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Quantum phase transition, quantum fidelity and fidelity susceptibility in the Yang–Baxter system

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Abstract In this paper, we investigate the ground-state fidelity and fidelity susceptibility in the many-body Yang–Baxter system and analyze their connections with quantum phase transition. The Yang–Baxter system was perturbed by a twist of $e^{i\varphi}$ at each bond, where the parameter φ originates from the q-deformation of the braiding operator U with $q = e^{-i\varphi}$ (Jimbo in Yang–Baxter equations in integrable systems, World Scientific, Singapore, 1990), and φ has a physical significance of magnetic flux (Badurek et al. in Phys. Rev. D 14:1177, 1976). We test the ground-state fidelity related by a small parameter variation φ which is a different term from the one used for driving the system toward a quantum phase transition. It shows that ground-state fidelity develops a sharp drop at the transition. The drop gets sharper as system size N increases. It has been verified that a sufficiently small value of φ used has no effect on the location of the critical point, but affects the value of $F(g_c, \varphi)$. The smaller the twist φ, the more the value of $F(g_c, \varphi)$ is close to 0. In order to avoid the effect of the finite value of φ, we also calculate the fidelity susceptibility. Our results demonstrate that in the Yang–Baxter system, the quantum phase transition can be well characterized by the ground-state fidelity and fidelity susceptibility in a special way.

Keywords Quantum phase transition · Quantum fidelity · Yang–Baxter system

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1 Introduction

Quantum phase transitions (QPTs) [1], especially in one and two dimensions, have drawn a considerable interest within many fields of physics till now. Recently, generous of effort [2–20] has been devoted to the role of fidelity, a concept emerging from quantum information theory [21], in quantum critical phenomena [22], demonstrating that fidelity is an efficient probe of quantum criticality [10–13]. Fidelity as well as Berry phase have also been used to analyze quantum phase transitions from a geometrical perspective recently. In [23], Venuti et al. unified these two approaches showing that the underlying mechanism is the critical singular behavior of a complex tensor over the Hamiltonian parameter space. The advantage of the fidelity is that it is a space geometrical quantity; no a priori knowledge of the order parameter and symmetry breaking is required in studies of QPTs.

In particular, the minimum of fidelity near a critical point has been established in several models [3,13]. For a recent review of the fidelity approach to QPTs, see Ref. [12], most of these studies consider the case where the system undergoes a quantum phase transition as the coupling λ is varied. The quantum fidelity is then defined corresponding to the same parameter. Apart from some studies [9,23–25], relatively little attention has been given to the case where the quantum fidelity is defined with respect to a parameter different from λ. Here, we consider this case in detail for the many-body Yang–Baxter system and show that the QPTs can be well characterized by the ground-state fidelity and fidelity susceptibility in a special way, in which we get the ground-state fidelity as a function of coupling g to study QPTs under the perturbation of a different meaningful spectrum parameter in the Yang–Baxter system.

YBE was originated from solving the δ-function interaction model by Yang [26,27] and statistical models by Baxter [28,29], respectively. Then, it was introduced by Faddeev [30] and Leningrad scholars to solve many quantum integrable models. In recent years, the YBE has been introduced to the field of quantum information and quantum computation. It has been shown that YBE has a very deep connection with entanglement swapping and topological quantum computation [31–42] in a series of papers. Unitary solutions of the quantum Yang–Baxter equation(QYBE) and unitary solutions of the braided Yang–Baxter (i.e., the braid group relation) can often be identified with universal quantum gates [43,44]. A Hamiltonian usually can be constructed from the unitary $\check{R}(\theta, \varphi)$ matrix through Yang–Baxterization approach. Yang–Baxterization [45,46] has been applied [38,47–50] to derive a Hamiltonian for the unitary evolution of entangled states. In Ref. [38], based on the unitary $\check{R}$ matrices, Chen et al. constructed a set of Hamiltonians, then explored the Berry phase and quantum criticality of the Yang–Baxter system. And in Ref. [51], our team studied the QPT-Like phenomenon in a Two-Qubit Yang–Baxter System.

Considering the special role of YBE in quantum information and crucial role of fidelity in quantum critical phenomena, so in this work, we extend our previous work to the many-body Yang–Baxter system. We investigate the fidelity and fidelity susceptibility for the many-body Yang–Baxter system and analyze their connections with QPTs. The paper is organized as follows. In Sect. 2, based on the YBE, via Yang–Baxter $\check{R}(\theta, \varphi)$ matrix, we get the many-body Yang–Baxter system. In Sect. 3, we calculate the ground-state fidelity and fidelity susceptibility of the Yang–Baxter model as a function of coupling g, while the twist φ is varied, to investigate the critical properties of QPTs in the Yang–Baxter system. It demonstrates that in the many-body Yang–Baxter system, the QPTs really happen and can be well characterized by the ground-state fidelity and fidelity susceptibility in a special way. We end with a summary in the last section.

2 R-matrices and Hamiltonians

In Refs. [34,48,49], the unitary $\breve{R}_{i,i+1}(\theta,\varphi)$-matrices have been introduced via the Yang–Baxterization approach [45,46] so as to investigate their properties and applications in physics. To make this paper be self-contained, we briefly review them as follows.

For the 4×4 Yang–Baxter systems of two qubits, the rational solution of the YBE, $\breve{R}(\mu)$ can a be written in terms of a unitary transformation U as the following way: $\breve{R}(\mu) = a(\mu)I + b(\mu)U$, with U satisfying the TLA as follows,

$$U_i^2 = dU_i, \quad U_{i+1}U_iU_{i+1} = U_{i+1}, \quad U_iU_{i+1} = U_{i+1}U_i \tag{1}$$

where$|i - j| \geq 2$, and d represents the single loop in the knot theory which does not depend on the sits in the lattices. When $d = \sqrt{2}$, the unitary matrix U is of the following form,

$$U = \begin{pmatrix} 1 & 0 & 0 & e^{i\varphi} \\ 0 & 1 & i\varepsilon & 0 \\ 0 & -i\varepsilon & 1 & 0 \\ e^{-i\varphi} & 0 & 0 & 1 \end{pmatrix} \tag{2}$$

where $\varepsilon = \pm$ and φ is real. According to the above U-matrix, we can get the unitary matrix $\breve{R}_{i,i+1}(\theta,\varphi)$ through the Yang–Baxterization approach [45,46] as follows:

$$\breve{R}_{i,i+1}(\theta,\varphi) = \begin{pmatrix} \cos\theta & 0 & 0 & -ie^{i\varphi}\sin\theta \\ 0 & \cos\theta & \varepsilon\sin\theta & 0 \\ 0 & \varepsilon\sin\theta & \cos\theta & 0 \\ -ie^{-i\varphi}\sin\theta & 0 & 0 & \cos\theta \end{pmatrix}. \tag{3}$$

The unitary $\breve{R}_{i,i+1}$-matrix satisfies the YBE which is of the form,

$$\breve{R}_{i,i+1}(\mu)\breve{R}_{i+1,i+2}(\mu+\upsilon)\breve{R}_{i,i+1}(\upsilon) = \breve{R}_{i+1,i+2}(\upsilon)\breve{R}_{i,i+1}(\mu+\upsilon)\breve{R}_{i+1,i+2}(\mu), \tag{4}$$

Based on the YBE, two-spin interaction Hamiltonians usually can be constructed. As $\breve{R}_{i,i+1}$ is unitary, it can define the evolution of a state $|\Psi(0)\rangle$

$$|\Psi(t)\rangle = \breve{R}_{i,i+1}(t)|\Psi(0)\rangle, \tag{5}$$

Taking the Schrödinger equation $i\hbar\partial|\Psi(\theta,\varphi)\rangle/\partial t = H(\theta,\varphi)|\Psi(\theta,\varphi)\rangle$ into account, we let $\breve{R}_{i,i+1}(\theta,\varphi)$ be time-independent, where parameters θ and φ are all time-independent. Let us consider a system of two spin-1/2 particles (particle i and i+1) described by an initial Hamiltonian H_0:

$$H_0 = \mu_i S_i^z + \mu_{i+1} S_{i+1}^z + g S_i^z S_{i+1}^z \tag{6}$$

Next we introduce two parameters $J_i = \frac{\mu_i - \mu_{i+1}}{2}$ and $B_i = \frac{\mu_i + \mu_{i+1}}{2}$ for convenience of calculations. According to Eq. (5), also by taking partial derivative of the state $|\Psi(t)\rangle$ with respect to time t, we get an equation

$$\begin{aligned} i\hbar\frac{\partial|\Psi(t)\rangle}{\partial t} &= \breve{R}_{i,i+1}(\theta,\varphi)H_0|\Psi(0)\rangle \\ &= H_{i,i+1}(\theta,\varphi)|\Psi(t)\rangle \\ &= \breve{R}_{i,i+1}(\theta,\varphi)H_0\breve{R}^{-1}_{i,i+1}(\theta,\varphi)|\Psi(t)\rangle, \end{aligned} \tag{7}$$

Then according to Eqs. (3), (6) and (7), and corresponding to the form of $\breve{R}_{i,i+1}(\theta,\varphi)$, the two-body interaction Hamiltonian $H_{i,i+1}$ can be written in the form of spin operators $S_i^+ = S_i^x + iS_i^y$ and $S_i^- = S_i^x - iS_i^y$ as follows,

$$\begin{aligned} H_{i,i+1}(\theta,\varphi) = {} & B\cos\theta\left(S_i^z + S_{i+1}^z\right) + J\cos\theta\left(S_i^z - S_{i+1}^z\right) + gS_i^zS_{i+1}^z \\ & + iB\sin\theta\left(e^{i\varphi}S_i^+S_{i+1}^+ - e^{-i\varphi}S_i^-S_{i+1}^-\right) \\ & - J\varepsilon\sin\theta\left(S_i^+S_{i+1}^- + S_i^-S_{i+1}^+\right) \end{aligned} \tag{8}$$

To simplify the calculation in the following, we let $\theta = \frac{\pi}{2}$, $B_i = \frac{1}{2}$, $J_i = 0$. Then we get the Yang–Baxter model,

$$H = \sum_{i=1}\left(gS_i^zS_{i+1}^z + \frac{i}{2}\left(e^{i\varphi}S_i^+S_{i+1}^+ - e^{-i\varphi}S_i^-S_{i+1}^-\right)\right) \tag{9}$$

3 Quantum phase transition and quantum fidelity

In Sect. 2, we have got the Yang–Baxter model as Eq. (9)

$$H(g,\varphi) = \sum_{i=1}\left(gS_i^zS_{i+1}^z + \frac{i}{2}\left(e^{i\varphi}S_i^+S_{i+1}^+ - e^{-i\varphi}S_i^-S_{i+1}^-\right)\right) \tag{10}$$

here we can see that just a twist of φ is applied at every bond comparing to the model $H = \sum_{i=1}(gS_i^zS_{i+1}^z + \frac{i}{2}(S_i^+S_{i+1}^+ - S_i^-S_{i+1}^-))$ (this is also a Yang–Baxter model, just $\varphi = 0$), the parameter φ is the spectrum parameter of the Yang–Baxter system which originates from the q-deformation of the braiding operator U with $q = e^{-i\varphi}$ [52], and φ has a physical significance of magnetic flux [53]. Next, we will calculate the ground-state fidelity of this model as a function of coupling g, while the twist φ is varied. It is

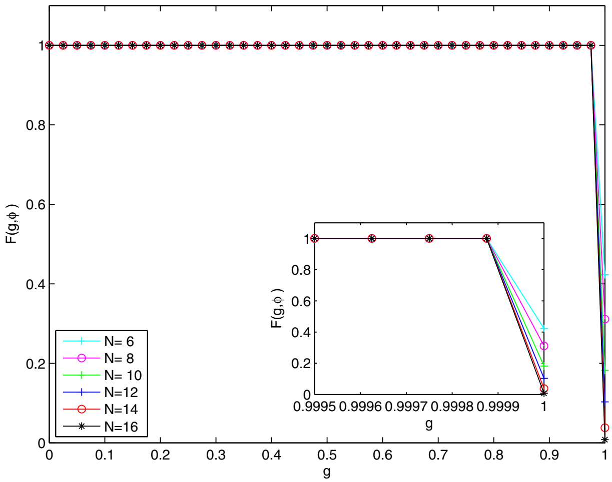

Fig. 1 The fidelity $F(g, \varphi)$ as a function of coupling g for system size from 6 to 16. The system size N is indicated in the legend, the twist $\varphi = 10^{-3}$. The drop gets sharper as system size N increases. The *inset* corresponds to enlarged picture for the parameter range of g near to g_c

special that here the quantum fidelity is not defined with respect to the same parameter g as most of studies have done, but with respect to the twist φ. It means that we get the ground-state fidelity as a function of coupling g to study QPTs under the perturbation of a different meaningful spectrum parameter φ in the Yang–Baxter system. This can be done through the overlap of the ground state with $\varphi = 0$ and a nonzero φ. Using $\Psi_0(g, \varphi)$ the ground state of this Yang–Baxter system, with respect to the twist in the limit where $\varphi \to 0$, the ground-state fidelity can then be written as [11]:

$$F(g, \varphi) = |\langle \Psi_0(g, 0)|\Psi_0(g, \varphi)\rangle| \tag{11}$$

A series expansion of the GS fidelity in φ yields,

$$F(g, \varphi) = 1 - \frac{(\varphi)^2}{2}\frac{\partial^2 F}{\partial g^2} + \cdots \tag{12}$$

where $\partial_g^2 F$ is called the fidelity susceptibility [12]. If the higher order terms are taken to be negligibly small then the fidelity susceptibility is defined as,

$$\chi(g) = \frac{2\,[1 - F(g, \varphi)]}{\varphi^2} \tag{13}$$

To compute $F(g, \varphi)$ and $\chi(g)$, we first calculate the ground state $|\Psi_0(g, 0)\rangle$ of the unperturbed Hamiltonian through numerical matrix exact diagonalization. Then, the system was perturbed by adding a twist of $e^{i\varphi}$ at each bond, and we recalculate the ground state $|\Psi_0(g, \varphi)\rangle$, here periodic boundary conditions were assumed. Through Eq. (11), we get $F(g, \varphi)$, the results are shown in Figs. 1 and 2, one can see that in the many-body Yang–Baxter system, the QPTs really happen and can be well

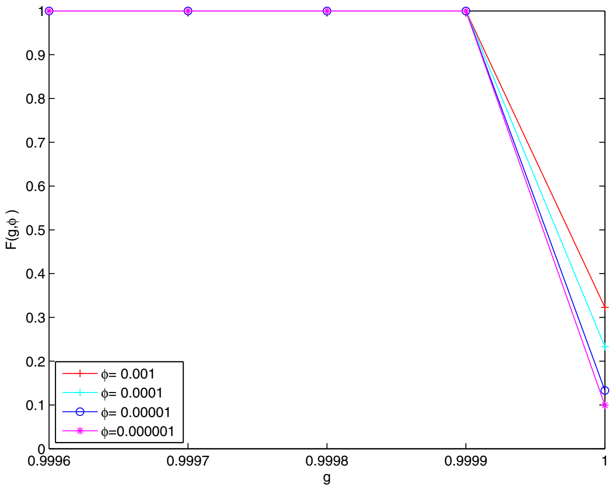

Fig. 2 The fidelity $F(g, \varphi)$ as a function of coupling g for system size $N = 8$ for four different twist φ. The smaller the twist φ, the more the value of $F(g_c, \varphi)$ is close to 0. The twist φ is indicated in the legend

characterized by the ground-state fidelity. In Fig. 1, for all data the twist φ was taken to be 10^{-3}, we observe that the critical region is clearly marked by a sudden drop of the fidelity $F(g, \varphi)$, it is almost constant and equal to 1 for all parameter ranges of g, apart from the very narrow range around the critical point g_c, where it has a significant sudden drop. Such behavior of the overlap function around the critical point can be ascribed to the fact that the ground state for $g = g_c$ becomes completely delocalized along one of two rotated axes, as opposed to the localized ground state outside of the critical point. From the inset of Fig. 1, we can see that the drop gets sharper as system size N increases. In all cases, it has been verified that a sufficiently small value of φ used has no effect on the location of the critical point, but it affects the value of $F(g_c, \varphi)$. In Fig. 2, we give an example for system size $N = 8$, plot the ground-state fidelity $F(g, \varphi)$ as a function of coupling g for four different twist φ, it demonstrates that the smaller the twist φ, the more the value of $F(g_c, \varphi)$ is close to 0. In order to avoid the effect of the finite value of φ, we also get the fidelity susceptibility from the corresponding fidelity. We calculate $\chi(g)$ through Eq. (13), the result for $\ln \chi(g)$ versus g is shown in Fig. 3. One can see that fidelity susceptibility also denotes the QPTs very well, the critical region is clearly marked by a sudden raise of the value of $\ln \chi(g)$, it is almost close to 0 in all parameter ranges of g, apart from the very narrow range around the critical point g_c, where it has a significant sudden raise.

4 Summary

In this paper, we move the discussion of QPTs into relatively new territory by engaging the mathematical sophistication of Yang–Baxter system analysis, especially into the many-body Yang–Baxter system. The numerical simulations performed together with our analytical arguments demonstrate that in the many-body Yang–Baxter system; the QPTs really happen and can be well characterized by the ground-state fidelity and

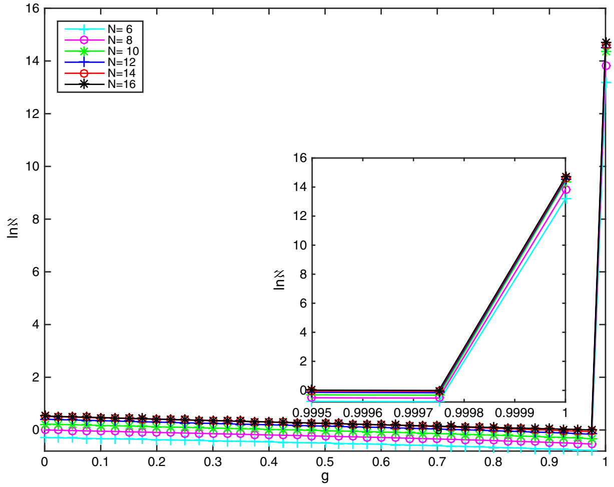

Fig. 3 The fidelity susceptibility $\chi(g)$ as a function of coupling g for system size from 6 to 16. The system size N is indicated in the legend, the twist $\varphi = 10^{-3}$. The raise gets sharper as system size N increases. The *inset* corresponds to enlarged picture for the parameter range of g near to g_c

fidelity susceptibility in a special way. Here, the quantum fidelity is not defined with respect to the same parameter g as most of studies have done, but with respect to the twist φ. In this way, one can see that here just the twist φ is the very small parameter perturbation (not δg), and then we can get the ground-state fidelity as a function of coupling g to study QPTs under the perturbation of a meaningful spectrum parameter φ in the Yang–Baxter system. Our work correlates the QPTs with the Yang–Baxter system through a twist of φ which is the spectrum parameter in the Yang–Baxter system and originates from the q-deformation of the braiding operator U with $q = e^{-i\varphi}$ [52], and φ has a physical significance of magnetic flux [53]. It shows that ground-state fidelity develops a sharp drop at the transition, the critical region is clearly marked by a sudden drop of the fidelity $F(g, \varphi)$, and the drop of $F(g, \varphi)$ gets sharper as system size N increases. In all cases, it has been verified that a sufficiently small value of φ used has no effect on the location of the critical point, but affects the value of $F(g_c, \varphi)$. The smaller the twist φ, the more the value of $F(g_c, \varphi)$ is close to 0. In order to avoid the effect of the finite value of φ, we also calculate the fidelity susceptibility from the corresponding fidelity. It shows that fidelity susceptibility also denotes the QPTs very well. Wish this work can provide valuable insights into the special role of YBE in quantum information and condensed matter physics.

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