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State estimation via Markov switching-channel network and application to suspension systems

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Abstract: The problem of H_∞ estimation for a class of networked non-linear systems is investigated. A practical scenario with multiple switching communication channels coexisting in the network is considered. System signals are exchanged over the multiple communication channels and each channel is subject to the two main transmission imperfections, network-induced time-varying delays and packet dropouts. The channel switching is assumed to be governed by a continuous-time Markov process, and a Markov jump non-linear system model is exploited to represent the overall networked system. Linear estimators are designed such that the underlying estimation error system is stochastically stable and the disturbance rejection attenuation satisfies an H_∞ performance bound. As a case study, a state estimation problem for an intelligent active suspension system is addressed to verify the theoretical findings.

1 Introduction

Over the past two decades, great efforts have been devoted to extensive studies on networked control systems (NCSs) in which the basic units (e.g. controllers, actuators, sensors and estimators) which realise a control (or estimation) loop are spatially distributed across diverse communication networks [1–3]. NCS applications emerge in various fields such as automobile systems [4], robotics [5], real-time communication systems [6], power networks [7], traffic and transportation systems [8, 9], and process control systems [10, 11]. On the other hand, network-induced imperfections such as packet dropouts and time delays which are unavoidable due to the networked communications can lead to system performance degradation or even system instability [12, 13]. Great efforts on overcoming such imperfections have been made in multiple disciplines, such as computer science, communication technology and control engineering. However, it is worth mentioning that most of the existing NCS studies (e.g. [14–16]) did not take into account the communication over multiple communication channels, which can potentially improve the communication performance and reliability.

Non-linear systems, which consist of a linear part and sector-bounded non-linearities, arise frequently in practical applications [17, 18]. Many control and estimation issues have been well researched including fundamental stability analysis [19, 20], handling of time delays [21]. Though the obtained results greatly enriched the non-linear control theories, the present advancements are still limited within the framework of point-to-point control or networked system with one single communication channel. Such systems when operated over a network with multiple communication channels have not been considered.

In this paper, we treat an H_∞ estimation problem for a class of networked systems with sector-bounded non-linearities. We consider the case of multiple switching communication channels. We assume that the channel switching is governed by a Markov process, which is a natural choice to describe random switching phenomena [22, 23]. The overall networked system is modelled as a Markov jump non-linear system (MJNS). A class of Lyapunov–Krasovskii functionals are exploited to obtain H_∞ estimators that account for time-varying delays and network-induced intermittent packet dropouts. While emphasis has been attached to the development of advanced control approaches for suspension systems and many results were reported (e.g. [24–26]), little attention has been given to the state estimation of automobiles with a few exceptions [27–29], where observers were designed for state re-construction or fault detection of certain states of ground vehicles. The above observation motivates us to apply the proposed method to suspension systems for the re-construction of the system states based on limited measurements. An example of an intelligent active suspension system is considered to demonstrate the effectiveness of the proposed estimator design approach.

This paper is organised as follows: in Section 2, the H_∞ estimation problem is formulated for a class of non-linear NCSs with multiple switching communication channels and some background results are reviewed. Section 3 is devoted to the stability and H_∞ performance analysis with a Lyapunov–Krasovskii-functional-based approach. Explicit expressions for the H_∞ estimator parameters are derived in Section 4. In Section 5, an example of an intelligent active suspension system is presented as a case study. Finally, the paper is concluded in Section 6.

Notation: The notation used throughout this paper is fairly standard. The superscripts 'T' and ' -1' stand for matrix transposition and inverse, respectively. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $L_2[0,\infty)$ is the space of square-integrable functions on $[0,\infty)$, and for $w(t)\in L_2[0,\infty)$, $\| w \|_2^2 = \int_0^\infty w(t)^T w(t)\,\mathrm{d}t$; in symmetric block matrices or long matrix expressions, we use $*$ as an ellipsis to represent symmetric terms and $\mathrm{diag}\{\cdot\ \cdot\ \cdot\}$ stands for a block-diagonal matrix. $\mathrm{Sym}(A)$ is a shorthand notation for $A + A^{\mathrm{T}}$. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible to perform

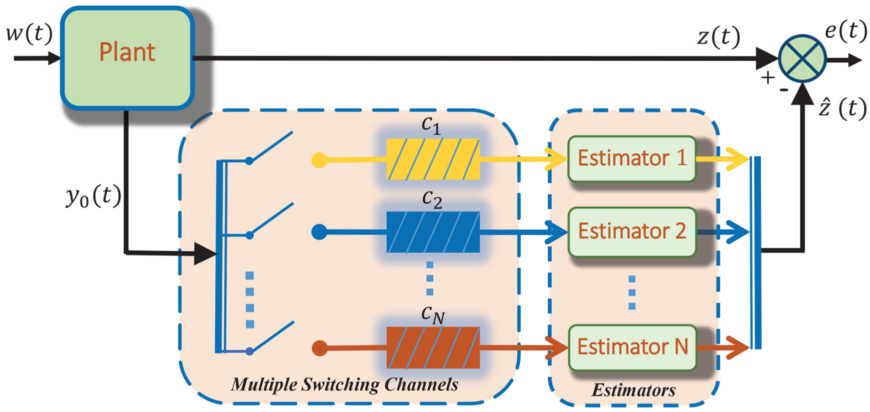

Fig. 1 *Networked estimation with multiple switching communication channels*

suitable algebraic operations. For a symmetric matrix, $P > 0\,(P \geq 0)$ means that P is positive-(semi-positive-)definite. I and 0 represent, respectively, the identity matrix and zero matrix.

2 Problem formulation and preliminaries

Consider the following class of continuous-time dynamic systems

$$\begin{cases} \dot{x}(t) = Ax(t) + B_w w(t) + Gg(x(t)), \\ y_0(t) = Cx(t) + Dw(t), \\ z(t) = Lx(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y_0(t) \in \mathbb{R}^r$ is the measured output, $z(t) \in \mathbb{R}^m$ is the controlled output, $w(t) \in \mathbb{R}^q$ is the process disturbance belonging to $L_2[0, \infty)$ and $g(\cdot): \mathbb{R}^n \to \mathbb{R}^n$ is a non-linear vector-valued function assumed to satisfy the following sector-bounded conditions

$$\begin{aligned} [g(a) - g(b) - R_1(a-b)]^{\mathrm{T}}[g(a) - g(b) - R_2(a-b)] \leq 0, \\ g(0) = 0, \quad \forall a, b \in \mathbb{R}^n, \end{aligned} \tag{2}$$

where $R_1, R_2 \in \mathbb{R}^{n \times n}$ and $R_1 - R_2$ is a positive definite matrix.

In this paper, system (1) is operated over a network with multiple communication channels (MCCs) (Fig. 1). Note that the measurement $y_0(t)$ is sent via the MCC network for which N communication channels are available, but only one of the channels is chosen and used at a time for signal transmission.

Let c_t denote the channel switching signal taking values in a finite set $\mathscr{C} = \{1, 2, \ldots, N\}$. We assume the overall channel switching is governed by a continuous-time Markov process $\{c_t \in \mathscr{C}, t \geq 0\}$ with a given infinitesimal generator

$$\Lambda = [\lambda_{ij}], \quad i, j \in \mathscr{C}, \tag{3}$$

where $\lambda_{ij} \geq 0, \forall j \neq i, \lambda_{ii} = -\sum_{j \neq i} \lambda_{ij}$. Then the transition probability of channel i to channel j is described as

$$\Pr\,(c_{t+\Delta} = j | c_t = i) = \begin{cases} \lambda_{ij}\Delta + o(\Delta), & j \neq i, \\ 1 + \lambda_{ii}\Delta + o(\Delta), & j = i, \end{cases} \tag{4}$$

where $o(\Delta)$ denotes second- or higher-order terms of Δ, i.e. $\lim_{\Delta \to 0} o(\Delta)/\Delta = 0$. The available channels have different characteristics in terms of time delays and packet dropout rates.

The following assumption is made about the communication channel delays.

Assumption 1: The communication delay of channel i, $i \in \mathscr{C}$, denoted as $\tau_i(t)$, is time varying and satisfies $\underline{\tau}_i \leq \tau_i(t) \leq \overline{\tau}_i$, where $\underline{\tau}_i$ and $\overline{\tau}_i$ are positive constants and represent the known lower and upper bound on the communication delay of channel i, respectively. Also, we assume that $\dot{\tau}_i \leq d < \infty$, $\forall i \in \mathscr{C}$, where d is a bound known *a priori*. For notational simplicity, we use τ_i to represent $\tau_i(t)$ when there is no confusion.

Let $y(t)$ denote the received measurement signal at time t via communication channel $i \in \mathscr{C}$. Thus

$$y(t) = \Theta_i y_0(t - \tau_i(t)), \tag{5}$$

where $\Theta_i := \mathrm{diag}\{\theta_{i1}, \theta_{i2}, \ldots, \theta_{ir}\}$ is a matrix with mutually independent random variables θ_{ij}, $i \in \mathscr{C}$ and $j = 1, 2, \ldots, r$, representing the packet arrival rate of sensor j via channel i. In the sequel, we denote $\bar{\Theta}_i = \mathbb{E}[\Theta_i] = \mathrm{diag}\{\mu_{i1}, \mu_{i2}, \ldots, \mu_{ir}\}$.

Remark 1: We note that the packet dropouts are often treated by using Bernoulli process. Our assumption in (5) includes Bernoulli process-based treatment as a special case.

Also, it is assumed that there are certain overlaps in terms of time delays and packet dropouts over all the MCCs, i.e.

$$\begin{aligned} &\bigcap\nolimits_{i \in \mathscr{C}} [\underline{\tau}_i, \overline{\tau}_i] \neq \varnothing, \\ &\Pr\,(\Theta_i < \Theta_j) \neq 0, \quad \forall i, j \in \mathscr{C}. \end{aligned} \tag{6}$$

By (6), we mean that generally there is no universally best or worst channel that can always be selected or abandoned.

Combining the assumptions from the above, (1) and (5), the overall networked system can be formulated as an MJNS as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_w w(t) + Gg(x(t)), \\ y(t) &= \Theta_{c_t} Cx(t - \tau_{c_t}(t)) + \Theta_{c_t} Dw(t), \\ z(t) &= Lx(t). \end{aligned} \tag{7}$$

For each of the N channels, $i \in \mathscr{C}$, we aim at designing a linear time-invariant estimator of the following form

$$\begin{aligned} \dot{\hat{x}}(t) &= K_A(i)\hat{x}(t) + K_B(i)y(t), \\ \hat{z}(t) &= K_C(i)\hat{x}(t), \end{aligned} \tag{8}$$

where $K_A(i)$, $K_B(i)$ and $K_C(i)$, $i \in \mathscr{C}$, are estimator parameters to be determined.

Then, combining (7) and (8), and letting $\eta(t) = \left[x^T(t) \quad \hat{x}^T(t)\right]^{\mathrm{T}}$ denote an extended state, the following augmented MJNS is obtained:

$$\begin{aligned} \dot{\eta}(t) &= \bar{A}(c_t)\eta(t) + \bar{A}_d(c_t)\eta(t - \tau_{c_t}(t)) \\ &\quad + \bar{B}(c_t)w(t) + \bar{G}g(x(t)), \\ e(t) &\triangleq z(t) - \hat{z}(t) = \bar{C}(c_t)\eta(t), \end{aligned} \tag{9}$$

where

$$\bar{A}(c_t) = \begin{bmatrix} A & 0_{n\times n} \\ 0_{n\times n} & K_A(c_t) \end{bmatrix}, \quad \bar{A}_d(c_t) = \begin{bmatrix} 0_{n\times n} & 0_{n\times n} \\ K_B(c_t)\Theta_{c_t}C & 0_{n\times n} \end{bmatrix},$$
$$\bar{B}_w(c_t) = \begin{bmatrix} B_w \\ K_B(c_t)\Theta_{c_t}D \end{bmatrix}, \quad \bar{G} = \begin{bmatrix} G \\ 0_{n\times n} \end{bmatrix},$$
$$\bar{C}(c_t) = [L \quad -K_C(c_t)].$$

Now we introduce the following definition which is essential for the subsequent analysis and filtering design.

Definition 1 [30]: System (9) with $w(t) \equiv 0$ is said to be stochastically stable (SS) if there exists a constant $T(c_0, \phi(\cdot)) > 0$, such that

$$\mathbb{E}[\,\|\eta(t)\|^2 \mid (c_0, \phi(\cdot))] \le T(c_0, \phi(\cdot))), \tag{10}$$

where $\phi(s) \in L_2[-\bar{\tau}, 0]$ is the initial condition of system (9) with $\bar{\tau} = \max_{i\in\mathscr{C}} \bar{\tau}_i$.

Then the H_∞ estimation problem addressed in this paper can be formulated as follows: given the system (7) and a prescribed level of noise attenuation $\gamma > 0$, determine linear estimators in the form (8) such that the underlying estimation error system is stochastically stable and under zero initial conditions, we have

$$\mathbb{E}[\,\|e(t)\|_2^2\,] \le \gamma^2 \,\|w\|_2^2. \tag{11}$$

3 Performance analysis

The following theorem is a sufficient condition on the stability of the estimation error system (9).

Theorem 1: Let $K_A(i)$, $K_B(i)$, $K_C(i)$, $i \in \mathscr{C}$, be given estimator parameters. Then the estimation error system in (9), with $w(t) \equiv 0$, is stochastically stable if there exist a positive scalar ϱ_1 and positive definite matrices $P(1)$, $P(2)$, …, $P(N)$, and Q such that

$$\begin{bmatrix} \Xi_i - \varrho_1 I_0^T \hat{R}_1 I_0 & \mathscr{P}(i) \\ * & \mathscr{Q} \end{bmatrix} < 0 \tag{12}$$

for all $i = 1, 2, \dots, N$, where

$$\Xi_i = \bar{A}^T(i)P(i) + P(i)\bar{A}(i) + \sum_{j=1}^{N} \lambda_{ij} P(j) + \kappa Q,$$
$$\mathscr{P}(i) = \left[P(i)\hat{A}_d(i) \quad P(i)\bar{G} - \varrho_1 I_0^T \hat{R}_2\right],$$
$$\hat{R}_1 = (R_1^T R_2 + R_2^T R_1)/2, \quad \hat{R}_2 = -(R_1^T + R_2^T)/2,$$
$$\hat{A}_d(i) = \begin{bmatrix} 0_{n\times n} & 0_{n\times n} \\ K_B(c_t)\bar{\Theta}_{c_t}C & 0_{n\times n} \end{bmatrix},$$
$$\mathscr{Q} = \mathrm{diag}\{-(1-d)Q, \ -\varrho_1 I_n\},$$
$$\kappa = 1 + \bar{\lambda}(\bar{\tau} - \underline{\tau}), \quad \bar{\lambda} = \max_{i\in\mathscr{C}} \{|\lambda_{ii}|\},$$
$$I_0 = [I_n \quad 0_{n\times n}], \quad \underline{\tau} = \min_{i\in\mathscr{C}} \{\underline{\tau}_i\}, \quad \bar{\tau} = \min_{i\in\mathscr{C}} \{\bar{\tau}_i\}.$$

Proof: To prove the result, we construct a Lyapunov–Krasovskii functional (LKF) as follows:

$$V(\eta(t), c_t) = \sum_{i=1}^{3} V_i(\eta(t), c_t), \tag{13}$$

with

$$V_1(\eta(t), c_t) = \eta^T(t)P(c_t)\eta(t),$$
$$V_2(\eta(t), c_t) = \int_{t-\tau_{c_t}}^{t} \eta^T(s)Q\eta(s)\,\mathrm{d}s,$$
$$V_3(\eta(t), c_t) = \int_{-\bar{\tau}}^{-\underline{\tau}} \int_{t+s}^{t} \eta^T(\xi)\bar{\lambda}Q\eta(\xi)\,\mathrm{d}\xi\,\mathrm{d}s,$$

where $\bar{\lambda}$ is defined in the statement of Theorem 1.

Let $\mathscr{L}$ be the weak infinitesimal generator of a random process $\{\eta(t), c_t\}$ and define

$$\mathscr{L}V(\eta(t), c_t) = \lim_{\Delta\to 0} \frac{1}{\Delta}\left\{\mathbb{E}[V(\eta(t+\Delta), c_{t+\Delta})] - V(\eta(t), c_t)\right\},$$

then we have

$$\begin{aligned}\mathscr{L}V_1(\eta(t), c_t) &= \dot{\eta}^T(t)P(c_t)\eta(t) + \eta^T(t)\mathscr{L}P(c_t)\eta(t) + \eta^T P(c_t)\dot{\eta}(t) \\ &= [\bar{A}(c_t)\eta(t) + \bar{A}_d(c_t)\eta(t-\tau_{c_t}) + \bar{B}(c_t)w(t) \\ &\quad + \bar{G}g(x(t))]^T P(c_t)\eta(t) + \eta^T(t)P(c_t)[\bar{A}(c_t)\eta(t) \\ &\quad + \bar{A}_d(c_t)\eta(t-\tau_{c_t}) + \bar{B}(c_t)w(t) + \bar{G}g(x(t))] \\ &\quad + \eta^T(t)\mathscr{L}P(c_t)\eta(t).\end{aligned} \tag{14}$$

Also,

$$\begin{aligned}\mathscr{L}P(c_t) &= \lim_{\Delta\to 0} \frac{\mathbb{E}[P(c_{t+\Delta})|c_t] - P(c_t)}{\Delta} \\ &= \lim_{\Delta\to 0} \frac{1}{\Delta}\Big[\sum_{j\ne c_t} \Pr\,(c_{t+\Delta} = j|c_t)P(j) \\ &\quad + \Pr\,(c_{t+\Delta} = c_t|c_t)P(c_t) - P(c_t)\Big]\end{aligned}$$

Combining with the transition probabilities defined in (4) yields

$$\mathscr{L}P(c_t) = \sum_{j=1}^{N} \lambda_{c_t j} P(j).$$

Consequently,

$$\eta^T(t)\mathscr{L}P(c_t)\eta(t) = \eta^T(t)\sum_{j=1}^{N} \lambda_{c_t j} P(j)\eta(t). \tag{15}$$

We next compute the infinitesimal generator of $V_2(\eta(t), c_t)$. Note that $\mathbb{E}[V_2(\eta(t+\Delta), c_{t+\Delta})|(\eta(t), c_t)]$ can be decomposed as

$$\begin{aligned}&\mathbb{E}[V_2(\eta(t+\Delta), c_{t+\Delta})|(\eta(t), c_t)] \\ &= \mathbb{E}\Big[I_{\{c_{t+\Delta}\ne c_t\}}\int_t^{t+\Delta} \eta^T(s)Q\eta(s)\,\mathrm{d}s\,|(\eta(t), c_t)\Big] \\ &\quad + \mathbb{E}\Big[I_{\{c_{t+\Delta}\ne c_t\}}\int_{t+\Delta-\tau_{c_{t+\Delta}}}^{t} \eta^T(s)Q\eta(s)\,\mathrm{d}s\,|(\eta(t), c_t)\Big] \\ &\quad + \mathbb{E}\Big[I_{\{c_{t+\Delta}= c_t\}}\int_{t+\Delta-\tau_{c_t}}^{t+\Delta} \eta^T(s)Q\eta(s)\,\mathrm{d}s\,|(\eta(t), c_t)\Big] \\ &:= V_{21} + V_{22} + V_{23},\end{aligned} \tag{16}$$

where $I_{\{\cdot\}}$ is the indicator function. It follows that

$$
\begin{aligned}
V_{21} &= \sum_{j \neq c_t} \Pr(c_{t+\Delta} = j | c_t) \int_t^{t+\Delta} \eta^T(s) Q \eta(s)\, \mathrm{d}s \\
&= \sum_{j \neq c_t} (\lambda_{c_t j}\Delta + o(\Delta)) \int_t^{t+\Delta} \eta^T(s) Q \eta(s)\, \mathrm{d}s \\
&= o(\Delta), \\
V_{22} &= \sum_{j \neq c_t} \Pr(c_{t+\Delta} = j | c_t) \int_{t+\Delta-\tau_j}^{t} \eta^T(s) Q \eta(s)\, \mathrm{d}s \\
&= \sum_{j \neq c_t} (\lambda_{c_t j}\Delta + o(\Delta)) \int_{t+\Delta-\tau_j}^{t} \eta^T(s) Q \eta(s)\, \mathrm{d}s, \\
V_{23} &= \Pr(c_{t+\Delta} = c_t) \int_{t+\Delta-\tau_{c_t}}^{t+\Delta} \eta^T(s) Q \eta(s)\, \mathrm{d}s \\
&= (1 + \lambda_{c_t c_t}\Delta + o(\Delta)) \int_{t+\Delta-\tau_{c_t}}^{t+\Delta} \eta^T(s) Q \eta(s)\, \mathrm{d}s.
\end{aligned}
$$

Thus, we have

$$
\begin{aligned}
&\mathscr{L}V_2(\eta(t), c_t) \\
&= \lim_{\Delta \to 0} \frac{1}{\Delta}\left\{\mathbb{E}[V_2(\eta(t+\Delta), c_{t+\Delta}) | (\eta(t), c_t)] - V_2(\eta(t), c_t)\right\} \\
&= \lim_{\Delta \to 0} \frac{1}{\Delta}\left\{V_{21} + V_{22} + V_{23} - V_2(\eta(t), c_t)\right\} \\
&= \sum_{j \neq c_t} \lambda_{c_t j} \int_{t-\tau_j}^{t} \eta^{\mathrm{T}}(s) Q \eta(s) ds + \lambda_{c_t c_t} \int_{t-\tau_{c_t}}^{t} \eta^{\mathrm{T}}(s) Q \eta(s) ds \\
&+ \lim_{\Delta \to 0} \frac{1}{\Delta}\{ \int_{t+\Delta-\tau_{c_t}}^{t+\Delta} \eta^{\mathrm{T}}(s) Q \eta(s) ds - \int_{t-\tau_{c_t}}^{t} \eta^{\mathrm{T}}(s) Q \eta(s) ds \}.
\end{aligned} \tag{17}
$$

Noting that the last term of (17) is the derivative of $\int_{t-\tau_{c_t}}^{t} \eta^T(s) Q \eta(s)\, \mathrm{d}s$, it follows that

$$
\begin{aligned}
\mathscr{L}V_2(\eta(t), c_t) = &\sum_{j \in \mathscr{C}} \lambda_{c_t j} \int_{t-\tau_j}^{t} \eta^T(s) Q \eta(s)\, \mathrm{d}s \\
&+ \eta^T(t) Q \eta(t) - (1 - \dot{\tau}_{c_t}) \eta^T(t - \tau_{c_t}) Q \eta(t - \tau_{c_t}).
\end{aligned} \tag{18}
$$

Direct computations yield

$$
\mathscr{L}V_3(\eta(t), c_t) = \bar{\lambda}(\bar{\tau} - \underline{\tau}) \eta^T(t) Q \eta(t) - \bar{\lambda} \int_{t-\bar{\tau}}^{t-\underline{\tau}} \eta^T(s) Q \eta(s)\, \mathrm{d}s. \tag{19}
$$

Also in (18),

$$
\begin{aligned}
&\sum_{j=1}^{N} \lambda_{c_t j} \int_{t-\tau_j}^{t} \eta^T(s) Q \eta(s)\, \mathrm{d}s \\
&= \sum_{j \neq c_t} \lambda_{c_t j} \int_{t-\tau_j}^{t} \eta^T(s) Q \eta(s)\, \mathrm{d}s + \lambda_{c_t c_t} \int_{t-\tau_{c_t}}^{t} \eta^T(s) Q \eta(s)\, \mathrm{d}s \\
&\leq \sum_{j \neq c_t} \lambda_{c_t j} \int_{t-\bar{\tau}}^{t} \eta^T(s) Q \eta(s)\, \mathrm{d}s + \lambda_{c_t c_t} \int_{t-\underline{\tau}}^{t} \eta^T(s) Q \eta(s)\, \mathrm{d}s
\end{aligned} \tag{20}
$$

Noting that $\sum_{j \neq c_t} \lambda_{c_t j} = -\lambda_{c_t c_t}$, it follows that

$$
\begin{aligned}
&\sum_{j=1}^{N} \lambda_{c_t j} \int_{t-\tau_j}^{t} \eta^T(s) Q \eta(s)\, \mathrm{d}s \\
&\leq -\lambda_{c_t c_t} \int_{t-\bar{\tau}}^{t-\underline{\tau}} \eta^T(s) Q \eta(s)\, \mathrm{d}s \leq \bar{\lambda} \int_{t-\bar{\tau}}^{t-\underline{\tau}} \eta^T(s) Q \eta(s)\, \mathrm{d}s.
\end{aligned} \tag{21}
$$

Combining (14), (15), (18), (19) and (21), we have

$$
\begin{aligned}
&\mathscr{L}V(\eta(t), c_t) \\
&= \mathscr{L}V_1(\eta(t), c_t) + \mathscr{L}V_2(\eta(t), c_t) + \mathscr{L}V_3(\eta(t), c_t) \\
&\leq \eta^{\mathrm{T}}(t)[\bar{A}^{\mathrm{T}}(c_t)P(c_t) + P(c_t)\bar{A}(c_t) + \sum_{j=1}^{N} \lambda_{c_t j} P(j)]\eta(t) \\
&+ 2\eta^{\mathrm{T}}(t)P(c_t)\bar{A}_d(c_t)\eta(t - \tau_{c_t}) + 2\eta^{\mathrm{T}}P(c_t)\bar{B}(c_t)w(t) \\
&+ 2\eta^{\mathrm{T}}(t)P(c_t)\bar{G}g(x(t)) + \eta^{\mathrm{T}}(t)Q\eta(t) \\
&- (1-d)\eta^{\mathrm{T}}(t - \tau_{c_t})Q\eta(t - \tau_{c_t}) + \bar{\lambda}(\bar{\tau} - \underline{\tau})\eta^{\mathrm{T}}(t)Q\eta(t).
\end{aligned} \tag{22}
$$

For the purpose of stability analysis, we let $w(t) \equiv 0$. Then without loss of generality, we assume $c_t = i$, denoted as i_t. Following (22), we have

$$
\mathscr{L}V(\eta(t), i_t) \leq \zeta_i^T(t) \Psi_i \zeta_i(t), \tag{23}
$$

where $\zeta_i(t) = [\eta^T(t)\ \eta^T(t - \tau_i)\ g^T(x(t))]^T$ and

$$
\Psi_i = \begin{bmatrix} \Xi_i & P(i)\bar{A}_d(i) & P(i)\bar{G} \\ * & -(1-d)Q & 0_{2n \times n} \\ * & * & 0_{n \times n} \end{bmatrix},
$$

with Ξ_i defined in the statement of Theorem 1.
Then we have

$$
\mathbb{E}[\mathscr{L}V(\eta(t), i_t)] \leq \zeta_i^T(t) \bar{\Psi}_i \zeta_i(t), \tag{24}
$$

$$
\bar{\Psi}_i = \begin{bmatrix} \Xi_i & P(i)\hat{A}_d(i) & P(i)\bar{G} \\ * & -(1-d)Q & 0_{2n \times n} \\ * & * & 0_{n \times n} \end{bmatrix},
$$

with $\hat{A}_d(i)$ defined in the statement of Theorem 1.
We next consider the sector-bounded non-linearity in (9). Following (2) and letting $a = x(t) = I_0\eta(t)$ and $b = 0$, we have

$$
\begin{aligned}
&[g(x(t)) - 0 - R_1 x(t)]^{\mathrm{T}}[g(x(t)) - 0 - R_2 x(t)] \\
&= \begin{bmatrix} \eta(t) \\ g(x(t)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} I_0^T \hat{R}_1 I_0 & I_0^T \hat{R}_2 \\ \hat{R}_2 I_0 & I \end{bmatrix} \begin{bmatrix} \eta(t) \\ g(x(t)) \end{bmatrix} \leq 0
\end{aligned} \tag{25}
$$

From (24) and (25), it follows that

$$
\mathbb{E}[\mathscr{L}V(\eta(t), i_t)] \leq \zeta_i^T(t) \Pi_i \zeta_i(t), \tag{26}
$$

where

$$
\Pi_i = \begin{bmatrix} \Xi_i - \varrho_1 I_0^T \hat{R}_1 I_0 & P(i)\hat{A}_d(i) & P(i)\bar{G} - \varrho_1 I_0^T \hat{R}_2 \\ * & -(1-d)Q & 0_{2n \times n} \\ * & * & -\varrho_1 I_n \end{bmatrix}, \tag{27}
$$

with ϱ_1 being a positive constant. It is clear that the conditions in Theorem 1 can guarantee $\Pi_i < 0$, which leads to $\mathbb{E}[\mathscr{L}V(\eta(t), i_t)] < 0$.
Following from (26), we have

$$
\mathbb{E}[\mathscr{L}V(\eta(t), i_t)] \leq -\min_{i \in \mathscr{C}} [\lambda_{\min}(-\Pi_i)] \eta^T(t) \eta(t), \tag{28}
$$

where $\lambda_{\min}(\cdot)$ represents the minimum eigenvalue operator. Then applying Dynkin's formula yields

$$\begin{aligned}
&\mathbb{E}[V(\eta(t), i_t)] - \mathbb{E}[V(\eta(0), 0)] \\
&= \mathbb{E}\Big[\int_0^t \mathscr{L}V(\eta(s), i_s)\,\mathrm{d}s\,|\,(c_0, \phi(\cdot))\Big] \\
&\le -\min_{i \in \mathscr{C}} [\lambda_{\min}(-\Pi_i)]\mathbb{E}\Big[\int_0^t \eta^T(s)\eta(s)\,\mathrm{d}s\,|\,(c_0, \phi(\cdot))\Big],
\end{aligned} \tag{29}$$

which implies that

$$\begin{aligned}
&\min_{i \in \mathscr{C}} [\lambda_{\min}(-\Pi_i)]\mathbb{E}\Big[\int_0^t \eta^T(s)\eta(s)\,\mathrm{d}s\,|\,(c_0, \phi(\cdot))\Big] \\
&\le \mathbb{E}[V(\eta(0), c_0)] - \mathbb{E}[V(\eta(t), i_t)] \\
&\le \mathbb{E}[V(\eta(0), c_0)]\,.
\end{aligned} \tag{30}$$

Then from (30), it follows that

$$\mathbb{E}\Big[\int_0^t \eta^T(s)\eta(s)\,\mathrm{d}s\,|\,(c_0, \phi(\cdot))\Big] \le \frac{\mathbb{E}[V(\eta(0), c_0)]}{\min_{i \in \mathscr{C}} [\lambda_{\min}(-\Pi_i)]}\,. \tag{31}$$

Based on the stability definition in (10), the error system in (9) is stochastically stable. □

We next give sufficient conditions for the existence of the estimators such that the estimation error system satisfies the H_∞ performance bound.

Theorem 2: Let $K_A(i)$, $K_B(i)$, $K_C(i)$, $i \in \mathscr{C}$, be given estimator parameters. Let γ be a prescribed positive scalar. Then the estimation error system in (9), with $w(t) \equiv 0$, is stochastically stable and the H_∞ performance in (11) is satisfied if there exist positive definite matrices $P(1), P(2), \ldots, P(N)$, Q and positive constant ϱ_1 such that

$$\begin{bmatrix} \Pi_i & \Phi_2(i) \\ * & \Phi_3(i) \end{bmatrix} < 0 \tag{32}$$

for all $i = 1, 2, \ldots, N$, where

$$\Phi_2(i) = \begin{bmatrix} P(i)\hat{B}_w & \bar{C}^T(i) \\ 0_{3n \times q} & 0_{3n \times m} \end{bmatrix}$$

$$\hat{B}_w(i) = \begin{bmatrix} B_w \\ K_B(i)\bar{\Theta}_i D \end{bmatrix}, \quad \Phi_3(i) = \mathrm{diag}\{-\gamma^2 I_q, -I_m\},$$

and Π_i, $\hat{R}_1$, $\hat{R}_2$, I_0 and $\hat{A}_d(i)$ are defined the same as in Theorem 1.

Proof: Using Schur complement, it is clear that (32) implies (12). According to Theorem 1, the estimation system in (9) is stochastically stable.

Now let us define the following H_∞ performance cost function

$$J_T = \mathbb{E}\Big[\int_0^T (e^T(t)e(t) - \gamma^2 w^T(t)w(t))\,\mathrm{d}t\Big], \quad \forall T > 0. \tag{33}$$

Then

$$\begin{aligned}
J_T &= \mathbb{E}\Big[\int_0^T (e^T(t)e(t) - \gamma^2 w^T(t)w(t)) + \mathscr{L}V(\eta(t), i_t)\,\mathrm{d}t\Big] \\
&\quad - \mathbb{E}\Big[\int_0^T \mathscr{L}V(\eta(t), i_t)\,\mathrm{d}t\Big] \\
&= \mathbb{E}\Big[\int_0^T (e^T(t)e(t) - \gamma^2 w^T(t)w(t)) + \mathscr{L}V(\eta(t), i_t)\,\mathrm{d}t\Big] \\
&\quad - \mathbb{E}[V(\eta(t), i_t)] + \mathbb{E}[V(\eta(0), i_0)]
\end{aligned} \tag{34}$$

Under zero initial conditions and using (25), we have

$$\begin{aligned}
J_T &\le \mathbb{E}\Big[\int_0^T (e^T(t)e(t) - \gamma^2 w^T(t)w(t)) + \mathscr{L}V(\eta(t), i_t)\,\mathrm{d}t\Big] \\
&\le \xi_i^T(t)\Gamma(i)\xi_i(t),
\end{aligned} \tag{35}$$

where

$$\xi_i(t) = [\eta^T(t),\ \eta^T(t - \tau_i(t)),\ g^T(x(t)),\ w^T(t)]^{\mathrm{T}}$$

$$\Gamma(i) = \begin{bmatrix} \Pi_i + \begin{bmatrix} \bar{C}^T(i) \\ 0_{3n \times m} \end{bmatrix}[\bar{C}(i) \quad 0_{m \times 3n}] & \begin{bmatrix} P(i)\hat{B}_w \\ 0_{3n \times q} \end{bmatrix} \\ * & -\gamma^2 I_q \end{bmatrix}.$$

with Π_i being defined in (27). Applying Schur complement, it is straightforward to show that (32) implies $J_T < 0$, $\forall T$, which can be used to show that

$$\mathbb{E}[\,\| e(t) \|_2^2\,] \le \gamma^2 \| w(t) \|_2^2\,. \tag{36}$$

This completes the proof of Theorem 2. □

Remark 2: When the number of channels $N = 1$, the setup reduces to the conventional case that only one single channel is used, cf. [1, 31–33]. Results in Theorem 2 are thus more general than those in the existing literature.

4 Estimator design

In this section, we solve the H_∞ estimation problem for system (1), that is, find estimator gains $K_A(i)$, $K_B(i)$ and $K_C(i)$ in forms of (8) such that the error system in (9) is stochastically stable with a guaranteed H_∞ bound. The following theorem gives sufficient conditions on the existence of such an estimator.

Theorem 3: Consider the non-linear system (1) and the coexistence of switching MCCs. Let γ be a given constant representing desired attenuation level. There exists a desired H_∞ estimator in forms of (8) such that the error system in (9) is stochastically stable and has an H_∞ performance index γ, if there exist positive-definite matrices

$$P(i) = \begin{bmatrix} P_1(i) & P_2(i) \\ * & P_2(i) \end{bmatrix}, \quad Q = \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix},$$

a positive scalar ϱ_1, and matrices $\bar{K}_A(i)$, $\bar{K}_B(i)$, $K_C(i)$, $i = 1, 2, \ldots, N$, such that

$$\begin{bmatrix} \Omega_1(i) & \Omega_2(i) & \Omega_3(i) \\ * & -(1-d)Q & 0_{2n \times (n+q+m)} \\ * & * & \Omega_4(i) \end{bmatrix} < 0 \tag{37}$$

for all $i = 1, 2, \ldots, N$, where

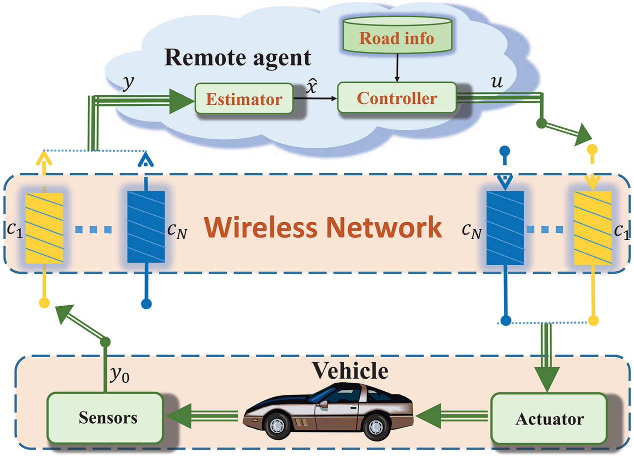

Fig. 2 *Active suspension control system with a remote agent*

$$\Omega_1(i) = \begin{bmatrix} \Omega_{11}(i) & \Omega_{12}(i) \\ * & \Omega_{22}(i) \end{bmatrix}, \quad \Omega_2(i) = \begin{bmatrix} \bar{K}_B(i)\bar{\Theta}_i C & 0_{n\times n} \\ \bar{K}_B(i)\bar{\Theta}_i C & 0_{n\times n} \end{bmatrix},$$

$$\Omega_3(i) = \begin{bmatrix} \Omega_{15}(i) & \Omega_{16}(i) & L^T \\ P_2(i)G & \Omega_{26}(i) & -K_C^T(i) \end{bmatrix},$$

$$\Omega_4(i) = \mathrm{diag}\{-\varrho_1 I_n, \quad -\gamma^2 I_q, \quad -I_m\},$$

$$\Omega_{11}(i) = \mathrm{Sym}(P_1(i)A) + \sum_{j=1}^{N} \lambda_{ij} P_1(j) + \kappa Q_1 - \varrho_1 \hat{R}_1,$$

$$\Omega_{12}(i) = \bar{K}_A(i) + A^T P_2(i) + \sum_{j=1}^{N} \lambda_{ij} P_2(j) + \kappa Q_2,$$

$$\Omega_{15}(i) = P_1(i)G - \varrho_1 \hat{R}_2,$$

$$\Omega_{16}(i) = P_1(i)B_w + \bar{K}_B(i)\bar{\Theta}_i D,$$

$$\Omega_{22}(i) = \mathrm{Sym}(\bar{K}_A(i)) + \kappa Q_3,$$

$$\Omega_{26}(i) = P_2(i)B_w + \bar{K}_B(i)\bar{\Theta}_i D.$$

Furthermore, if $(P(i), Q, \bar{K}_A(i), \bar{K}_B(i), K_C(i), \varrho_1)$, $i \in \mathscr{C}$ is a feasible solution of (37), then the estimator parameters in (8) are given as

$$K_A(i) = P_2^{-1}(i)\bar{K}_A(i), \quad K_B(i) = P_2^{-1}(i)\bar{K}_B(i). \tag{38}$$

Proof: According to Theorem 2, the error system in (9) is stochastically stable and $\mathbb{E}[\, \| e(t) \|_2 \,] < \gamma \mathbb{E}[\, \| w(t) \|_2 \,]$ for all $w(t) \in L_2[0, \infty)$, if there exist positive matrices $P(1), P(2), \ldots, P(N)$, Q, and a positive scalar ϱ_1 such that (12) holds. For the estimator synthesis procedure,we first partition the Lyapunov matrices $P(i)$ as

$$P(i) = \begin{bmatrix} P_1(i) & P_2(i) \\ * & P_3(i) \end{bmatrix}. \tag{39}$$

Then performing a congruence transformation to $P(i)$ by $\mathrm{diag}\{I, P_2(i)P_3^{-1}(i)\}$ yields

$$\begin{bmatrix} P_1(i) & P_2(i)P_3^{-1}(i)P_2^T(i) \\ * & P_2(i)P_3^{-1}(i)P_2^T(i) \end{bmatrix}.$$

As a result, we can, without loss of generality, directly specify the matrices as

$$P(i) = \begin{bmatrix} P_1(i) & P_2(i) \\ * & P_2(i) \end{bmatrix}. \tag{40}$$

(40) into (32) and introducing

$$\bar{K}_A(i) = P_2(i)K_A(i), \quad \bar{K}_B(i) = P_2(i)K_B(i),$$

then we are ready to show that (37) is equivalent to (32). We note that $P(i) > 0$ requires $P_2(i) > 0$. Then the estimator gains can be constructed by (38). This completes the proof. □

Remark 3: In Theorem 3, the designed estimators are mode dependent, i.e. the estimator gains switch as channels switch.

5 Case study

In this section, we consider a network-based state estimation problem for an agent-based active suspension system as illustrated in Fig. 2. The estimated states are used in the suspension control module in the remote agent, where the road profile information is stored a priori. Road information from the agent is used for suspension control, enabling the functionality to plan and optimally respond to the road information. In addition, control optimisation is performed in the remote server which has much higher computation capability compared with onboard computation units. More details on this configuration of the investigated suspension control system can be referred to [34, 35], in which cloud-aided suspension control problems have been studied.

In Fig. 2, the remote vehicle software agent receives vehicle sensor measurements. The signals are sent to the remote agent via N available wireless channels $c_1, \ldots, c_N$. When the vehicle moves, it is supposed that only one channel is selected based on a (sub)optimal scheduler and the overall channel switching is assumed to be governed by a continuous-time Markov process.

Here a quarter-car active suspension model, with 2 degrees of freedom (DOF) as shown in Fig. 3, is used. The M_s and M_{us} represent the car body (sprung) mass and the tire and axles (unsprung mass), respectively. The spring, shock absorber and an actuator constitute the suspension system, connecting sprung (body) and unsprung (wheel assembly) masses. The tire is modelled as a spring with stiffness k_{us} and its damping ratio is assumed to be negligible in the suspension formulation. From Fig. 3, we have the following equations of motion:

$$\begin{aligned} \dot{x}_1 &= x_2 - w, \\ M_{us}\dot{x}_2 &= -k_{us}x_1 + k_s x_3 + S(c_s(x_4 - x_2)) + u, \\ \dot{x}_3 &= x_4 - x_2, \\ M_s \dot{x}_4 &= -k_s x_3 - S(c_s(x_4 - x_2)) - u, \end{aligned} \tag{41}$$

where x_1 is the tire deflection from equilibrium, x_2 is the unsprung mass velocity, x_3 is the suspension deflection from equilibrium, x_4 is the sprung mass velocity, w represents the road disturbance, c_s is the constant damping coefficient, u is the controlled actuator force and k_s is suspension stiffness.

It is worth mentioning that, in practice, suspension dampers are subject to saturations and are typically sector bounded in nature [36]. This phenomenon is captured in the non-linear term $S(c_s(x_4 - x_2))$ and we assume $S(\alpha(t)) = \alpha(t) + 0.1\alpha(t)\sin(\alpha(t))$.

By defining $x = [x_1 \quad x_2 \quad x_3 \quad x_4]^{\mathrm{T}}$, the suspension system model in (41) can be written as

$$\dot{x} = Ax + Bu + B_0\omega + Gg(x), \tag{42}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_{us}}{M_{us}} & -\frac{c_s}{M_{us}} & \frac{k_s}{M_{us}} & \frac{c_s}{M_{us}} \\ 0 & -1 & 0 & 1 \\ 0 & \frac{c_s}{M_s} & -\frac{k_s}{M_s} & -\frac{c_s}{M_s} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{1}{M_{us}} \\ 0 \\ -\frac{1}{M_s} \end{bmatrix}, \tag{43}$$

$$G = \begin{bmatrix} 0 & 0.1c_sC_0^T & 0 & -0.1c_sC_0^T \end{bmatrix}^{\mathrm{T}}, \quad B_0 = [-1\ 0\ 0\ 0]^{\mathrm{T}},$$

$$C_0 = [0 \quad -1 \quad 0 \quad 1], \quad g(x) = \sin(c_sC_0x)x\,.$$

It can be shown that $g(x)$ is sector bounded with $R_1 = I_4$ and $R_2 = -I_4$, where R_1 and R_2 are matrix bounds defined in (2).

For vehicles equipped with active suspension, in general, the measurements of suspension deflection (x_3) and body velocity (x_4) are available, while vertical wheel velocity (x_2) and tire deflection (x_1) are not measured. Let y_0 denote the onboard measurements and z denote the objective signal to be estimated, we have

$$\begin{aligned} \dot{x} &= Ax + Bu + B_0\omega + Gg(x), \\ y_0 &= [x_3 \quad x_4]^{\mathrm{T}} = Cx + D_0v, \\ z &= x, \end{aligned} \tag{44}$$

where $C = [0_{2\times 2} \quad I_2]$ and v is the zero-mean unit-density white noise.

The estimators on the agent will use the received measurement y for state estimation. The received measurements will be delayed and possibly lost during the transmission via the wireless communication channels. Thus, we have

$$y(t) = \Theta_{c_t}Cx(t - \tau_{c_t}(t)) + D_0v(t), \tag{45}$$

where $\tau_{c_t}(t)$ and Θ_{c_t} are, respectively, the time-varying delay and packet dropout matrix defined in Section 2.

Defining a disturbance vector as $\xi = [w^T \quad v^T]^{\mathrm{T}}$ and letting $u \equiv 0$ for estimator design purposes, we have the following system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_w\xi(t) + Gg(x(t)), \\ y(t) &= \Theta_{c_t}Cx(t - \tau_i) + \Theta_{c_t}D\xi(t), \\ z(t) &= x(t), \end{aligned} \tag{46}$$

with $B_w = [B_0 \quad 0], \quad D = [0 \quad D_0]$, which is now in the form of (7).

Remark 4: Automobiles with wireless connections are increasingly popular in the industry, see for example [37, 38]. The mobility of the automobiles demands high reliability of the communication channels and the MCC framework is quite practical and advantageous for this architecture.

For simulation purposes, the system parameters used are listed in Table 1 and we specify

$$R_1 = I_4, \quad R_2 = -I_4, \quad D_0 = [0.1 \quad 0.2]^{\mathrm{T}},$$

where R_1 and R_2 are matrix bounds defined in (2).

We consider three available channels with the following switching and delay characteristics:

$$\Lambda = \begin{bmatrix} -3 & 2 & 1 \\ 2 & -5 & 3 \\ 3 & 4 & -7 \end{bmatrix}, \quad d = 0.5, \quad \bar{\tau}_1 = 0.15, \quad \underline{\tau}_1 = 0.6,$$

$$\bar{\tau}_2 = 0.2, \quad \underline{\tau}_2 = 0.55, \quad \bar{\tau}_3 = 0.25, \quad \underline{\tau}_3 = 0.4,$$

where time delay of each channel i, $i = 1, 2, 3$ is set to be uniformly distributed over $[\underline{\tau}_i, \bar{\tau}_i]$. The packet arrival rate matrix of each channel is specified as

Table 1 Suspension system parameters

m_s	m_{us}	k_s	k_{us}	c_s
290 kg	60 kg	16, 800 N/m	19, 000 N/m	200 N · s/m

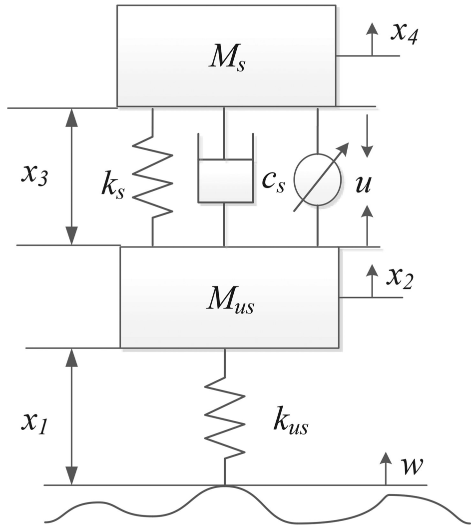

Fig. 3 *Active suspension dynamics*

$$\bar{\Theta}_1 = \mathrm{diag}\{0.9,\ 0.92\}, \quad \bar{\Theta}_2 = \mathrm{diag}\{0.91,\ 0.89\},$$

$$\bar{\Theta}_3 = \mathrm{diag}\{0.89,\ 0.93\}\,.$$

The road disturbance over a 10-s horizon is modelled as follows:

$$w(t) = \begin{cases} 0.15 \cdot \sin\pi(t-1) & 1s \le t \le 3s, \\ 0.05 \cdot \sin\pi/2t & 4s \le t \le 8s, \\ 0 & \text{otherwise}\,. \end{cases} \tag{47}$$

We assume that the disturbance to the suspension is with a similar frequency of the spring in a passenger car suspension. Moreover, in [39], the disturbance to the suspension is modelled as known uneven road information. Taking into account both the frequency range of passenger car suspensions, and the frequency and the magnitude of the modelled disturbance in terms of uneven road conditions in [39], we assume that the disturbance to the suspension system studied in this work is in the form of (47).

Using Theorem 3 and the Matlab LMI toolbox, simulation results are generated. The parameters of the designed estimators for the three communication channels are calculated as

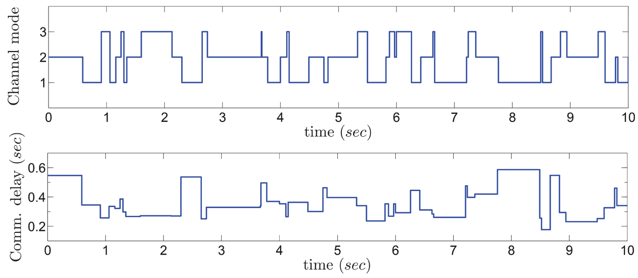

Fig. 4.

Fig. 4 *Channel switching and communication delays*

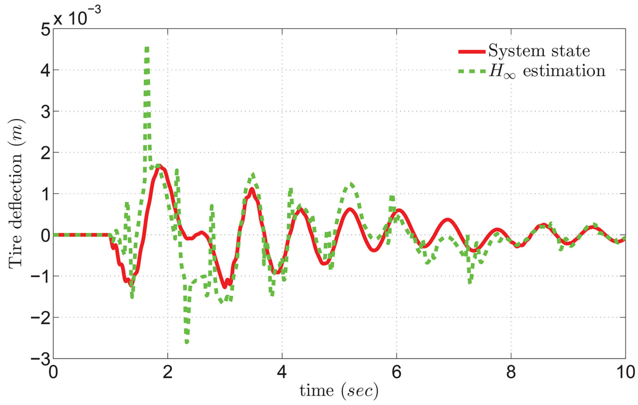

Fig. 5.

Fig. 5 *Tire deflection x_1 estimation*

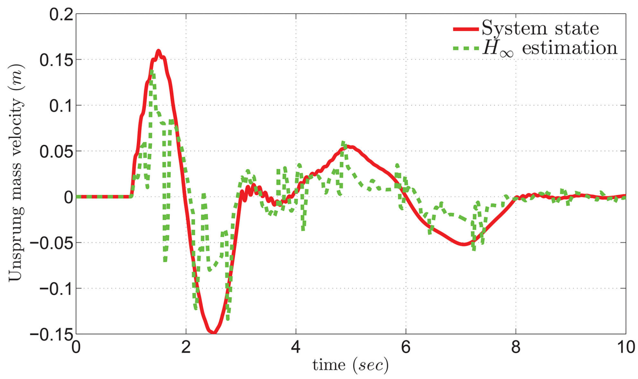

Fig. 6.

Fig. 6 *Unsprung mass velocity x_2 estimation*

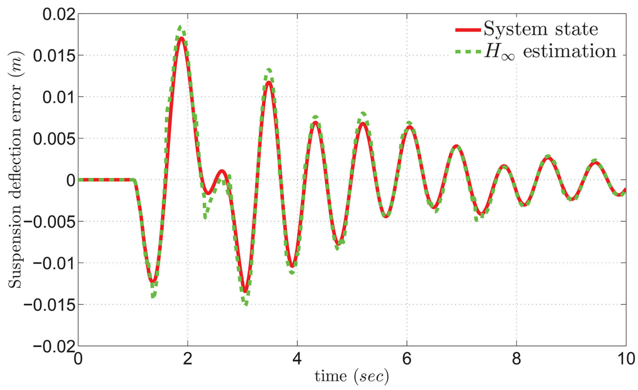

Fig. 7.

Fig. 7 *Suspension deflection x_3 estimation*

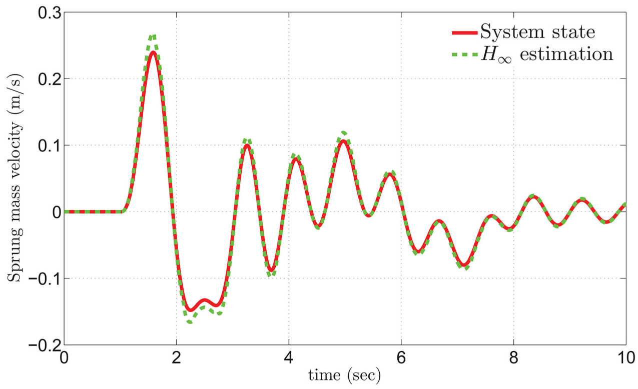

Fig. 8.

Fig. 8 *Sprung mass velocity x_4 estimation*

$$KA(1) = \begin{bmatrix} 0.2375 & -0.1948 & -1.2690 & 0.0940 \\ -0.8577 & 0.7220 & 4.6922 & -0.3318 \\ 0.1599 & -0.1353 & -0.8783 & 0.0622 \\ -0.2241 & 0.1868 & 1.2069 & -0.0933 \end{bmatrix} \times 10^4$$

$$KB(1) = \begin{bmatrix} 0.5412 & -0.4838 \\ -1.9838 & 1.6876 \\ 0.3716 & -0.3169 \\ -0.5559 & 0.4917 \end{bmatrix} \times 10^4$$

$$KA(2) = \begin{bmatrix} 0.2360 & -0.1922 & -1.2444 & 0.0891 \\ -0.8523 & 0.7102 & 4.6549 & -0.3146 \\ 0.1604 & -0.1333 & -0.8712 & 0.0589 \\ -0.2226 & 0.1833 & 1.1981 & -0.0886 \end{bmatrix} \times 10^4$$

$$KB(2) = \begin{bmatrix} 0.5307 & -0.4623 \\ -1.9443 & 1.6112 \\ 0.3643 & -0.3026 \\ -0.5456 & 0.4704 \end{bmatrix} \times 10^4$$

$$KA(3) = \begin{bmatrix} 0.2151 & -0.0809 & -0.6805 & 0.0533 \\ -0.7775 & 0.3055 & 2.5344 & -0.1896 \\ 0.1449 & -0.0574 & -0.4747 & 0.0355 \\ -0.2024 & 0.0732 & 0.6174 & -0.0553 \end{bmatrix} \times 10^4$$

$$KB(3) = \begin{bmatrix} 2.5878 & -1.8385 \\ -9.4865 & 6.4207 \\ 1.7805 & -1.2126 \\ -2.7322 & 2.1207 \end{bmatrix} \times 10^3$$

The channel switching signal and the communication delays are shown in Fig.4, while the state estimates obtained based on the developed state estimators are presented in Figs. 5–8. In Figs. 5–8, we use red solid lines to represent the actual states of the suspension system and use blue-dashed lines to depict the trajectories of the obtained state estimates. It can be seen that the designed estimators are able to collaboratively give sufficiently accurate state estimates in the presence of road disturbances within the channel-switching framework. The results also imply that time-varying communication delays and intermittent packet dropouts can be well handled.

By examining the trajectories of the state estimates and the actual system states in Figs. 5–8, we can see that the estimation performance for the states x_3 and x_4 is better than that of the rest two states x_1 and x_2. This is mainly because that the states x_3 and x_4 are measured even though the measurements are subject to measurement noise. Therefore, direct information with respect to these two states is provided to the estimators for the state estimate evaluation.

6 Conclusions

The problem of network-based H_∞ estimation for a class of non-linear systems with MCCs subject to Markovian switching has been treated. Time-varying communication delays, random packet dropouts and sector-bounded non-linearities have been considered in a unified framework. The Lyapunov–Krasovskii approach and LMI techniques were exploited to establish the existence of the estimator and further derive the estimator parameters. A case study of state estimation for an intelligent active suspension system has been investigated and simulation results have been reported to demonstrate the effectiveness of the proposed approach.

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