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Design of lenses with curved Petzval image surfaces

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ABSTRACT

Design strategies have been devoted to simplify and miniaturize optical systems. In this paper, by constraining image surface to coincide with the Petzval surface, we achieve a compact f/2.8 lens system design with a curved Petzval image surface. Arc distortion is proposed to accurately measure the distortion relative to a curved image surface. The optical performance of our curved image surface lens is analysed and compared. Results show that modulation transfer function (MTF) of our curved Petzval design over 69% at 100 cycles/mm for entire fields is achievable with 100-mm effective focal length, 40° full field of view, >92.4% edge relative illumination, <0.5% arc distortion. Comparisons with a traditional lens with a planar image plane demonstrate that a curved Petzval image surface is an excellent strategy to simplify and miniaturize optical systems, compensate field curvature and benefit astigmatism correction, increase off-axis illumination and improve MTF. Furthermore, the lens with a curved Petzval image surface has a more uniform optical power distribution and greater degree of lens symmetry.

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Lens system design; aberration compensation; aberrations

1. Introduction

Nowadays, with the rapid development of digital photography, there is an increasing requirement for simpler, more compact and lower cost digital cameras with high optical performance, thereby favouring more civilian and military applications. To reach this goal, some design strategies have been studied in the literature. For instance, simple lenses computational imaging which needs complicated image post-processing (*1*), multichannel imaging systems which combine the key advantages of the human single-aperture eye and the insects' compound eye (*2, 3*), optical systems with curved image surfaces (*4–7*) and so on.

As far as optical design is concerned, using curved image surface has many advantages. Generally speaking, allowing the image surfaces of optical systems to be curved will reduce the number of elements, increase field of view (FOV) as well as off-axis relative illumination remarkably, improve modulation transfer function (MTF), reduce complexity and lower the cost (*4–8*).

To sense the curved image surface well, two main approaches are proposed and developed in the past decades. The first approach is the fibre-coupled focal planes, which can transfer the curved image over a dense array of optical fibres with a curved input face and a planar output face, so that the curved image can be sensed by traditional commercialized flat detectors (*9*). The second approach is the curved focal plane arrays. A number of researchers and companies have been devoted to create practical curved focal plane arrays, and they have got some achievement lately (*8, 10–13*).

In this paper, a fast f-number, long effective focal length (EFL), wide FOV imaging system with a curved Petzval image surface is presented. An image surface which coincides with the Petzval surface is called as the curved Petzval image surface. By means of making the image surface coincide with the curved Petzval surface, the field curvature is directly compensated, resulting in a simplified lens design with a better image performance. In addition, for the purpose of accurately describing the distortion aberration relative to a curved image surface, we propose and derive the arc distortion which is measured in image arc length instead of image height.

The remaining part of the paper is organized as follows. In Section 2, we review the Petzval surface and field curvature. The arc distortion is proposed and derived to accurately measure distortion of optical systems with curved image surfaces. We build macros to control aberrations. In

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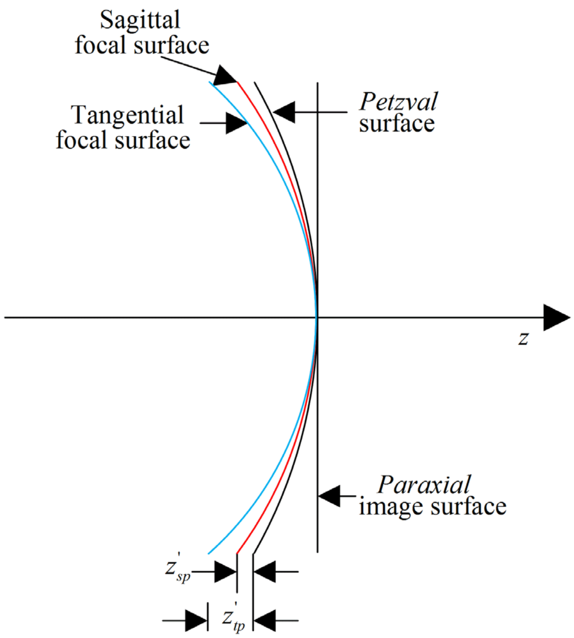

Figure 1. Relationships of the Petzval surface, the tangential focal surface and the sagittal focal surface. z'_{tp} and z'_{sp} are the tangential and sagittal field curvature relative to the Petzval surface.

Section 3, the design examples for both planar and curved image surfaces are described, and optical performance of the two designs are analysed and compared in detail. In the end, the conclusion is drawn in Section 4.

2. Design principles and aberration control

2.1. Petzval surface and field curvature

In the absence of astigmatism, a surface on which the image is formed is called the Petzval surface. The curvature of the Petzval surface depends simply on the optical powers and refractive indices of the lens elements. According to the Petzval's theorem (*14*), the radius of curvature of the Petzval surface is given by the following equation:

$$\frac{1}{R_p} = n' \sum_j \frac{1}{r_j}\left(\frac{1}{n'_j} - \frac{1}{n_j}\right). \tag{1}$$

where r_j is the radius of curvature of the jth surface, n_j and n'_j are the refractive indices before and after the jth surface, n' is the refractive index in image space.

If a lens has no astigmatism, the sagittal and tangential images coincide with each other and lie on the Petzval surface. In the presence of astigmatism, the tangential focal surface is three times as far from the Petzval surface as the sagittal focal surface. As illustrated in Figure 1, when taking the Petzval surface as the reference surface, the tangential curvature and the sagittal field curvature are described by (*15*)

$$z'_{tp} = -\frac{3}{2n'u'^2} S_{\mathrm{III}}. \tag{2}$$

$$z'_{sp} = -\frac{1}{2n'u'^2} S_{\mathrm{III}}. \tag{3}$$

where S_{III} is the Seidel coefficient for astigmatism, u' is the paraxial marginal ray slope in image space.

By constraining the image surface to coincide with the Petzval surface, the Petzval curvature can be suppressed directly, and more degrees of freedom are used for simplifying optical configuration and improving optical performance.

2.2. Arc distortion

Distortion is the only aberration that does not result in image blur, since it is a lateral displacement of the image point from its ideal paraxial position (*16*). Traditionally, distortion is defined as

$$D = \frac{y_{\mathrm{real}} - y_{\mathrm{para}}}{y_{\mathrm{para}}}. \tag{4}$$

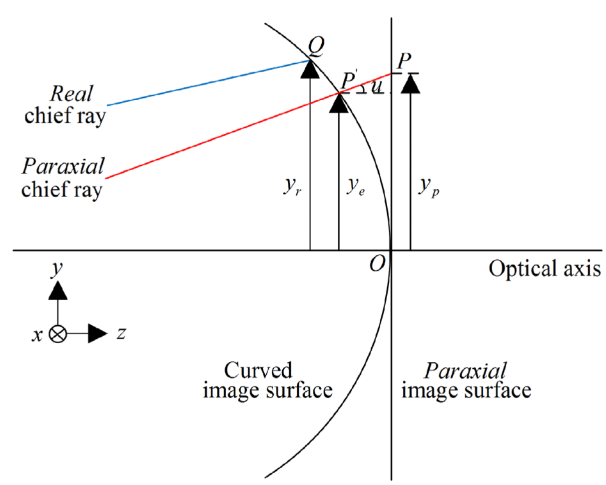

Figure 2. Illustration of arc distortion. Q is the real image point, the equivalent image point P' is mapped from the paraxial image point P along the paraxial chief ray, u is the paraxial chief ray slope in image space.

Table 1. Optical specifications of curved Petzval design.

Parameters	Specifications
EFL	100 mm
F-number	2.8
Full FOV	40°
Wavelength	486.1 ~ 656.3 nm
Arc distortion	<0.5%
MTF(all fields)	>0.6 at 100 cycles/mm
Edge relative illumination	>90%

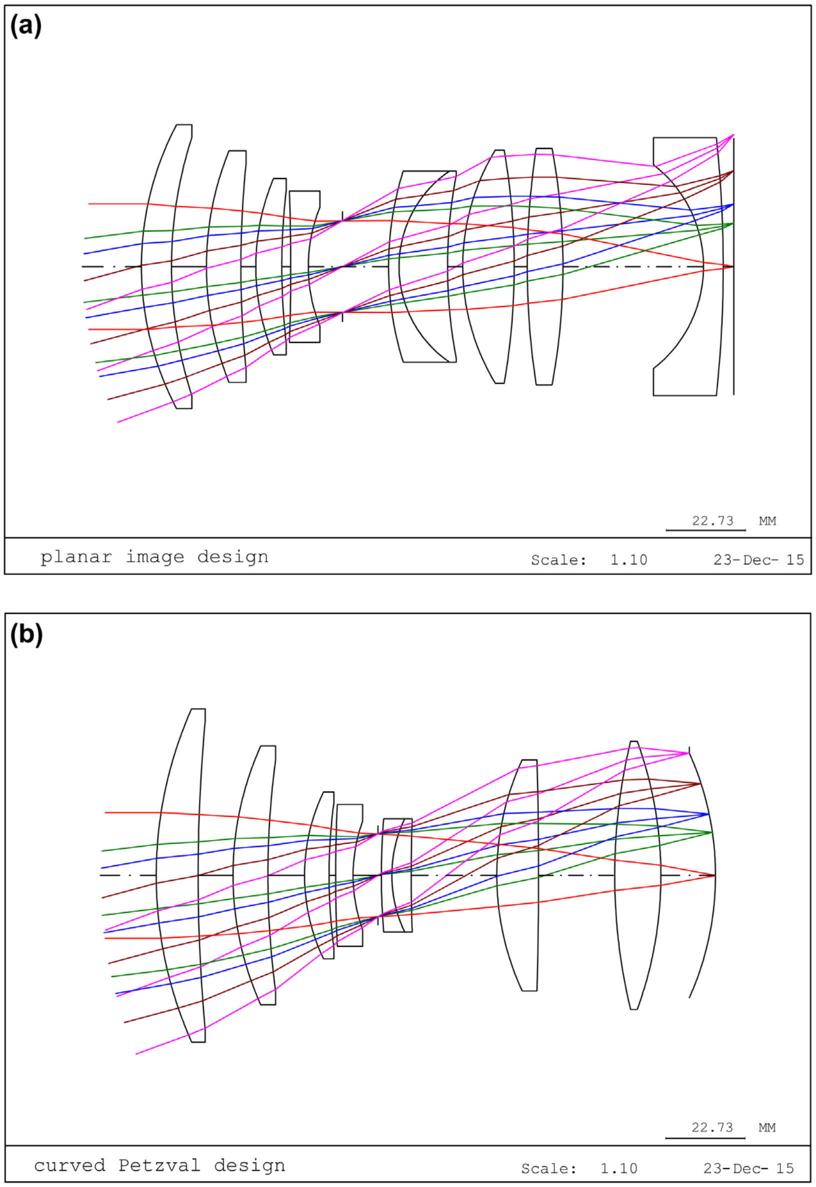

Figure 3. Layouts of (a) planar image design and (b) curved Petzval design.

where y_{real} is the real image height and y_{para} is the paraxial image height.

For a lens with a planar image surface, Equation (4) is very valid. However, when it comes to a lens with a curved image surface, the meaning of distortion aberration changes and the planar image distortion definition is not suitable any more. It is obvious that image arc length is more valid than image height to measure the distortion relative to a curved image surface.

In order to accurately measure the distortion aberration of lenses with curved image surfaces, we propose the arc distortion which is measured in image arc length instead of image height. As illustrated schematically in Figure 2, the real chief ray intersects the curved image surface at Q. The paraxial chief ray intersects with the paraxial image surface at P. The point $P'P$ on the real curved image surface, which is mapped from the paraxial image point P along the paraxial chief ray, is called the equivalent image point. y_{r} and y_{p} can be obtained through ray tracing. y_{e} is the equivalent image height. u is the paraxial chief ray slope in image space.

The arc distortion D_l is defined by

$$D_l = \frac{l_r - l_e}{l_e}. \tag{5}$$

where l_{r} and l_{e} are the arc lengths of real image OQ and equivalent image OP', respectively, given by

$$l_{\mathrm{r}} = \arcsin\left(\frac{y_{\mathrm{r}}}{R}\right) \cdot R. \tag{6}$$

Table 2. Prescription data for planar image design.

Surface no.	Radius (mm)	Thickness (mm)	Glass
Object	Infinity	Infinity	
1	85.75	8.59	HLAF53_CDGM
2	117.84	10.00	
3	86.57	9.86	HFK61_CDGM
4	285.69	4.28	
5	64.28	7.43	HFK61_CDGM
6	182.54	2.59	
7	−563.70	5.00	TF3_CDGM
8	44.10	9.77	
Stop	Infinity	13.20	
10	89.60	3.00	TF3_CDGM
11	32.72	13.88	HZK3_CDGM
12	136.14	4.34	
13	63.43	14.75	HFK61_CDGM
14	−191.36	3.71	
15	218.40	10.25	HFK61_CDGM
16	−180.11	40.21	
17	−36.40	5.65	HZK9A_CDGM
18	−339.39	3.00	
Image	Infinity	0.00	

Table 3. Prescription data for curved Petzval design.

Surface no.	Radius (mm)	Thickness (mm)	Glass
Object	Infinity	Infinity	
1	115.34	12.00	HFK61_CDGM
2	464.91	10.00	
3	88.72	10.00	HFK61_CDGM
4	241.04	10.51	
5	54.25	7.02	HFK61_CDGM
6	152.88	1.89	
7	420.92	5.00	TF3_CDGM
8	44.37	7.11	
Stop	Infinity	1.00	
10	191.72	3.00	TF3_CDGM
11	36.93	5.56	HLAK50_CDGM
12	445.17	24.52	
13	77.58	12.00	HFK61_CDGM
14	−1099.24	21.79	
15	160.91	13.26	HFK61_CDGM
16	−110.09	15.65	
Image	−84.04	0.00	

$$l_e = \arcsin\left(\frac{y_e}{R}\right) \cdot R. \qquad (7)$$

In Equations (6) and (7), R is the radius of curvature of the curved image surface.

2.3. Aberration control

According to the above design principles, we build Code V user-defined macros in the process of our curved Petzval lens design. The first macro is used to calculate the radius of Petzval surface and constrain the image surface to coincide with the Petzval surface throughout each optimization circle, thereby suppressing the field curvature directly. The second macro is established to obtain the y-coordinate of the equivalent image point and the real image point on a curved image surface at all FOV. According to the definition of arc distortion given by Equations (5)–(7), the arc distortion of our curved image surface lens is calculated and controlled. Besides, another macro programmed with the predefined optimization operands is built to restrict the first-order lens parameters and the third-order aberrations. By applying the Code V macro manager that runs our macros, we fulfil our curved Petzval lens design.

3. Design examples and analysis

In this section, the lens example with a planar image plane (planar image design for short) is given for comparison, and the lens example with a curved Petzval image surface (curved Petzval design for short) is presented to demonstrate optical advantages of the curved Petzval image surface. The design examples are analysed and optimized by the optical design software Code V (*17*).

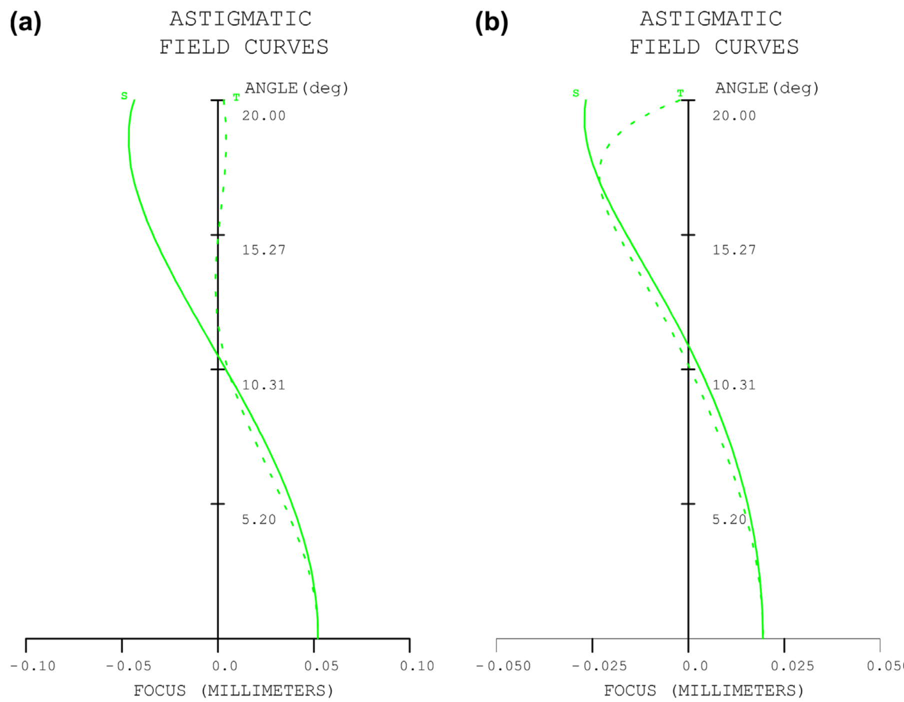

Figure 4. Astigmatic field curves of (a) planar image design and (b) curved Petzval design.

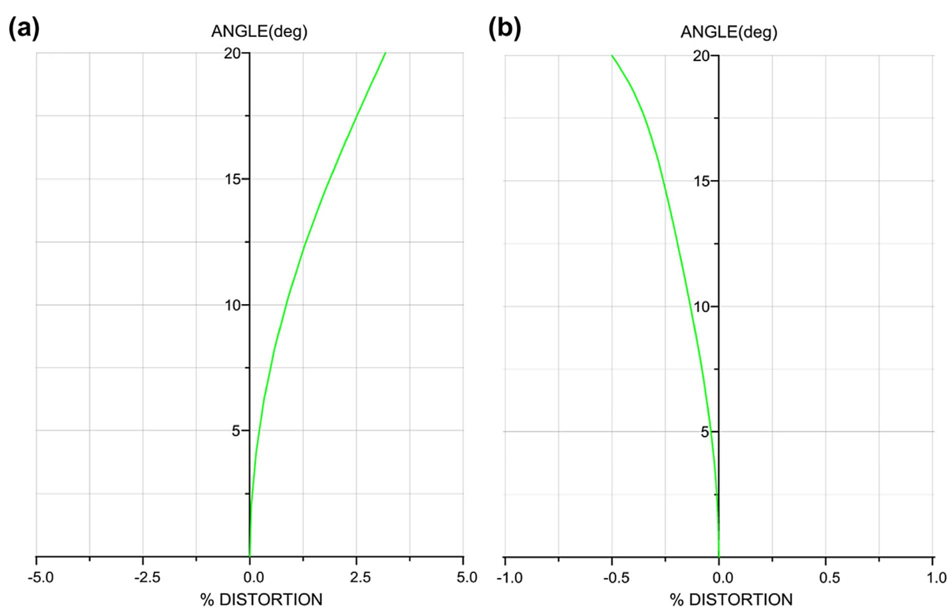

Figure 5. Distortion curves (a) distortion of planar image design and (b) arc distortion of curved Petzval design.

3.1. Specifications and layouts

It is well known that an important step in optical design is to select an appropriate initial configuration. U. S. Patent 4621909 (*18*) provides a lens with specifications close to the requirements listed in Table 1, and we choose it as a starting point for both our planar image design and the curved Petzval design.

We set lens curvature, thickness, airspace and glass material as variables for optimization in the process of the planar image lens design. Additional lenses are added to correct aberrations and improve image quality. The final layout of the planar image design is shown in Figure 3(a). The prescription data are summarized in Table 2.

In the process of the curved Petzval image lens design, the radius of the image surface is selected particularly as a

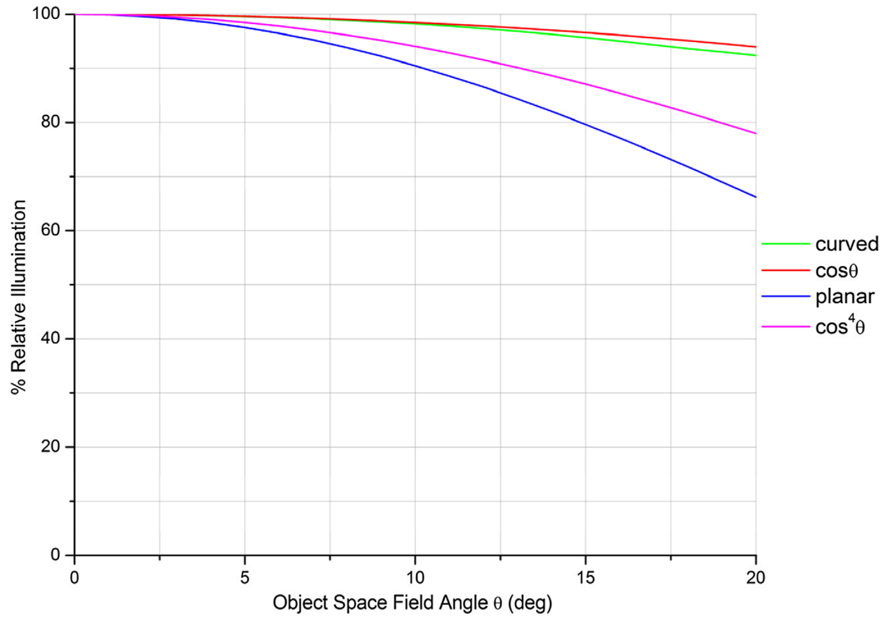

Figure 6. Comparisons of the relative illumination of curved Petzval design and planar image design. $\cos\theta$ and $\cos^4\theta$ curves are also given.

variable for optimization besides the default variables. The image surface is constrained to coincide with the Petzval surface, so that the Petzval curvature can be compensated directly. Through running the arc distortion macro mentioned in Section 2, distortion is calculated and controlled. Finally, we achieve a compact curved Petzval design with high optical performance. The layout is shown in Figure 3(b). The prescription data are summarized in Table 3.

In addition, we select HFK61 ($n_d = 1.497$ $\nu_d = 81.6$) and TF3 ($n_d = 1.612$ $\nu_d = 44.1$) from the CDGM glass catalogue as a combination to correct the severe secondary spectrum which is proportional to the EFL and the square of the relative aperture. We also use CDGM HLAF53 ($n_d = 1.743$ $\nu_d = 49.2$), HZK3 ($n_d = 1.589$ $\nu_d = 61.3$), HZK9A ($n_d = 1.620$ $\nu_d = 60.3$), and HLAK50 ($n_d = 1.651$ $\nu_d = 58.4$).

3.2. Analysis and comparison

3.2.1. First-order lens parameter

The two designs with similar optical configuration share the same f-number of 2.8, EFL of 100 mm, and full FOV of 40°. The traditional planar image design consists of nine optical elements and a planar image plane. The curved Petzval design is composed of eight optical elements and a curved Petzval image surface. As shown in Figure 3, the curved Petzval design can eliminate the field lens without image quality degradation. The overall length of the planar image design is 170 mm, while that of the curved Petzval design is only 158 mm. In addition, the curved Petzval design has a longer back focal length, which is desirable for the long working distance applications. Therefore, lenses with curved image surfaces may have more simplified and miniaturized optical configurations.

3.2.2. Field curvature and astigmatism

For all lenses with planar image surface, field curvature is an intrinsic aberration due to the existence of Petzval sum. When FOV of a lens is narrow, correspondingly, field curvature is small, then astigmatism can be well corrected; but the wider the FOV is, the more severe the field curvature will be. So, it is difficult to achieve a wide FOV lens system design with a planar image plane. The astigmatic field curves of the planar image design are illustrated in Figure 4(a).

However, by constraining the image surface to coincide with the curved Petzval surface, field curvature is compensated directly, thereby benefiting the correction of astigmatism and other aberrations. Therefore, our curved Petzval design with a wide FOV is fulfilled. The radius of image surface is −84.04 which is very close to the radius of Petzval surface: −83.53. This obviously suggests that the Petzval curvature is compensated by the curved image surface. As shown in Figure 4(b), the third-order tangential field curvature is very close to the third-order sagittal field curvature over all fields, so that the third-order astigmatism is very tiny. The residual third-order field astigmatism is balanced with higher orders astigmatism.

3.2.3. Distortion

Traditional definition of distortion is very useful to evaluate distortion of optical systems with planar image planes. Figure 5(a) shows the distortion of the planar image

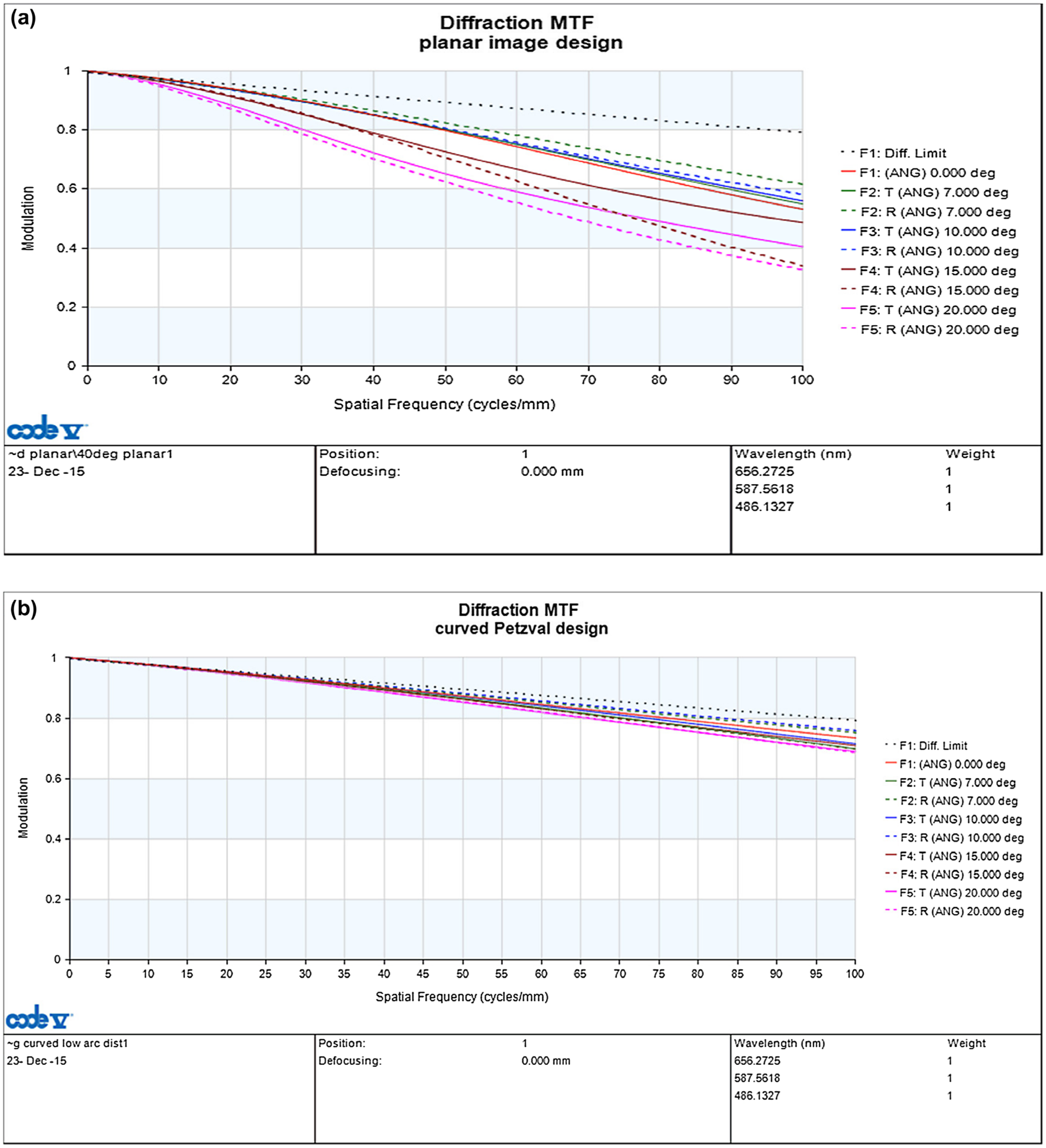

Figure 7. Modulation transfer functions (MTFs) of (a) planar image design and (b) curved Petzval design.

design. However, it is not suitable any more for lenses with curved image surfaces. To accurately evaluate the distortion of optical systems with curved image surfaces, we proposed the arc distortion in Section 2. According to Equations (5)–(7), arc distortion of the curved Petzval design is calculated and controlled in 0.5%. As illustrated in Figure 5(b), the maximum arc distortion is about 0.5%, which corresponds to 2.7% of distortion calculated in Code V. We believe that our proposed arc distortion will benefit the image post-processing of lenses with curved image surfaces in the future.

3.2.4. Relative illumination

The illumination of off-axis image points is usually lower than that of the axial image point. The relative illumination is influenced by the chief ray incidence angle on the image surface and the existence of distortion. Generally, the relative illumination fall-off follows a 'cosine fourth law'.

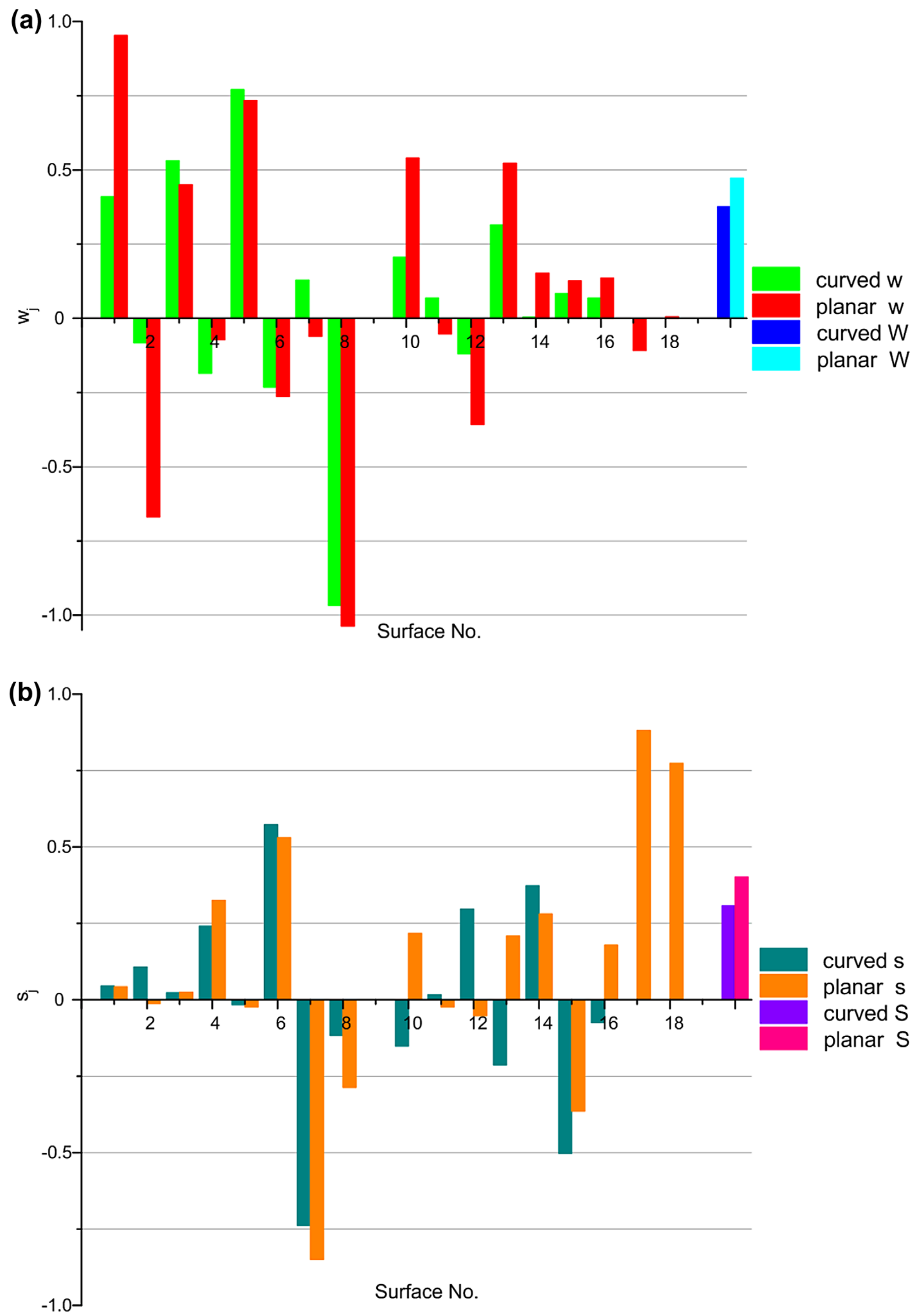

Figure 8. Comparisons of *W* and *S* values of planar image design and curved Petzval design (a) weighted surface refractive power w_j for each surface and *W* value and (b) degree of lens symmetry parameter s_j for each surface and *S* value.

Since the chief ray incidence angle is measured from the image surface normal, the chief ray incidence angle on a curved image surface is decreased remarkably; so that the off-axis illumination of a lens system with a curved image surface can be dramatically increased and do not obey the 'cosine fourth law' any longer. As illustrated schematically in Figure 6, the relative illumination curve of the planar image design is far lower than the $\cos^4\theta$ curve, while that of the curved Petzval design is greatly higher than the $\cos^4\theta$ curve and slightly lower than the $\cos\theta$ curve; this obviously suggests that the off-axis illumination of the curved Petzval design is significantly improved. The relative illumination of the curved Petzval design at 20° FOV is about 92.4%.

3.2.5. Modulation transfer function

The MTF is a comprehensive optical performance criterion for imaging systems. The MTF for the planar image design and the curved Petzval design is shown in Figure 7. It is obvious that the planar image design performs badly compared with the curved Petzval design. As shown in Figure 7(a), the MTFs of the planar image design vary greatly with different field angles and their values are smaller. On the other hand, the curved Petzval design is superior to the planar image design. As shown in Figure 7(b), MTFs of the curved Petzval design shift slightly with different field angles. The MTFs are over 69% at 100 cycles/mm for all fields, and it can be seen that all of them nearly reach the diffraction limit. MTFs of the curved Petzval design are more uniform and higher than those of the planar image design, which demonstrates excellent optical performance for both on-axis and off-axis fields.

3.2.6. Optical power distribution and degree of lens symmetry

Two lens form parameters, W and S, for quantifying the optical power distribution and the degree of lens symmetry of optical systems are proposed by Sasian and Descour (*19*). W and S are powerful criteria to evaluate and compare our designs, for they are independent of conjugate distance, aperture, field angle and size of optical systems. Lenses with smaller values of W and S are inclined to possess better image quality. The values of W and S of the two designs are shown in Figure 8. It is clear that the curved Petzval design performs better than the planar image design. It can be seen in Figure 8(a) that the majority of the weighted surface refractive powers of the curved Petzval design are smaller than those of the traditional design. The W value of the curved Petzval design is 0.38, which is lower than 0.47 of the planar image design. This demonstrates that the optical power distribution of the curved Petzval design is more uniform, thereby benefiting the higher order aberrations' correction. Figure 8(b) shows the s values for each surface and the S values of the two lenses. The last two surfaces of the traditional design have large s values, which worsen the symmetry of the lens system, while they are deleted in the curved Petzval design. The S value for the curved Petzval design is 0.31, which is smaller than 0.40 for the planar image design. This means that the optical structure of the curved Petzval design has greater degree of lens symmetry, leading to well-balanced lateral aberrations.

4. Conclusion

In conclusion, by constraining the image surface to coincide with the curved Petzval surface, we design a compact f/2.8 lens system with 100-mm EFL and 40° full FOV. Arc distortion relative to a curved image surface is reasonable, and its value of the curved Petzval design is controlled in 0.5%. The optical performance for the designed lens with a curved Petzval image surface is analysed and compared with a traditional planar image lens. Results show that a curved Petzval image surface offers a strategy to simplify and miniaturize optical systems, compensate field curvature and benefit astigmatism correction, increase off-axis illumination and improve MTF. Smaller values of W and S for the curved Petzval design demonstrate a more uniform optical power distribution and greater degree of lens symmetry, corresponding to a better balanced aberration and higher image quality. In addition, the radius of the image surface in our design is approximately 84 mm, which is very close to the radius of the Petzval surface. This provides a guide for the required radius of the detector for digital camera lenses.

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