants denoting the strength of the ith neurons on the jth neurons; $\tilde{f}_j(u_j(t,x))$, $\tilde{h}_j(u_j(t,x))$ denote the activation functions of the jth neuron at time t and in space x; $\tau_j(t)$ denotes the unbounded time-varying delay; the delay kernels k_{ij} are real valued nonnegative continuous functions; I_i denotes external input to the ith neuron.

To obtain our main results, we give the following assumptions.

Assumption 1. $\tau_j(t)$ $(j=1,2,\ldots,n)$ is a nonnegative continuous differential function and satisfies $\dot{\tau}_j(t)\le\rho<1, \rho$ is a positive constant.

Assumption 2. Delay kernels $k_{ij}:[0,+\infty]\to[0,+\infty]$ $(i,j=1,2,\ldots,n)$ and satisfy $\int_0^{+\infty}k_{ij}(s)\,ds=k_i<\infty, i=1,2,\ldots,n$.

Assumption 3. There exist positive diagonal matrices $l=\mathrm{diag}(l_1,l_2,\ldots,l_n)$ and $m=\mathrm{diag}(m_1,m_2,\ldots,m_n)$ such that

$$l_i=\sup_{\eta_1\ne\eta_2}\left|\frac{\tilde{f}_j(\eta_1)-\tilde{f}_j(\eta_2)}{\eta_1-\eta_2}\right|,\quad m_i=\sup_{\eta_1\ne\eta_2}\left|\frac{\tilde{h}_j(\eta_1)-\tilde{h}_j(\eta_2)}{\eta_1-\eta_2}\right|, \tag{5}$$

for any $\eta_1,\eta_2\in R, \eta_1\ne\eta_2, j=1,2,\ldots,n$.

A constant vector $u^*=(u_1^*,u_2^*,\ldots,u_n^*)^T\in R^n$ is assumed to be an equilibrium point of system (1)–(4). To simplify the calculations, now let $y_i(t,x)=u_i(t,x)-u_i^*, i=1,2,\ldots,n$. It is easy to see that system (1)–(4) can be transformed into

$$\begin{aligned}\frac{\partial(y_i(t,x))}{\partial t}=&\sum_{s=1}^m\frac{\partial}{\partial x_s}\left(D_{is}\frac{\partial y_i(t,x)}{\partial x_s}\right)-a_iy_i(t,x)+\sum_{j=1}^n b_{ij}f_j(y_j(t,x))\\&+\sum_{j=1}^n c_{ij}f_j(y_j(t-\tau_j(t),x))\\&+\sum_{j=1}^n d_{ij}\int_0^{+\infty}k_{ij}(s)f_j(y_j(t-s,x))\,ds,\\&t>0,\ t\ne t_k,\ k\in N,\ x\in\Omega,\ i=1,2,\ldots,n,\end{aligned} \tag{6}$$

$$\Delta y_i(t,x)=d_iy_i(t^-,x)+\sum_{j=1}^n e_{ij}h_j(y_j(t^- -\tau_j(t),x)),\quad t=t_k,\ k\in N,\ x\in\Omega,\ i=1,2,\ldots,n, \tag{7}$$

$$\frac{\partial y_i}{\partial n}=\left(\frac{\partial y_i}{\partial x_1},\frac{\partial y_i}{\partial x_2},\ldots,\frac{\partial y_i}{\partial x_m}\right)^T=0,\quad \forall t\ge0,\ x\in\partial\Omega,\ i=1,2,\ldots,n, \tag{8}$$

$$y_i(s,x)=\varphi_i(s,x),\quad -\infty<s\le0,\ x\in\Omega,\ i=1,2,\ldots,n, \tag{9}$$

where $f_j(y_j(t,x))=\tilde{f}_j(y_j(t,x)+u_j^*)-\tilde{f}_j(u_j^*)$, $j=1,2,\ldots,n$, $h_j(y_j(t-\tau_j(t),x))=\tilde{h}_j(y_j(t-\tau_j(t),x)+u_j^*)-\tilde{h}_j(u_j^*)$, $j=1,2,\ldots,n$, $\varphi_i(s,x)=\phi_i(s,x)-u^*, n\ge2$. Then, it is easy to see that $f_j(\cdot)$ and $h_j(\cdot)$ satisfy Assumption 3.

Definition 1 (*Chen et al. [44], Liu and Chen [45]*). Suppose that $\mu(t)$ is a nonnegative continuous function and satisfy $\mu(t)\to+\infty$ when $t\to+\infty$. For impulsive neural networks (6) with reaction–diffusion terms and both time-varying unbounded delays and distributed delays, the solution is said to be globally μ-stable, if there exists a constant $M>0$ such that when $t>T$

$$\|u(t,x)-u^*\|_2^2\le\frac{M}{\mu(t)}. \tag{10}$$

Remark 1. This Definition 1 unifies various stabilities. For example, when $\mu(t)=e^{\beta t}, \beta>0$, the global μ-stability becomes the global exponential stability; when $\mu(t)=t$, the global μ-stability becomes the global asymptotic stability; when $\mu(t)=t^\gamma$, the global μ-stability becomes the global power stability; when $\mu(t)=\ln(1+t)$, the global μ-stability becomes the global log stability; when $\mu(t)=\ln\ln(3+t)$, the global μ-stability becomes the global log–log stability. We will give a concrete discussion about the relationship between the μ-stability and other stabilities in Section 4.

To obtain our main results, we need the following Lemma 1.

Lemma 1 (*Lu [5]*). *Let Ω be a cube $|x_i|<q_i$ $(i=1,2,\ldots,m)$ and let $w(x)$ be a real-valued function belonging to $S^1(\Omega)$ which vanishes on the boundary $\partial\Omega$ of Ω, i.e. $w(x)|_{\partial\Omega}=0$. Then,*

$$\int_\Omega w^2(x)\,dx\le q_i^2\int_\Omega\left|\frac{\partial w}{\partial x_i}\right|^2dx. \tag{11}$$

3. Main results

Theorem 1. *Suppose that when $t>T>0$ the function $\mu(t)$ satisfies*

$$\frac{\dot{\mu}(t)}{\mu(t)}\le\beta_1,\quad \frac{\mu(t-\tau(t))}{\mu(t)}\ge\beta_2,\quad \frac{\int_0^{+\infty}k_{ij}(\sigma)\mu(t+\sigma)\,d\sigma}{\mu(t)}\le\beta_3, \tag{12}$$

where β_1, β_2 and β_3 are nonnegative scalars.

The solution of impulsive neural networks (6) with reaction–diffusion terms and both time-varying unbounded delays and distributed delays is said to be globally μ-stable, if under Assumptions 1–3, there exists a matrix $P\in R^{n\times n}$ whose corresponding elements to principal diagonal as the axis of symmetry form opposite numbers and whose elements on the principal diagonal are all greater than zero (i.e., $p_{ij}=-p_{ji}, i\ne j$; $p_{ij}>0$, $i=j$) such that

$$\begin{aligned}&\sum_{i=1}^n\left(\beta_1p_i-2p_i\sum_{s=1}^m\frac{D_{is}}{q_i^2}\right)+\sum_{j=1}^n\left(\left(\frac{p_ib_{ji}^2l_i^2}{a_j}-a_ip_i\right)\right.\\&\left.\quad+(1-\rho)^{-1}\beta_2^{-1}p_i^2c_{ij}^2+\frac{k_i}{4}d_{ij}^2p_i^2+l_i^2+\beta_3l_i^2\right)\le0,\end{aligned} \tag{13}$$

$$\begin{bmatrix}(E+D_k)^TP(E+D_k)-\lambda_kP & (E+D_k)^TPE_k\Gamma_k\\ \Gamma_k^TE_k^TP(E+D_k) & \Gamma_k^TE_k^TPE_k\Gamma_k\end{bmatrix}\le0, \tag{14}$$

where

$$E_k=\begin{bmatrix}a_{11}&\cdots&a_{1n}\\ \vdots&\ddots&\vdots\\ a_{n1}&\cdots&a_{nn}\end{bmatrix}(a_{ij}=1;i,j=1,2,\ldots,n), \tag{15}$$

$$\Gamma_k=\mathrm{diag}(m_1,m_2,\ldots,m_n), \tag{16}$$

$$D_k=\mathrm{diag}(d_1,d_2,\ldots,d_n), \tag{17}$$

$$1<e^{\alpha_1(t_k-t_{k-1})}\le\lambda_k\le e^{\alpha_2(t_k-t_{k-1})}\quad(0<\alpha_1<\alpha_2). \tag{18}$$

Proof. Define a novel Lyapunov–Krasovskii functional:

$$V(t,x)=\int_\Omega(V_1(t,x)+V_2(t,x)+V_3(t,x))\,dx, \tag{19}$$

where

$$V_1(t,x)=\sum_{i=1}^n p_i\mu(t)y_i^2(t,x), \tag{20}$$

$$V_2(t,x)=\sum_{i=1}^n\sum_{j=1}^n\int_{t-\tau_j(t)}^t\mu(s)f_j^2(y_j(s,x))\,ds, \tag{21}$$

$$V_3(t,x)=\sum_{i=1}^n\sum_{j=1}^n\int_0^{+\infty}k_{ij}(\sigma)\int_{t-\tau_j(t)}^t\mu(s+\sigma)f_j^2(y_j(s,x))\,ds\,d\sigma. \tag{22}$$

For any $t\in[t_{k-1},t_k)$, the time derivative of $V(t,x)$ along the trajectory of system (6) is

$$\begin{aligned}\dot{V}(t,x)|_{(6)}=&\int_\Omega\left[\sum_{i=1}^n p_i\dot{\mu}(t)y_i^2(t,x)\right]dx\\&+\int_\Omega2\sum_{i=1}^n p_i\mu(t)y_i(t,x)\left(\sum_{s=1}^m\frac{\partial}{\partial x_s}\left(D_{is}\frac{\partial y_i(t,x)}{\partial x_s}\right)-a_iy_i(t,x)\right.\\&\left.+\sum_{j=1}^n b_{ij}f_j(y_j(t,x))+\sum_{j=1}^n c_{ij}f_j(y_j(t-\tau_j(t),x))\right.\end{aligned}$$

$$+\sum_{j=1}^{n} d_{ij}\int_0^{+\infty} k_{ij}(s)f_j(y_j(t-s,x))\,ds\Bigg)\,dx$$
$$+\int_\Omega\sum_{i=1}^{n}\sum_{j=1}^{n}[\mu(t)f_j^2(y_j(t,x))-\mu(t-\tau_j(t))f_j^2(y_j(t-\tau_j(t),x))(1-\dot{\tau}_j(t))]\,dx$$
$$+\int_\Omega\sum_{i=1}^{n}\sum_{j=1}^{n}k_{ij}(\sigma)[\mu(t+\sigma)f_j^2(y_j(t,x))-\mu(t)f_j^2(y_j(t-\sigma,x))]\,d\sigma\,dx. \tag{23}$$

From Assumption 1, we can obtain

$$\dot{V}(t,x)|_{(6)}\le\int_\Omega\mu(t)\left[\sum_{i=1}^{n}p_i\frac{\dot{\mu}(t)}{\mu(t)}y_i^2(t,x)\right]dx$$
$$+\int_\Omega 2\mu(t)\sum_{i=1}^{n}p_iy_i(t,x)\left(\sum_{s=1}^{m}\frac{\partial}{\partial x_s}\left(D_{is}\frac{\partial y_i(t,x)}{\partial x_s}\right)-a_iy_i(t,x)\right.$$
$$+\sum_{j=1}^{n}b_{ij}f_j(y_j(t,x))+\sum_{j=1}^{n}c_{ij}f_j(y_j(t-\tau_j(t),x))$$
$$\left.+\sum_{j=1}^{n}d_{ij}\int_0^{+\infty}k_{ij}(s)f_j(y_j(t-s,x))\,ds\right)dx$$
$$+\int_\Omega\mu(t)\sum_{i=1}^{n}\sum_{j=1}^{n}\left[f_j^2(y_j(t,x))-\frac{\mu(t-\tau_j(t))}{\mu(t)}f_j^2(y_j(t-\tau_j(t),x))(1-\rho)\right]dx$$
$$+\int_\Omega\mu(t)\sum_{i=1}^{n}\sum_{j=1}^{n}k_{ij}(\sigma)\left[\frac{\mu(t+\sigma)}{\mu(t)}f_j^2(y_j(t,x))-f_j^2(y_j(t-\sigma,x))\right]d\sigma\,dx. \tag{24}$$

By the boundary condition of (8), we get

$$\sum_{s=1}^{m}\int_\Omega y_i(t,x)\frac{\partial}{\partial x_s}\left(D_{is}\frac{\partial y_i(t,x)}{\partial x_s}\right)dx$$
$$=\int_\Omega y_i(t,x)\left(D_{is}\frac{\partial y_i(t,x)}{\partial x_s}\right)_{s=1}^{m}dx=\int_\Omega\nabla\bullet\left(y_i(t,x)D_{is}\frac{\partial y_i(t,x)}{\partial x_s}\right)_{s=1}^{m}dx$$
$$-\int_\Omega\left(D_{is}\frac{\partial y_i(t,x)}{\partial x_s}\right)_{s=1}^{m}\bullet\nabla y_i(t,x)\,dx=\int_{\partial\Omega}\left(y_i(t,x)D_{is}\frac{\partial y_i(t,x)}{\partial x_s}\right)_{s=1}^{m}dx$$
$$-\sum_{s=1}^{m}\int_\Omega D_{is}\left(\frac{\partial y_i(t,x)}{\partial x_s}\right)^2dx=-\sum_{s=1}^{m}\int_\Omega D_{is}\left(\frac{\partial y_i(t,x)}{\partial x_s}\right)^2dx, \tag{25}$$

in which

$$\nabla=\left(\frac{\partial}{\partial x_1},\frac{\partial}{\partial x_2},\dots,\frac{\partial}{\partial x_s}\right)^T$$

is the gradient operator, and

$$\left(D_{is}\frac{\partial(y_i(t,x))}{\partial x_s}\right)_{s=1}^{m}=\left(D_{i1}\frac{\partial(y_i(t,x))}{\partial x_1},\dots,D_{im}\frac{\partial(y_i(t,x))}{\partial x_m}\right)^T.$$

From Lemma 1, we can obtain

$$\int_\Omega\sum_{s=1}^{m}y_i(t,x)\frac{\partial}{\partial x_s}\left(D_{is}\frac{\partial y_i(t,x)}{\partial x_s}\right)dx=$$
$$-\sum_{s=1}^{m}\int_\Omega D_{is}\left(\frac{\partial y_i(t,x)}{\partial x_s}\right)^2dx\le-\int_\Omega\sum_{s=1}^{m}D_{is}\frac{y_i^2(t,x)}{q_i^2}\,dx. \tag{26}$$

From (12) and Assumption 3, we can obtain

$$\sum_{i=1}^{n}\mu(t)\left[-(1-\rho)\sum_{j=1}^{n}\left(\frac{\mu(t-\tau_j(t))}{\mu(t)}f_j^2(y_j(t-\tau_j(t),x))\right)\right.$$
$$\left.+2p_iy_i(t,x)\left(\sum_{j=1}^{n}c_{ij}f_j(y_j(t-\tau_j(t),x))\right)\right]$$
$$\le\sum_{i=1}^{n}\mu(t)\left[-(1-\rho)\beta_2\sum_{j=1}^{n}f_j^2(y_j(t-\tau_j(t),x))+2p_iy_i(t,x)\right.$$
$$\left.\left(\sum_{j=1}^{n}c_{ij}f_j(y_j(t-\tau_j(t),x))\right)\right]=\sum_{i=1}^{n}\mu(t)$$
$$\left\{-\left[(1-\rho)^{1/2}\beta_2^{1/2}f_j(y_j(t-\tau_j(t),x))-(1-\rho)^{-1/2}\beta_2^{-1/2}p_ic_{ij}y_i(t,x)\right]^2\right.$$
$$\left.+(1-\rho)^{-1}\beta_2^{-1}p_i^2c_{ij}^2y_i^2(t,x)\right\}\le\sum_{i=1}^{n}\mu(t)(1-\rho)^{-1}\beta_2^{-1}p_i^2c_{ij}^2y_i^2(t,x). \tag{27}$$

From (12) and Assumption 2, we get

$$\mu(t)\left\{\left[2\sum_{i=1}^{n}p_iy_i(t,x)\sum_{j=1}^{n}d_{ij}\int_0^{+\infty}k_{ij}(s)f_j(y_j(t-s,x))\,ds\right]\right.$$
$$\left.-\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^{+\infty}k_{ij}(\sigma)f_j^2(y_j(t-\sigma,x))\,d\sigma\right\}=\mu(t)$$
$$\left\{\left[2\sum_{i=1}^{n}p_iy_i(t,x)\sum_{j=1}^{n}d_{ij}\int_0^{+\infty}k_{ij}(s)f_j(y_j(t-s,x))\,ds\right]\right.$$
$$\left.-\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^{+\infty}k_{ij}(s)f_j^2(y_j(t-s,x))\,ds\right\}$$
$$=\mu(t)\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^{+\infty}k_{ij}(s)[2p_id_{ij}y_i(t,x)f_j(y_j(t-s,x))$$
$$-f_j^2(y_j(t-s,x))]\,ds=\mu(t)\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^{+\infty}k_{ij}(s)$$
$$\left\{-\left[\frac{1}{2}p_id_{ij}y_i(t,x)-f_j(y_j(t-s,x))\right]^2+\left(\frac{1}{2}p_id_{ij}y_i(t,x)\right)^2\right\}$$
$$\le\frac{\mu(t)}{4}\sum_{i=1}^{n}\sum_{j=1}^{n}p_i^2d_{ij}^2\int_0^{+\infty}k_{ij}(s)y_i^2(t,x)\,ds$$
$$\le\frac{\mu(t)}{4}\sum_{i=1}^{n}\sum_{j=1}^{n}k_ip_i^2d_{ij}^2y_i^2(t,x). \tag{28}$$

By Assumption 3, we can obtain

$$\mu(t)\left[2\sum_{i=1}^{n}p_iy_i(t,x)\sum_{j=1}^{n}b_{ij}f_j(y_j(t,x))-\sum_{i=1}^{n}\sum_{j=1}^{n}2a_ip_iy_i^2(t,x)\right]$$
$$\le\mu(t)\left[\sum_{i=1}^{n}2p_iy_i(t,x)\sum_{j=1}^{n}b_{ij}l_jy_j(t,x)-\sum_{i=1}^{n}\sum_{j=1}^{n}a_ip_iy_i^2(t,x)\right]$$
$$-\mu(t)\sum_{i=1}^{n}\sum_{j=1}^{n}a_ip_iy_i^2(t,x)=\mu(t)\left[\sum_{i=1}^{n}a_ip_i\sum_{j=1}^{n}\right.$$
$$\left\{-\left[y_i(t,x)-\frac{b_{ij}l_j}{a_i}y_j(t,x)\right]^2+\left(\frac{b_{ij}l_j}{a_i}\right)^2y_j^2(t,x)\right\}$$
$$\left.-\sum_{i=1}^{n}\sum_{j=1}^{n}a_ip_iy_i^2(t,x)\right]\le\mu(t)\left[\sum_{i=1}^{n}a_ip_i\sum_{j=1}^{n}\frac{b_{ij}^2l_j^2}{a_i^2}y_j^2(t,x)\right.$$
$$\left.-\sum_{i=1}^{n}\sum_{j=1}^{n}a_ip_iy_i^2(t,x)\right]=\mu(t)\sum_{i=1}^{n}\sum_{j=1}^{n}\left[\frac{p_ib_{ji}^2l_i^2}{a_j}-a_ip_i\right]y_i^2(t,x). \tag{29}$$

Substituting (26)–(29) into (24), we further have

$$\dot{V}(t,x)|_{(6)}\le\int_\Omega\mu(t)\sum_{i=1}^{n}\left\{\beta_1p_iy_i^2(t,x)-2\sum_{s=1}^{m}D_{is}p_i\frac{y_i^2(t,x)}{q_i^2}\right.$$
$$+\sum_{j=1}^{n}\left[\frac{p_ib_{ji}^2l_i^2}{a_j}-a_ip_i\right]y_i^2(t,x)+(1-\rho)^{-1}\beta_2^{-1}p_i^2c_{ij}^2y_i^2(t,x)$$
$$\left.+\frac{k_i}{4}d_{ij}^2p_i^2y_i^2(t,x)+(1+\beta_3)l_j^2y_j^2(t,x)\right\}dx=\int_\Omega\mu(t)$$
$$\left\{\sum_{i=1}^{n}\left(\beta_1p_i-2p_i\sum_{s=1}^{m}\frac{D_{is}}{q_i^2}\right)+\sum_{j=1}^{n}\left(\left(\frac{p_ib_{ji}^2l_i^2}{a_j}-a_ip_i\right.\right.\right.$$

$$+(1-\rho)^{-1}\beta_2^{-1}p_i^2c_{ij}^2+\frac{k_i}{4}d_{ij}^2p_i^2+(1+\beta_3)l_i^2\Big)\Big\}y_i^2(t,x)\,dx. \tag{30}$$

From (13), we can get

$$\dot V(t,x)|_{(6)}\le 0,\quad t\in[t_{k-1},t_k)\cap[T,+\infty),\quad k\in N=\{1,2,\cdots\}. \tag{31}$$

When $t=t_k^+$

$$V(t_k^+,x)=\int_\Omega\Bigg[\sum_{i=1}^n p_i\mu(t_k^+)y_i^2(t_k^+,x)+\sum_{i=1}^n\sum_{j=1}^n\int_{t_k^+-\tau_j(t_k^+)}^{t_k^+}\mu(s)f_j^2(y_j(s,x))\,ds+\sum_{i=1}^n\sum_{j=1}^n\int_0^{+\infty}k_{ij}(\sigma)\int_{t_k^+-\sigma}^{t_k^+}\mu(s+\sigma)f_j^2(y_j(s,x))\,ds\,d\sigma\Bigg]dx$$
$$=\int_\Omega\Bigg[\sum_{i=1}^n p_i\mu(t_k^-)(y_i(t_k^-,x)+\Delta y_i(t_k,x))^2+\sum_{i=1}^n\sum_{j=1}^n\int_{t_k^--\tau_j(t_k^-)}^{t_k^-}\mu(s)f_j^2(y_j(s,x))\,ds+\sum_{i=1}^n\sum_{j=1}^n\int_0^{+\infty}k_{ij}(\sigma)\int_{t_k^--\sigma}^{t_k^-}\mu(s+\sigma)f_j^2(y_j(s,x))\,ds\,d\sigma\Bigg]dx$$
$$=\int_\Omega\Bigg\{\sum_{i=1}^n p_i\mu(t_k^-)\Big[(d_i+1)y_i(t_k^-,x)+\sum_{j=1}^n e_{ij}h_j(y_j(t_k^--\tau_j(t_k),x))\Big]^2+\sum_{i=1}^n\sum_{j=1}^n\int_{t_k^--\tau_j(t_k^-)}^{t_k^-}\mu(s)f_j^2(y_j(s,x)\Big)\,ds+\sum_{i=1}^n\sum_{j=1}^n\int_0^{+\infty}k_{ij}(\sigma)\int_{t_k^--\sigma}^{t_k^-}\mu(s+\sigma)f_j^2(y_j(s,x))\,ds\,d\sigma\Bigg\}dx. \tag{32}$$

By the matrix form and from Assumption 3, we can get

$$\sum_{i=1}^n p_i\mu(t_k^-)\Big[(d_i+1)y_i(t_k^-,x)+\sum_{j=1}^n e_{ij}h_j(y_j(t_k^--\tau_j(t_k),x))\Big]^2$$
$$=\mu(t_k^-)[(E+D_k)y(t_k^-,x)+E_kH(y(t_k^--\tau(t_k),x))]^TP[(E+D_k)y(t_k^-,x)+E_kH(y(t_k^--\tau(t_k),x))]=\mu(t_k^-)[y^T(t_k^-,x)(E+D_k)^TP(E+D_k)y(t_k^-,x)$$
$$+y^T(t_k^-,x)(E+D_k)^TPE_kH(y(t_k^--\tau(t_k),x))+H^T(y(t_k^--\tau(t_k),x))E_k^TP(E+D_k)y(t_k^-,x)+H^T(y(t_k^--\tau(t_k),x))E_k^TPE_kH(y(t_k^--\tau(t_k),x))]$$
$$\le\mu(t_k^-)[y^T(t_k^-,x)(E+D_k)^TP(E+D_k)y(t_k^-,x)+y^T(t_k^-,x)(E+D_k)^TPE_k\Gamma_ky(t_k^--\tau(t_k),x)+y^T(t_k^--\tau(t_k),x)\Gamma_k^TE_k^TP(E+D_k)y(t_k^-,x)+y^T(t_k^--\tau(t_k),x)\Gamma_k^TE_k^TPE_k\Gamma_ky(t_k^--\tau(t_k),x)]$$
$$=\mu(t_k^-)\Bigg[\begin{bmatrix}y(t_k^-,x)\\y(t_k^--\tau(t_k),x)\end{bmatrix}^T\begin{bmatrix}(E+D_k)^TP(E+D_k)-\lambda_kP & (E+D_k)^TPE_k\Gamma_k\\ \Gamma_k^TE_k^TP(E+D_k) & \Gamma_k^TE_k^TPE_k\Gamma_k\end{bmatrix}\begin{bmatrix}y(t_k^-,x)\\y(t_k^--\tau(t_k),x)\end{bmatrix}+\lambda_ky^T(t_k^-,x)Py(t_k^-,x)\Bigg]. \tag{33}$$

From (12) and (18), we can get

$$\sum_{i=1}^n p_i\mu(t_k^-)\Big[(d_i+1)y_i(t_k^-,x)+\sum_{j=1}^n e_{ij}h_j(y_j(t_k^--\tau_j(t_k),x))\Big]^2\le\lambda_k\mu(t_k^-)y^T(t_k^-,x)Py(t_k^-,x)=\lambda_k\mu(t_k^-)\sum_{i=1}^n p_iy_i^2(t_k^-,x). \tag{34}$$

Therefore,

$$V(t_k^+,x)\le\int_\Omega\Bigg[\lambda_k\mu(t_k^-)\sum_{i=1}^n p_iy_i^2(t_k^-,x)+\sum_{i=1}^n\sum_{j=1}^n\int_{t_k^--\tau_j(t_k^-)}^{t_k^-}\mu(s)f_j^2(y_j(s,x))\,ds+\sum_{i=1}^n\sum_{j=1}^n\int_0^{+\infty}k_{ij}(\sigma)\int_{t_k^--\sigma}^{t_k^-}\mu(s+\sigma)f_j^2(y_j(s,x))\,ds\,d\sigma\Bigg]dx$$
$$\le\lambda_kV(t_k^-,x). \tag{35}$$

Let $T_1=\max\{t_0,T\}$ and $V_0=\sup_{0\le s\le T_1}V(s,x)$ and $V_{max}=\max\{V_0,\lambda_{k-1}\cdots\lambda_0V(t_0^-,x)\}$. When $t\in[t_{k-1},t_k)\cap[T,+\infty),k\in N=\{1,2,\ldots\}$ we can obtain

$$\mu(t)\lambda_{min}(P)\|u(t,x)-u^*\|_2^2\le V_1(t,x)\le V(t,x)\le V(t_{k-1}^+,x)\le\lambda_{k-1}V(t_{k-1}^-,x)\le\lambda_{k-1}\cdots\lambda_0V(t_0^-,x), \tag{36}$$

$$\text{i.e.}\quad\|u(t,x)-u^*\|_2^2\le\frac{V_{max}}{\mu(t)\lambda_{min}(P)}. \tag{37}$$

The proof is completed. □

4. Discussions

In what follows, along with the properties of time delays, we give several corollaries by specifying the functional $\mu(t)$, which represents several types of stabilities (also see [23,24]).

We suppose that

$$k_{ij}(s)=e^{-(\beta_1+1)s},\qquad\int_0^{+\infty}k_{ij}(s)\,ds=\int_0^{+\infty}e^{-(\beta_1+1)s}\,ds=\frac{1}{1+\beta_1}=k_i. \tag{38}$$

Corollary 1 (*Global exponential stability*). *Suppose that $\tau(t)\le\tau^*,\mu(t)=e^{\beta_1t},\beta_1>0$. If (13)–(14) hold, then the solution of neural networks (6) is globally exponentially stable.*

Proof. Factually, we just need to verify the condition (12). Since

$$\frac{\dot\mu(t)}{\mu(t)}=\frac{\beta_1e^{\beta_1t}}{e^{\beta_1t}}=\beta_1, \tag{39}$$

$$\frac{\mu(t)}{\mu(t-\tau(t))}=e^{\beta_1\tau(t)}\le e^{\beta_1\tau^*}=\frac{1}{\beta_2}, \tag{40}$$

$$\frac{\int_0^{+\infty}e^{-(1+\beta_1)\sigma}e^{\beta_1(t+\sigma)}d\sigma}{e^{\beta_1t}}=\int_0^{+\infty}e^{-\sigma}\,d\sigma=1. \tag{41}$$

Let $\beta_1'\ge\beta_1,\beta_2\le e^{-\beta_1\tau^*},\beta_3\ge1$. Similar to the proof procedure of Theorem 1, finally, we can obtain

$$\|u(t,x)-u^*\|_2^2\le\frac{V_{max}}{e^{\beta_1t}\lambda_{min}(P)}, \tag{42}$$

i.e. if under Assumptions 1–3 there exists a matrix P such that (13) and (14) hold, the solution of impulsive neural networks (6) is said to be globally exponentially stable according to Definition 1 and Remark 1. □

Corollary 2 (*Global power stability*). *Suppose that $\tau(t)\le kt$, $0<k<1,\mu(t)=t^\gamma,\gamma>0$. If (13)–(14) hold, then the solution of neural networks (6) is globally power stable.*

Proof. Factually, we just need to verify the condition (12). Since when $t\ge T$,

$$\frac{\dot\mu(t)}{\mu(t)}=\frac{\gamma t^{\gamma-1}}{t^\gamma}=\frac{\gamma}{t}\le\frac{\gamma}{T}, \tag{43}$$

$$\frac{\mu(t)}{\mu(t-\tau(t))}=\frac{t^\gamma}{(t-\tau(t))^\gamma}\le\frac{t^\gamma}{(t-kt)^\gamma}=(1-k)^{-\gamma}, \tag{44}$$

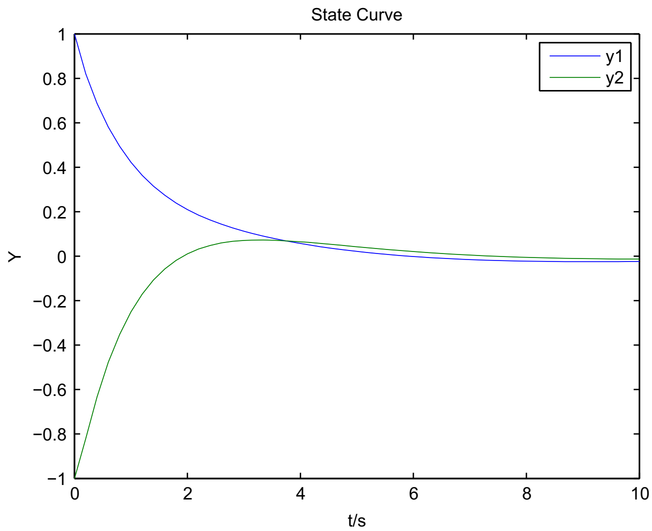

Fig. 1. The response curves of system (64)–(69) without impulsive effects.

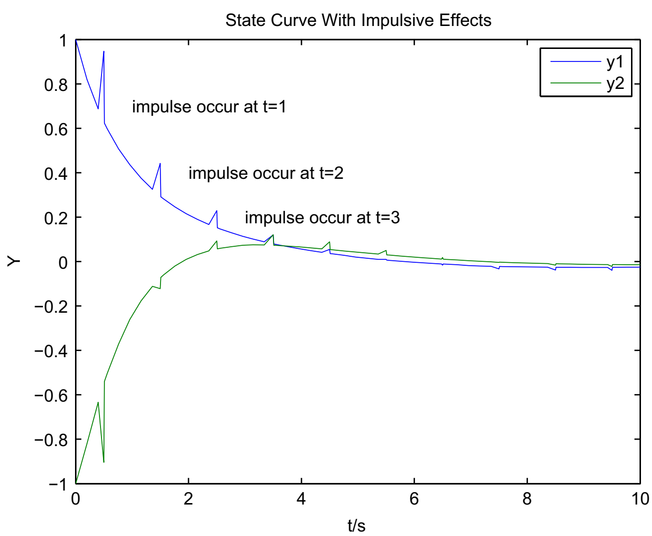

Fig. 2. The response curves of system (64)–(69) with impulsive effects.

$$\frac{\int_0^{+\infty} e^{-(1+\beta_1)\sigma}(t+\sigma)^\gamma\, d\sigma}{t^\gamma} \le \frac{\gamma!}{(1+\beta_1)^{\gamma+1}t^\gamma} \le \frac{\gamma!}{(1+\beta_1)^{\gamma+1}T^\gamma}. \tag{45}$$

Let $\beta_1 \ge \gamma/T, \beta_2 \le (1-k)^\gamma, \beta_3 \ge \gamma!/(1+\beta_1)^{\gamma+1}T^\gamma$. Similar to the proof procedure of Theorem 1, finally, we can obtain

$$\| u(t,x)-u^* \|_2^2 \le \frac{V_{max}}{t^\gamma \lambda_{min}(P)}, \tag{46}$$

i.e. if under Assumptions 1–3 there exists a matrix P such that (13) and (14) hold, the solution of impulsive neural networks (6) is said to be globally power stable according to Definition 1 and Remark 1. □

Corollary 3 (*Global log stability*). *Suppose that* $\tau(t) \le t - t/\ln t, \mu(t) = \ln(1+t)$. *If* (13)–(14) *hold, then the solution of neural networks* (6) *is globally log stable.*

Proof. Factually, we just need to verify the condition (12). Since when $t \ge T$,

$$\frac{\dot{\mu}(t)}{\mu(t)} = \frac{1/(1+t)}{\ln(1+t)} = \frac{1}{(1+t)\ln(1+t)} < \frac{1}{t\ln t} < \frac{1}{(\ln t)^2} \le \frac{1}{(\ln T)^2}, \tag{47}$$

$$\frac{\mu(t)}{\mu(t-\tau(t))} \le \frac{\ln(1+t)}{\ln(1+t/\ln t)}|_{t \to +\infty} = 1.478, \tag{48}$$

$$\begin{aligned}&\frac{\int_0^{+\infty} e^{-(1+\beta_1)\sigma}\ln(1+t+\sigma)\, d\sigma}{\ln(1+t)}\\ &\le \frac{1}{(1+\beta_1)} + \frac{1}{(1+\beta_1)\ln(1+t)}\int_0^{+\infty} e^{-(1+\beta_1)\sigma}\frac{1}{1+t+\sigma}\, d\sigma\\ &\le \frac{1}{1+\beta_1} + \frac{1}{(1+\beta_1)^2\ln(1+t)} \le \frac{1}{1+\beta_1} + \frac{1}{(1+\beta_1)^2\ln(1+T)}.\end{aligned} \tag{49}$$

Let $\beta_1 \ge (\ln T)^{-2}$, $\beta_2 \le 1.478^{-1}$, $\beta_3 \ge 1/(1+\beta_1) + 1/(1+\beta_1)^2\ln(1+T)$. Similar to the proof procedure of Theorem 1, finally, we can obtain

$$\| u(t,x)-u^* \|_2^2 \le \frac{V_{max}}{\ln(1+t)\lambda_{min}(P)}, \tag{50}$$

i.e. if under Assumptions 1–3 there exists a matrix P such that (13) and (14) hold, the solution of impulsive neural networks (6) is said to be globally log stable according to Definition 1 and Remark 1. □

Corollary 4 (*Global log–log stability*). *Suppose that* $\tau(t) \le t - t^\alpha$, $0 < \alpha < 1, \mu(t) = \ln\ln(3+t)$. *If* (13)–(14) *hold, then the solution of neural networks* (6) *is globally log–log stable.*

Proof. Factually, we just need to verify the condition (12). Since when $t \ge T$,

$$\begin{aligned}&\frac{\dot{\mu}(t)}{\mu(t)} = \frac{1/\ln(3+t)\cdot(1/(3+t))}{\ln\ln(3+t)} = \frac{1}{(3+t)\ln(3+t)\ln\ln(3+t)}\\ &\le \frac{1}{(3+T)\ln(3+T)\ln\ln(3+T)},\end{aligned} \tag{51}$$

$$\frac{\mu(t)}{\mu(t-\tau(t))} = \frac{\ln\ln(3+t)}{\ln\ln(3+t^\alpha)}|_{t\to+\infty} = 1.628, \tag{52}$$

$$\begin{aligned}&\frac{\int_0^{+\infty} e^{-(1+\beta_1)\sigma}\ln\ln(3+t+\sigma)\, d\sigma}{\ln\ln(3+t)} \le \frac{1}{1+\beta_1} + \frac{1}{(1+\beta_1)^2\ln\ln(3+t)}\\ &\le \frac{1}{1+\beta_1} + \frac{1}{(1+\beta_1)^2\ln\ln(3+T)},\end{aligned} \tag{53}$$

Let $\beta_1 \ge 1/(3+T)\ln(3+T)\quad\ln\ln(3+T), \beta_2 \le 1.628^{-1}, \beta_3 \ge 1/(1+\beta_1) + 1/(1+\beta_1)^2\ln\ln(3+T)$. Similar to the proof procedure of Theorem 1, finally, we can obtain

$$\| u(t,x)-u^* \|_2^2 \le \frac{V_{max}}{\ln\ln(3+t)\lambda_{min}(P)}, \tag{54}$$

i.e. if under Assumptions 1–3 there exists a matrix P such that (13) and (14) hold, the solution of impulsive neural networks (6) is said to be globally log–log stable according to Definition 1 and Remark 1. □

5. Three illustrate examples

Example 1. Consider the following system:

$$\begin{aligned}\frac{\partial(y_1(t,x))}{\partial t} &= \sum_{s=1}^{2}\frac{\partial}{\partial x_s}\left(\frac{\partial y_1(t,x)}{\partial x_s}\right) - 1.2y_1(t,x) + 0.2f_1(y_1(t,x))\\ &\quad -0.3f_2(y_2(t,x)) + 0.1f_1(y_1(t-\tau_1(t),x))\\ &\quad +0.2f_2(y_2(t-\tau_2(t),x)) + 0.3\int_0^{+\infty}k_{11}(s)f_1(y_1(t-s,x))\, ds - 0.4\\ &\quad \times\int_0^{+\infty}k_{12}(s)f_2(y_2(t-s,x))\, ds,\end{aligned} \tag{55}$$

$$\begin{aligned}\frac{\partial(y_2(t,x))}{\partial t} &= \sum_{s=1}^{2}\frac{\partial}{\partial x_s}\left(\frac{\partial y_2(t,x)}{\partial x_s}\right) - y_2(t,x) - 0.1f_1(y_1(t,x))\\ &\quad +0.1f_2(y_2(t,x)) - 0.15f_1(y_1(t-\tau_1(t),x))\\ &\quad +0.12f_2(y_2(t-\tau_2(t),x)) - 0.13\int_0^{+\infty}k_{21}(s)f_1(y_1(t-s,x))\, ds\\ &\quad +0.2\times\int_0^{+\infty}k_{22}(s)f_2(y_2(t-s,x))\, ds,\end{aligned} \tag{56}$$

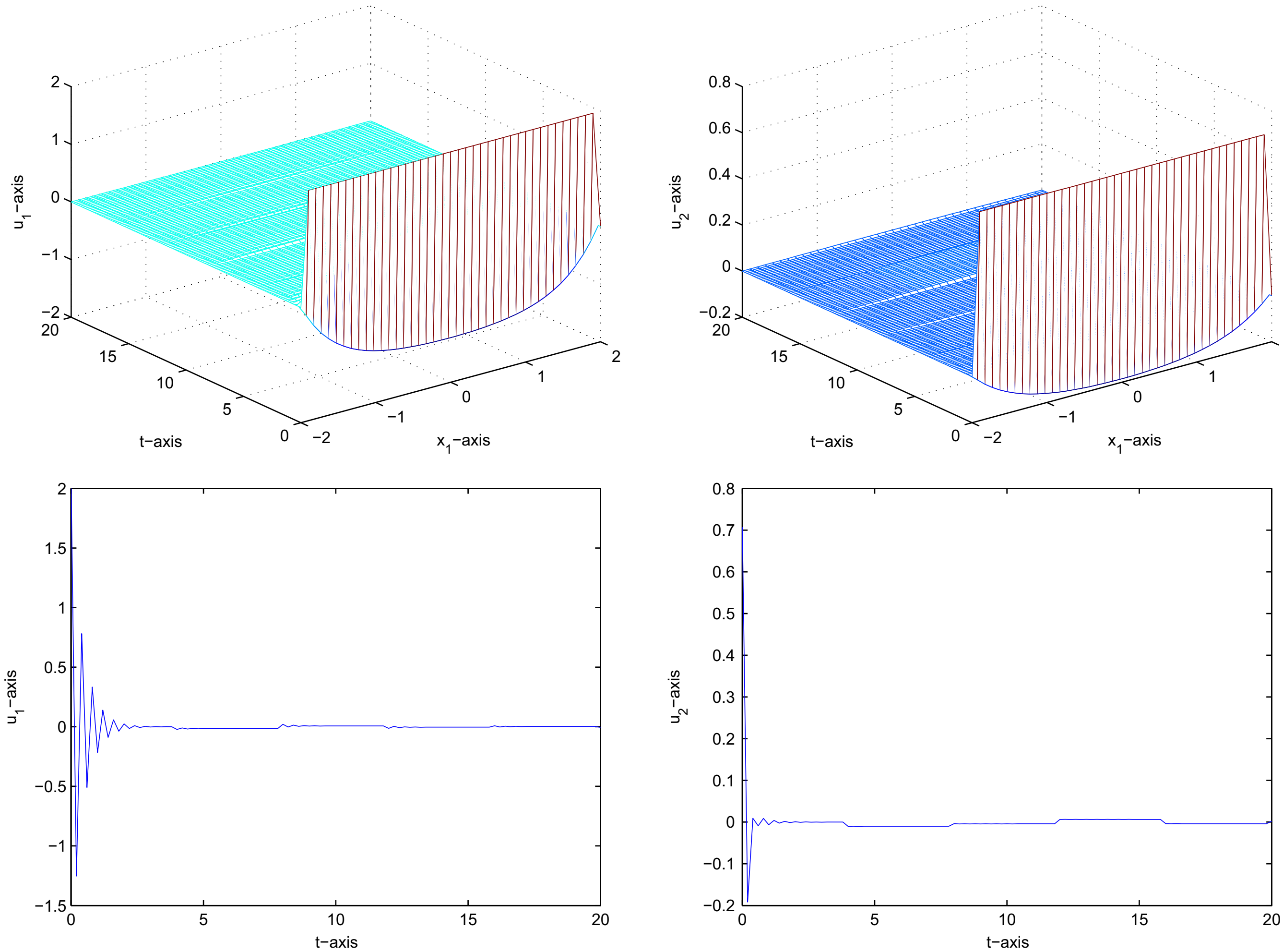

Fig. 3. State trajectories of system (72)–(75) when $u_1(t,x_1)=2,\ u_2(t,x_1)=\sin(pi/4),\ t_{max}=20$.

$$y_1(t_k^+,x)=y_1(t_k^-,x)-(1+e^{0.0025k})y_1(t_k^-,x)+\sum_{j=1}^{n}e_{1j}h(y_1(t_k^- -\tau_j(t),x)), \tag{57}$$

$$y_2(t_k^+,x)=y_2(t_k^-,x)-(1+e^{0.025k})y_2(t_k^-,x) + \sum_{j=1}^{n}e_{2j}h(y_2(t_k^- -\tau_j(t),x)), \tag{58}$$

$$h(y_i(t-\tau_j(t),x))=y_i(t-\tau_j(t),x)-(1+e^{0.025k})y_i(t-\tau_j(t),x),$$
$$i,j=1,2,x\in\Omega=\{(x_1,x_2)^T\,|\,0<x_1,x_2<1\}\subset R^2,\quad t_0=0,$$
$$t_k=t_{k-1}+0.5k,\ k\in N, \tag{59}$$

$$\frac{\partial y_i}{\partial n}=\left(\frac{\partial y_i}{\partial x_1},\frac{\partial y_i}{\partial x_2},\dots,\frac{\partial y_i}{\partial x_m}\right)^T=0,\quad \forall t\ge 0,\ x\in\partial\Omega,\ i,m=1,2, \tag{60}$$

$$y_i(s,x)=\varphi_i(s,x),\quad -\infty<s\le 0,\ x\in\Omega,\ i=1,2, \tag{61}$$

where the coefficients and functions are taken as

$$A=\begin{bmatrix}1.2 & 0\\ 0 & 1\end{bmatrix},\quad B=\begin{bmatrix}0.2 & -0.3\\ -0.1 & 0.1\end{bmatrix},\quad C=\begin{bmatrix}0.1 & 0.2\\ -0.15 & 0.12\end{bmatrix},$$

$$D=\begin{bmatrix}0.3 & -0.4\\ -0.13 & 0.2\end{bmatrix},E_k=\begin{bmatrix}e_{11} & e_{12}\\ e_{21} & e_{22}\end{bmatrix}=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix},$$

$$L=\begin{bmatrix}l_1 & 0\\ 0 & l_2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\quad \Gamma_k=\begin{bmatrix}e^{0.025k} & 0\\ 0 & e^{0.025k}\end{bmatrix},\quad k\in N,$$

$$E=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\quad D_k=\begin{bmatrix}d_1 & 0\\ 0 & d_2\end{bmatrix}$$
$$=\begin{bmatrix}-(1+e^{0.0025k}) & 0\\ 0 & -(1+e^{0.025k})\end{bmatrix},\quad k\in N,$$

$$(k_{ij}(s))_{2\times 2}=\begin{bmatrix}e^{-s} & e^{-s}\\ e^{-s} & e^{-s}\end{bmatrix},\quad D_{is}=1,\quad i,$$

$$s=1,2,f_i(x)=0.5(|x+1|-|x-1|),\quad i=1,2,\tau_1(t)=\tau_2(t)=0.5t.$$

Let $T=1,\alpha_1=0.01,\alpha_2=0.1,|\Omega|=1,\mu(t)=t$. Then $\beta_1=1,\beta_2=2,\beta_3=0.25,q_1=q_2=1$. It is easy to verify that Assumptions 1–3 are satisfied, and by using the Matlab LMI toolbox, we obtain $P=\left[\begin{smallmatrix}p_1 & 0\\ 0 & p_2\end{smallmatrix}\right]=\left[\begin{smallmatrix}1 & 0\\ 0 & 1\end{smallmatrix}\right]$ which satisfies that

$$\sum_{i=1}^{2}\left(\beta_1 p_i-2p_i\sum_{s=1}^{2}\frac{D_{is}}{q_i^2}\right)+\sum_{j=1}^{2}\left(\left(\frac{p_i b_{ji}^2 l_i^2}{a_j}-a_i p_i\right)\right.$$
$$\left.+(1-\rho)^{-1}\beta_2^{-1}p_i^2c_{ij}^2\quad+\frac{k_i}{4}d_{ij}^2p_i^2+l_i^2+\beta_3 l_i^2\right)=-5.1081<0, \tag{62}$$

$$\begin{bmatrix}(E+D_k)^TP(E+D_k)-\lambda_kP & (E+D_k)^TPE_k\Gamma_k\\ \Gamma_k^TE_k^TP(E+D_k) & \Gamma_k^TE_k^TPE_k\Gamma_k\end{bmatrix}\le 0. \tag{63}$$

From Definition 1 and Theorem 1, the above discussed system is globally asymptotically stable.

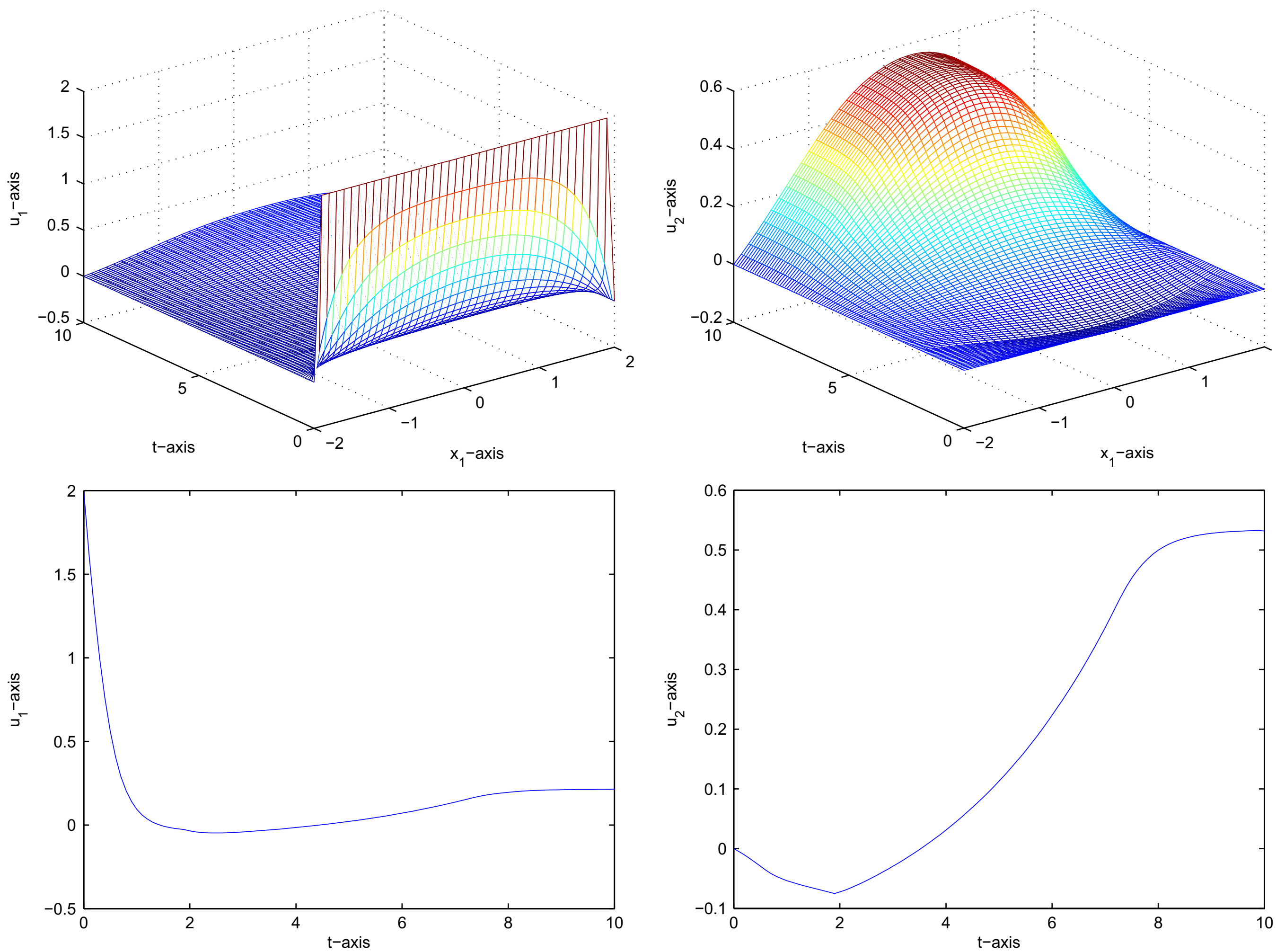

Fig. 4. State trajectories of system (72)–(75) when $u_1(t,x_1)=2,\ u_2(t,x_1)=\sin(pi),\ t_{max}=10$.

Example 2. If we let $f_i(\xi)=h_i(\xi), D_{is}=0, d_{ij}=0$, for $i,j=1,2,\xi\in R$. System (6) will be reduced to the following system:

$$\frac{dy_1}{dt}=-0.9y_1(t)+0.2h_1(y_1(t))+0.3h_2(y_2(t))$$
$$-0.1h_1(y_1(t-\tau_1(t)))-0.2h_2(y_2(t-\tau_2(t))), \tag{64}$$

$$\frac{dy_2}{dt}=-0.86y_1(t)+0.2h_1(y_1(t))+0.1h_2(y_2(t))$$
$$+0.1h_1(y_1(t-\tau_1(t)))-0.2h_2(y_2(t-\tau_2(t))), \tag{65}$$

$$y_1(t_k^+)=0.5y_1(t_k^-), \tag{66}$$

$$y_2(t_k^+)=0.64y_2(t_k^-), \tag{67}$$

$$t_k=t_{k-1}+k, \quad k\in N,\ t_0=0, \tag{68}$$

$$y_i(s)=\varphi_i(s), \quad -\infty<s\le 0,\ i=1,2,$$
$$y\in\Omega=\{(y_1,y_2)^T \mid 0<y_1,y_2<1\}\subset R^2. \tag{69}$$

Let $\mu(t)=t, h_i(\xi)=\frac{1}{2}(|\xi+1|-|\xi-1|), \tau_1(t)=\tau_2(t)=0.5t, T=1, \alpha_1=0.01, \alpha_2=0.1$. Then $\beta_1=1, \beta_2=2, \beta_3=0.25, q_1=q_2=1$.

$$A=\begin{bmatrix}0.9 & 0\\ 0 & 0.86\end{bmatrix},\quad B=\begin{bmatrix}0.2 & 0.3\\ 0.2 & 0.1\end{bmatrix},\quad C=\begin{bmatrix}-0.1 & -0.2\\ 0.1 & -0.2\end{bmatrix},$$

$$D=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix},\quad E=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, E_k=\begin{bmatrix}e_{11} & e_{12}\\ e_{21} & e_{22}\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix},$$

$$D_k=\begin{bmatrix}d_1 & 0\\ 0 & d_2\end{bmatrix}=\begin{bmatrix}0.5 & 0\\ 0 & 0.64\end{bmatrix},\quad k\in N,\quad L=\begin{bmatrix}l_1 & 0\\ 0 & l_2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}.$$

Similar to Example 1, Assumptions 1–3 are satisfied. By using the Matlab LMI toolbox, we obtain $P=\begin{bmatrix}p_1 & 0\\ 0 & p_2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ which satisfies that

$$\sum_{i=1}^{2}\left(\beta_1 p_i-2p_i\sum_{s=1}^{2}\frac{D_{is}}{q_i^2}\right)+\sum_{j=1}^{2}\left(\left(\frac{p_i b_{ji}^2 l_i^2}{a_j}-a_i p_i\right)\right.$$
$$\left.+(1-\rho)^{-1}\beta_2^{-1}p_i^2c_{ij}^2 \quad +\frac{k_i}{4}d_{ij}^2p_i^2+l_i^2+\beta_3 l_i^2\right)=-4.2132<0, \tag{70}$$

$$\begin{bmatrix}(E+D_k)^TP(E+D_k)-\lambda_kP & (E+D_k)^TPE_k\Gamma_k\\ \Gamma_k^TE_k^TP(E+D_k) & \Gamma_k^TE_k^TPE_k\Gamma_k\end{bmatrix}\le 0. \tag{71}$$

Therefore, the conditions of Theorem 1 are satisfied, which means that the above discussed system is globally asymptotically stable. The state responses of the system simulation discussed in Example 2 are shown in Figs. 1 and 2.

In fact, this model is also discussed in [34] where the stability criteria depend on the upper bound of time delays. Hence, theorems in [34] are not applicable to ascertain the stability of the model in Example 2. This point implies that our results are more general and universal.

Example 3. Consider the following impulsive delayed reaction–diffusion cellular neural network:

$$\frac{\partial(u_i(t,x))}{\partial t}=\sum_{s=1}^{m}\frac{\partial}{\partial x_s}\left(D_{is}\frac{\partial u_i(t,x)}{\partial x_s}\right)-a_iu_i(t,x)+\sum_{j=1}^{n}b_{ij}\tilde{f}_j(u_j(t,x))$$
$$+\sum_{j=1}^{n}c_{ij}\tilde{f}_j(u_j(t-\tau_j(t),x)),t>0,\ t\neq t_k,\ x\in\Omega,$$
$$i=1,2,\ldots,n,\ k=1,2,\ldots, \tag{72}$$

$$u_i(t_k+0,x)=u_i(t_k,x)+1.343u_i(t_k,x),\quad x\in\Omega,$$
$$i=1,2,\ldots,n,\ k=1,2,\ldots, \tag{73}$$

$$\frac{\partial u_i}{\partial n}=\left(\frac{\partial u_i}{\partial x_1},\frac{\partial u_i}{\partial x_2},\frac{\partial u_i}{\partial x_3},\frac{\partial u_i}{\partial x_4}\right)^T=0,$$
$$\forall t\geq 0,x_1,x_2,x_3,x_4\in\partial\Omega,\ i=1,2,\ldots,n, \tag{74}$$

$$u_i(s,x)=\varphi_i(s,x),\quad -\infty<s\leq 0,\ x\in\Omega,\ i=1,2,\ldots,n. \tag{75}$$

Let $n=2,\ m=4,\ \tau(t)=0.95t,\ t\in(0,t_{max}),\quad \Omega=\{(x_1,\ldots,x_4)^T \mid \Sigma_{i=1}^4 x_i^2<4\},\ a_i=6.5,\ \alpha_1=36,\ \alpha_2=40,\ t_k-t_{k-1}=0.05k,\ \tilde{f}_j(u_j)=\frac{1}{4}(|u_j+1|-|u_j+1|),\quad \mu(t)=e^{mt},\ m>0,\quad \rho=0.95,\ q_i=2,\quad l_i=0.5,\ d_i=1.343,\ e_{ij}=0,\ d_{ij}=0,\ i,j=1,2,$

$$(D_{is})_{2\times4}=\begin{bmatrix}1.2 & 2.3 & 2.5 & 3.1\\ 1.8 & 3.2 & 2.7 & 3.4\end{bmatrix},\quad (b_{ij})_{2\times2}=\begin{bmatrix}-0.23 & 1.3\\ -0.1 & 3\end{bmatrix},$$
$$(c_{ij})_{2\times2}=\begin{bmatrix}-0.1 & -0.2\\ 0.1 & -0.3\end{bmatrix}.$$

Then, $\beta_1\geq m,\ \beta_3\geq 1,\ \beta_2\leq e^{-t_{max}m}$. Obviously, Assumptions 1–3 are satisfied. By using the Matlab LMI toolbox, matrix $P=\left[\begin{smallmatrix}2 & 1.3\\ -1.3 & 4\end{smallmatrix}\right]$ which satisfies (13)–(14) can be obtained easily. Then, according to Corollary 1 the above discussed system is globally exponentially stable.

But we note that

$$\dot{\tau}(t)=0.95<1-(1/h)\ (h>0),\quad \rho=n\max_{i=1,\cdots,n}(l_i^2)=\frac{1}{2},$$
$$\lambda=\max_{i=1,\ldots,n}\left(-\chi-2a_i+\sum_{j=1}^{n}(b_{ij}^2+c_{ij}^2)\right)+\rho=-7.4698.$$

Since $h>20$, $\lambda=-7.4698>-10>-h\rho$. The conditions of Theorem 3 in [41] are not satisfied, which implies that our proposed method in this paper weaken some conservatism of the existing results in [41]. Considering $u(t,x_1),u(t,x_2),u(t,x_3)$, and $u(t,x_4)$ have similar state trajectories, here, we only take $u(t,x_1)$ as an example to show the superiority of the proposed method more clearly. The corresponding numerical simulations of the system discussed in Example 3 have been described in Figs. 3 and 4 under different initial conditions and parameters.

6. Conclusions

In this paper, a new model that is more general than those investigated in the previous literature has been outlined first. Then, by establishing a novel Lyapunov–Krasovskii functional and using some initial conditions, two concise sufficient conditions guaranteeing the global μ-stability of impulsive neural networks with reaction–diffusion terms and both unbounded time-varying delays and bounded continuously distributed time delays have been obtained, which unifies the exponential stability, power stability, log-stability, log–log stability, etc. Moreover, the new and simple LMI-based stability criteria have involved the reaction–diffusion terms and the regional feature, which weakens some conservatism of the existing results in previous literature. However, they are independent of boundaries of variable time delays. On the other hand, our method can also be used to investigate the stability for neural networks with bounded delays, either constant or time-varying. Finally, several corollaries and three concrete numerical examples have been given to verify the feasibility and superiority of our results. The results have generalized and improved the existing ones. In the future, we will try to search for sufficient conditions for the global stability of the neural systems under parameter uncertainties or stochastic perturbations and apply these obtained results to other problems such as combinatorial optimization.

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