

Random two-step phase shifting interferometry based on Lissajous ellipse fitting and least squares technologies

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Abstract: To accurately obtain the phase distribution of an optical surface under test, the accurate phase extraction algorithm is essential. To overcome the phase shift error, a random two-step phase shifting algorithm, which can be used in the fluctuating and non-uniform background intensity and modulation amplitude, Lissajous ellipse fitting, and least squares iterative phase shifting algorithm (LEF&LSI PSA), is proposed; pre-filtering interferograms are not necessary, but they can get relatively accurate phase distribution and unknown phase shift value. The simulation and experiment verify the correctness and feasibility of the LEF & LSI PSA.

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1. Introduction

Interferometry is the industry standard for optical measurement [1]. The phase-shifting interferometer (PSI) was introduced by Brunning [2] to achieve accurate optical metrology in 1974, PSI and its variations have been widely used [1,3,4]. For the standard phase-shifting algorithm (PSA), its accuracy depends on the accuracy of the phase shift [4–6], which should be a special constant (e.g. $\pi/2$). However, the actual phase shift may be different from the preset value because of the errors caused by the miscalibration of piezo-transducer (PZT), vibrational error, air turbulence in the working environment, instability of the laser frequency, and so on [7–9].

To overcome the phase shift error, several PSAs have been proposed [10,11] and they can be divided to two types. The first type is non-iterative method which can deal with the random phase shifted interferograms. While it is relatively fast to obtain the phase, the accuracy may not be high enough. In 1992 Farrell and Player [12] utilized Lissajous figures and ellipse fitting to calculate the phase difference between two interferograms, but the correction result is not accurate if the intensity distribution is non-uniform. From 2003 to 2014, Cai et al. [13–21] proposed a series of statistical algorithms which can extract the phase shifts and tested phase, however most of these algorithms need to know the intensity of object and reference. In 2016, Xu et al. [22] proposed a simple and rapid Euclidean matrix norm algorithm to retrieve the unknown phase shifts and phase in three-frame generalized phaseshifting interferometry, however this algorithm needs three phase-shifted interferograms. In 2016, Liu et al. [23] proposed a PSA which can simultaneously extract the tested phase and phase shift from only two interferograms using Lissajous figure and ellipse fitting technology, but the two interferograms used in this algorithm need to be filtered by the Hilbert-Huang pre-filtering.

The second type is the iterative method which can extract the unknown phase shift and tested phase from a series of phase shifting interferograms. It takes more time than the noniterative algorithm, but its accuracy is higher generally. In 2004, an advanced random PSA based on a least-squares iterative procedure was proposed [24], it copes with the limitation of the existing iterative algorithms by separating a frame-to-frame iteration from a pixel-to-pixel iteration, and it provides stable convergence and accurate phase extraction even when the

phase shifts are completely random. In 2008, Xu et al. [25] presented an advance iterative algorithm to extract phase distribution from random and spatial non-uniform phase-shifted interferograms, this algorithm divides the interferograms into small blocks and retrieves local phase shifts accurately by iterations. In 2013, an iterative phase-shifting algorithm based on the least-squares principle was developed to overcome the random piston and tilt wavefront errors generated from the phase shifter [26]. However, all the above iterative algorithms need at least three interferograms.

Recently, we proposed a method which can correct fringe-print-through (FPT) error in snapshot phase-shifting interference microscope based on the 4-step PSA and Lissajous ellipse fitting [27], this algorithm uses all four phase-shifted interferograms and can be applied in the different intensity distribution conditions, even the intensity distribution is non-uniform, it can also correct the FPT error by only one measurement. However, it is only suitable for the 4-step PSA, and the phase shift should be a constant ($\pi/2$).

For general two-step PSA, especially when the phase shift is unknown, it is difficult to obtain high accurate phase because of the non-uniform background intensity and modulation amplitude of different pixels and interferograms. In this paper, we will discuss the fast and accurate two-step PSA with unknown phase shift. Section 2 presents the principle and process of the proposed PSA - Lissajous ellipse fitting and least squares iterative phase shifting algorithm (LEF & LSI-PSA). In Section 3 the simulation of the LEF&LSI-PSA is discussed, Section 4 evaluates the novel algorithm with the experimental data. The conclusion is finally drawn in Section 5.

2. Principles

The intensity of two-frame interferograms can be expressed as

$$I_{1}(x, y) = A_{1}(x, y) + B_{1}(x, y)\cos(\varphi(x, y))$$

$$I_{2}(x, y) = A_{2}(x, y) + B_{2}(x, y)\cos(\varphi(x, y) + \alpha).$$
(1)

where $I_1(x, y)$ and $I_2(x, y)$ are the intensity of two interferograms, $A_1(x, y), A_2(x, y), B_1(x, y)$ and $B_2(x, y)$ respectively represent the background intensity and the modulation amplitude of the two interferograms, φ is the tested phase, and α is the phase shift.

2.1 Principle of two step Lissajous ellipse fitting phase shifting algorithm

Providing that background intensity $A_1(x, y)$ and $A_2(x, y)$ and modulation amplitude $B_1(x, y)$ and $B_2(x, y)$ are independent on the frame of the interferogram and the position of the pixel, we can set $A_1(x, y) = A_2(x, y) = a$, $B_1(x, y) = B_2(x, y) = b$. Equation (1) is rewritten as

$$I_1(x, y) = a + b \cos(\varphi(x, y))$$

$$I_2(x, y) = a + b \cos(\varphi(x, y) + \alpha).$$
(2)

In the above equations, there are four unknowns a, b, $\varphi(x, y)$ and α , typically we need four interferograms to extract the phase. In the following paragraphs, we will introduce the Lissjous ellipse LEF-PSI which can calculate the tested phase and phase shift from only two phase-shifted interferograms.

According to Eq. (2), we compute the sum and difference of $I_1(x, y)$ and $I_2(x, y)$,

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$$I_{dif} = I_1 - I_2 = 2b \sin\left(\varphi + \frac{\alpha}{2}\right) \sin\left(\frac{\alpha}{2}\right)$$

$$I_{sum} = I_1 + I_2 = 2a + 2b \cos\left(\varphi + \frac{\alpha}{2}\right) \cos\left(\frac{\alpha}{2}\right).$$
(3)

where the spatial dependency (x,y) has been omitted to simplify the equations. Then we can obtain

$$\sin\left(\varphi + \frac{\alpha}{2}\right) = \frac{I_{dif}}{2b\sin\left(\frac{\alpha}{2}\right)}$$

$$\cos\left(\varphi + \frac{\alpha}{2}\right) = \frac{I_{sum} - 2a}{2b\cos\left(\frac{\alpha}{2}\right)}.$$
(4)

Because $\sin^2\left(\varphi + \frac{\alpha}{2}\right) + \cos^2\left(\varphi + \frac{\alpha}{2}\right) = 1$, Eq. (3) can be rewritten as

$$\left(\frac{I_{dif} - x_0}{a_x}\right)^2 + \left(\frac{I_{sum} - y_0}{a_y}\right)^2 = 1.$$
(5)

Note that Eq. (5) is just an ellipse equation. If no error exists,

$$a_x = 2b\sin\left(\frac{\alpha}{2}\right), a_y = 2b\cos\left(\frac{\alpha}{2}\right), x_0 = 0, y_0 = 2a.$$
 (6)

According to Eq. (5), a general conic function can be obtained

$$\frac{I_{dif}^{2}}{a_{x}^{2}} + \frac{I_{sum}^{2}}{a_{y}^{2}} - 2\frac{x_{0}I_{dif}}{a_{x}^{2}} - 2\frac{y_{0}I_{sum}}{a_{y}^{2}} + \frac{x_{0}^{2}}{a_{x}^{2}} + \frac{y_{0}^{2}}{a_{y}^{2}} - 1 = 0.$$
 (7)

A general conic function can be also expressed by the following second order polynomial:

$$F = ax^{2} + bxy + cy^{2} + dx + fy + g.$$
 (8)

For an ellipse, Eq. (8) needs to meet the conditions of F = 0 and $b^2 - 4ac < 0$. According to Eqs. (7) and (8), the semi-major amplitude a_x , semi-minor amplitude a_y , the center offset x_0 and y_0 can be calculated as

$$a_{x} = \sqrt{2\frac{af^{2} + cd^{2} + gb^{2} - bdf - 4acg}{(b^{2} - 4ac)(\sqrt{(a - c)^{2} + b^{2}} - (a + c))}}, a_{y} = \sqrt{2\frac{af^{2} + cd^{2} + gb^{2} - bdf - 4acg}{(b^{2} - 4ac)(-\sqrt{(a - c)^{2} + b^{2}} - (a + c))}}$$
(9)
$$x_{0} = \frac{2cd - bf}{b^{2} - 4ac}, y_{0} = \frac{2af - bd}{b^{2} - 4ac}.$$

From Eqs. (6) and (9), the unknown random phase shift α and tested phase φ can be easily calculated as

$$\alpha = 2\tan^{-1} \left(\frac{a_x}{a_y} \right). \tag{10}$$

$$\varphi = \tan^{-1} \left(\frac{I_{dif} - x_0}{I_{sum} - y_0} \cdot \frac{a_y}{a_x} \right) - \tan^{-1} \left(\frac{a_x}{a_y} \right).$$
(11)

Provided that there are no background intensity and modulation amplitude errors, x_0 will be equal to zero, Eq. (11) can be rewritten as

$$\varphi = \tan^{-1} \left(\frac{I_{dif}}{I_{sum} - y_0} \cdot \frac{a_y}{a_x} \right)$$
(12)

because $\tan^{-1}\left(\frac{a_x}{a_y}\right)$ is a piston which doesn't affect the result and can be ignored. However,

the background intensity and modulation amplitude vary between the phase shifted interferograms and individual pixels, leading to the errors in calculated phase and phase shift.

2.2 Principle of least squares algorithm with preset phase

The least squares algorithm (LSA) with preset phase introduced in this section is directly adapted for the completeness of this paper.

Provided that the phase of the tested optical surface is preset as a column vector $[\varphi_1, \varphi_2, \dots, \varphi_k, \dots, \varphi_N]$, the intensity of the phase-shifted interferogram is

$$I'_{i,j} = A_{i,j} + B_{i,j} \cos(\varphi_j + \alpha_i).$$
(13)

where *i* is the number of the interferogram (i = 1, 2, 3, ..., M), *j* is the number of the pixel in one interferogram (j = 1, 2, 3, ..., N), φ_j is the phase of the pixel *j*, and α_i is the phase shift of the interferogram *i*.

Provided that the background intensity $A_{i,j}$ and modulation amplitude $B_{i,j}$ are irrelevant to *j*, only relevant to *i*, so $A_{i,1}=A_{i,2}=\cdots=A_{i,N}=A_i$, $B_{i,1}=B_{i,2}=\cdots=B_{i,N}=B_i$. By setting $a_i = A_i$, $b_i = B_i \cos \alpha_i$, and $c_i = -B_i \sin \alpha_i$, Eq. (13) becomes

$$I'_{i,j} = a_i + b_i \cos \varphi_j + c_i \sin \varphi_j.$$
⁽¹⁴⁾

The squared sum of the differences between the theoretical intensity and actual intensity of the interferogram can be expressed as

$$S_{i} = \sum_{j=1}^{N} \left(I_{i,j}' - I_{i,j} \right)^{2} = \sum_{j=1}^{N} \left(a_{i} + b_{i} \cos \varphi_{j} + c_{i} \sin \varphi_{j} - I_{i,j} \right)^{2}.$$
 (15)

According to the least squares theory [25, 26], S_i should be minimum when $\partial S_i / \partial a_i = 0$, $\partial S_i / \partial b_i = 0$, $\partial S_i / \partial c_i = 0$, so

$$X_i = S_i^{-1} R_i. aga{16}$$

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$$S_{i} = \begin{bmatrix} N & \sum_{j=1}^{N} \cos \varphi_{j} & \sum_{j=1}^{N} \sin \varphi_{j} \\ \sum_{j=1}^{N} \cos \varphi_{j} & \sum_{j=1}^{N} \cos^{2} \varphi_{j} & \sum_{j=1}^{N} \sin \varphi_{j} \cos \varphi_{j} \\ \sum_{j=1}^{N} \sin \varphi_{j} & \sum_{j=1}^{M} \sin \varphi_{j} \cos \varphi_{j} & \sum_{j=1}^{N} \sin^{2} \varphi_{j} \end{bmatrix}.$$
 (17)

$$X_i = \begin{bmatrix} a_i & b_i & c_i \end{bmatrix}^T.$$
 (18)

$$R_{i} = \left[\sum_{j=1}^{N} I_{i,j} \quad \sum_{j=1}^{N} I_{i,j} \cos \varphi_{j} \quad \sum_{j=1}^{N} I_{i,j} \sin \varphi_{j}\right]^{T}.$$
 (19)

 b_i and c_i can be obtained by Eq. (16), and the phase shift can be calculated by

$$\alpha_i = \tan^{-1} \left(-\frac{c_i}{b_i} \right). \tag{20}$$

We can also extract the background intensity and modulation amplitude from Eq. (16), $A_i = a_i$, $B_i = \sqrt{b_i^2 + c_i^2}$.

If we obtain only two phase-shifted interferograms, the phase shift α_1 and α_2 of these two interferograms can be calculated by Eq. (20), the relative phase shift is $\Delta \alpha = \alpha_2 - \alpha_1$.

The LSA has obvious advantage that it can calculate the random phase shift without more than three interferograms, however the accuracy of the preset tested phase distribution is important to obtain accurate phase shift.

2.3 Principle of Lissajous ellipse fitting and least squares iterative phase shifting algorithm

Based on the principles of LEF-PSA and LSA, we propose a novel PSA, namely Lissajous ellipse fitting and least squares iterative phase shifting algorithm (LEF & LSI-PSA), which uses only two phase-shifted interferograms without other information. In order to improve the accuracy of calculation, the iteration is introduced. In the following, we will introduce the algorithm in detail:

- 1) Plot an approximate ellipse with $I_1 I_2$ as the x coordinate and $I_1 + I_2$ as the y coordinate;
- 2) calculate the semi-major amplitude a_x , semi-minor amplitude a_y , the center offset x_0 and y_0 of the Lissajous ellipse using LEF-PSA;
- 3) estimate the initial phase distribution using Eq. (11);
- 4) using the initial phase distribution as the known phase distribution, calculate the relative phase shift $\Delta \alpha$, the background intensity A_1 and A_2 , and modulation amplitude B_1 and B_2 using LSA, then use Eqs. (21) and (22) to calculate the new phase distribution;

$$\cos \varphi = \frac{I_1 - A_1}{B_1}$$

$$\sin \varphi = \frac{\left(\frac{B_2 I_1}{B_1} \cos \Delta \alpha - I_2\right) - \left(\frac{B_2}{B_1} A_1 \cos \Delta \alpha - A_2\right)}{B_2 \sin \Delta \alpha}.$$

$$\varphi = \tan^{-1} \left(\frac{\left(B_2 I_1 \cos \Delta \alpha - B_1 I_2\right) - \left(B_2 A_1 \cos \Delta \alpha - B_1 A_2\right)}{(I_1 - A_1) B_2 \sin \Delta \alpha}\right).$$
(21)

5) repeat step 4) with the new phase distribution until $RMS(\varphi^k - \varphi^{k-1}) < \xi$, the final phase distribution and relative phase shift $\Delta \alpha$ can be obtained.

where ξ is the predefined converging threshold of iteration, i.e., 0.1 nm, and k presents the iterative times.

The whole procedure of the LEF & LSI PSA is illustrated in Fig. 1.



Fig. 1. Flow chart of LEF & LSI-PSA.

3. Simulation

To validate the effectiveness and robustness of the proposed LEF & LSI PSA, we perform 4 simulations under different conditions with the different background intensity and modulation amplitude distribution. We will also compare LEF & LSI-PSA with LEF-PSA.

In the following simulations, we first simulate a tested phase distribution (101 pixels*101pixels) using the Zernike polynomials with 2^{nd} , 3^{rd} , 5^{th} and 10^{th} coefficients of the Zernike polynomials as 1, -1, 0.2, and 0.3 and others as zero as shown in Fig. 2(a), then we use Eq. (1) to generate two interferograms by assigning a random phase shift (e.g., 1.2217 rad) between them (Fig. 2(b) and 2(c)). In the end we calculate the phase distribution and phase shift value using LEF and the proposed LEF & LSI-PSA.

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Fig. 2. Simulated tested surface and two phase shifting interferograms. (a) The tested surface, (b) the first interferogram, and (c) the second interferogram.

In the first simulation, the background intensity and modulation amplitude distribution of two interferograms are uniform $A_1 = A_2 = 1$ and $B_1 = B_2 = 1$. In the second simulation, there are fluctuations in the background intensity and modulation amplitude distribution between different interferograms. In the simulation, we set $A_1 = 1.09$, $A_2 = 1.25$, $B_1 = 0.94$, $B_2 = 1.15$. The third simulation is similar to the first one with $A_1 = A_2 = 1$ and $B_1 = B_2 = 1$, but we add the random noise using *rand* function in Matlab to the two interferograms (the SNR of noise is 20dB). In the last simulation, we add noise with SNR of 20dB to the second simulation.

The results of 4 different simulations are as shown in Table 1, every simulation has two results calculated by LEF and LEF&LSI PSI respectively. The calculated phase distributions are shown in the third row, and the phase errors are displayed in the fourth row. For the LEF & LSI PSA, the last row shows the iterative curves.

From the first simulation, we can see that the phase error is approximately equal to zero for two PSAs when the background intensity and modulation amplitude distribution are perfect, and the calculated phase shift is also equal to the pre-set value (1.2217 rad). Only one iteration is needed for LEF & LSI PSA to converge. In the second simulation, the phase distribution calculated by LEF is not very smooth and the phase error is relatively large (PV = 24.5684 nm, RMS = 7.9062 nm), the obtained phase shift value 1.1704 rad is also away from the pre-set value. In contrast after 7 iterations with LEF&LSI PSA, the retrieved surface is smoother than that from LEF PSA and phase error is decreased to a very small value with a PV value of 0.2098 nm and a RMS value of 0.0643 nm. In addition, the calculated phase shift value (1.2213 rad) is almost the same as to the pre-set value.

The third simulation is a little complex because the background intensity and modulation amplitude distribution are non-uniform for each pixel. The additional noise causes the additional phase error (PV = 31.5175 nm, RMS = 6.6464 nm) and incorrect phase shift value (1.1491 rad) for LEF PSI. Due to the non-uniform background intensity and modulation amplitude distribution, the calculated phase error (PV = 15.8666 nm, RMS = 2.4459 nm) from LEF & LSI PSA is relatively large, but the estimated phase shift value (1.2221 rad) after 6 iterations is still close to the pre-set value. The last simulation is more complex, it includes both the fluctuations and noise. For LEF PSA, the above mixed errors cause large phase error (PV = 43.1340 nm, RMS = 10.4027 nm) and the calculated phase shift value (1.1483 rad) is quite different from the pre-set value. In contrast, the LEF & LSI PSA can still obtain relatively accurate result, the phase error after 7 iterations can be corrected to a smaller value (PV = 16.6617 nm, RMS = 2.6393 nm) and the phase shift value after correction is 1.2204 rad.

Table 2 shows the phase shift error, RMS phase error, and processing time with different PSAs in 4 different simulations. For LEF PSA, the more complex the design error, the larger the phase shift error. However, for LEF & LSI PSA, the phase shift error is approximately equal to zero in all 4 simulations. In addition, the phase errors from LEF PSA are larger than that from LEF & LSI PSA. LEF & LSI PSA is not very sensitive to the fluctuations of the

background intensity and modulation amplitude distribution (simulation 2), moreover, we find that the non-uniform of the background intensity and modulation amplitude distribution plays an important role in generating phase error according to the similar phase error in the third and last simulations. Hence, if we can calibrate and correct the non-uniform of the background intensity and modulation amplitude distribution, the phase error will decrease further. Moreover, the processing time for LEF PSA in 4 different simulations are similar, indicating that different background intensity, modulation amplitude distribution and noise will not affect the processing time. The processing time for LEF & LSI PSA are longer than LEF PSA since LSI PSA costs more time. For LEF & LSI PSA, the time of the first and third simulations are lower than the second and fourth simulations because there are less iterations. Generally, to obtain higher accuary the time in LEF & LSI PSA are longer than those in LEF PSA.

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Table 1. The calculated phase distribution, phase error and iterative curve LEF & LSI method in different simulations.

	Simulation index	1	2	3	4
Phase shift error	LEF	0	0.0513	0.0726	0.0734
(rad)	LEF & LSI	0	0.0004	0.0004	0.0013
RMS Phase error	LEF	0	7.9062	6.6464	10.4027
(nm)	LEF & LSI	0	0.0643	2.4459	2.6393
Processing time	LEF	3.36	3.34	3.37	3.35
(s)	LEF & LSI	3.96	5.82	5.34	5.80

Table 2. The phase shift error, RMS phase error and processing time of LEF and LEF & LSI PSAs in 4 different simulations

We know that the phase shift value is important to PSAs, hence, it is necessary to discuss the robustness of LEF & LSI PSA with different phase shift values. We uniformly set the distribution of the phase shift value in the range of 0.1 rad to 2.6 rad, other simulated conditions are same as the fourth simulation discussed above. Under normal circumstances, the range should be 0 to pi, we cannot choose 0 rad because it means no phase shift, and we choose 2.6 rad as the max phase shift value since the phase error is too large when the phase shift value is more than 2.6 rad for LEF PSA. Figure 3 represents the phase error with different phase shift values in uniform distribution, we can see that, for LEF PSA the phase error is less than 10 nm RMS only when the phase shift value is between 0.54 rad and 1.72 rad, other phase shift values will generate large phase error. For LEF & LSI PSA, the phase error is less than 5 nm when the phase shift value is more than 0.25 rad, moreover, the smallest phase error is 2.3 nm when the phase shift value is $\pi/2$ rad. Therefore, LEF & LSI PSA is more stable than LEF PSA. In addition it is better to choose phase shift value which is close to $\pi/2$ rad to obtain the more accurate phase distribution.



Fig. 3. The phase errors of LEF and LEF & LSI PSAs for different phase shift values.

In order to understand the effect of the fluctuating background intensity and modulation amplitude distribution between different interferograms, different background intensity and modulation amplitude distributions are simulated. We analyze 8 situations as shown in Table 3. In 8 different situations, A_1 and B_1 are same, but A_2 and B_2 are different. Table 4 displays that the phase shift errors and RMS phase errors with 8 different situations for LEF and LEF & LSI PSAs, LEF & LSI PSA is almost insensitive to the fluctuations of the background intensity and modulation amplitude.

Table 3. The background intensity and modulation amplitude in 8 different situations

	1	2	3	4	5	6	7	8
A_{1}	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09
B_1	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94
A_2	1.1	1.15	1.2	1.25	1.3	1.35	1.4	1.45
B_2	1	1.05	1.1	1.15	1.2	1.25	1.3	1.35

	Simulation index	1	2	3	4	5	6	7	8
Phase shift	LEF	0.0052	0.0163	0.0321	0.0513	0.0727	0.0955	0.1191	0.1431
error (rad)	LEF & LSI	0.0004	0.0006	0.0007	0.0005	0.0005	0.0005	0.0003	0.0002
RMS Phase	LEF	2.3461	4.2315	6.0809	7.9062	9.7110	11.4945	13.2538	14.9859
error (nm)	LEF & LSI	0.0743	0.0677	0.0977	0.0643	0.0793	0.0942	0.0550	0.0623

Table 4. The phase shift errors and RMS phase errors of LEF and LEF & LSI PSAs with different fluctuations in the background intensity and modulation amplitude distribution

Through the above four different simulations, we know that the noise which is added to the background intensity and modulation amplitude distribution will affect the accuracy of the phase calculation. We analyze the phase error and phase shift extraction for different noises from 20 dB to 80 dB with an interval of 10dB. The phase error is plotted in Fig. 4(a) and the phase shift error is plotted in Fig. 4(b). The phase error decreases with the increase of SNR from 20 dB to 80 dB, the phase error can be ignored when the SNR is more than 50 dB. In addition, LEF PSA is more sensitive to the noise than LEF & LSI PSA. Figure 4(b) shows that the phase shift error is less than 0.08 rad for LEF PSA when the noise varies from 20 dB to 80 dB. LEF & LSI PSA can better suppress the noise, the phase shift error is approximately equal to zero.



Fig. 4. (a) The phase error (RMS) and (b) phase shift error of LEF and LEF & LSI PSAs with different noises.

Based on the above different simulations, the advantages of proposed LEF & LSI PSA can be summarized as: 1) It has a higher accuracy than LEF PSA; 2) it is less sensitive to the phase shift value; 3) it is almost insensitive to the fluctuations of the background intensity and modulation amplitude; and 4) it can partially suppress the effect introduced by the non-uniform of the background intensity and modulation amplitude distribution.

4. Demonstration with experimental data

To demonstrate the proposed algorithm, we measured a half-inch diamond turned copper surface, 4 phase shifted interferograms were collected as shown in Fig. 5. We used standard 4-step PSA, LEF PSA and LEF & LSI PSA to calculate the phase distribution of the tested surface. Only the first and second interferograms in Fig. 5 were used to obtain the phase distribution in LEF PSA and LEF & LSI PSA. The 2D map of the phase distribution calculated from standard 4 step PSA is plotted in Fig. 6(a) (PV = 452.7nm, RMS = 44.3 nm). Figure 6(b) shows the calculated phase distribution using LEF PSA, which is different from the result using 4 step PSA. In addition both PV and RMS value (PV = 703.4 nm, RMS = 54.3 nm) are quite different from that calculated by standard 4-step PSA, the main reason is

that the LEF PSA is sensitive to the non-uniform intensity between different pixels. The result from LEF & LSI PSA is plotted in Fig. 6(c), the surface shape is similar to the result of the 4step PSA. There are small differentce PV and RMS values (PV = 493.3 nm, RMS = 48.4 nm) from the 4-step PSA due to the non-uniform intensity. The converging curve is shown in Fig. 7(a), showing that the converging threshold of iteration (0.1 nm RMS) is achieved after 6 iterations. The curves of the calculated phase and phase shift (Figs. 7(b) and 7(c)) are relatively smooth and steady. Through the experimental result, we demonstrate that: 1) the proposed LEF & LSI PSA without pre-filtering can obtain relatively accurate result by only two interferograms, it can partially suppress the effect introduced by the non-uniform of the background intensity; and 2) LEF PSA cannot obtain accurate result when the background intensity distribution is especially non-uniform.



Fig. 5. Four phase shifted interferograms with $\pi/2$ phase shift.



Fig. 6. The 2D maps of the calculated phase distributions by (a) 4 step PSA (PV = 452.7nm, RMS = 44.3 nm), (b) LEF PSA (PV = 703.4 nm, RMS = 54.3 nm) and (c) LEF & LSI PSA (PV = 493.3 nm, RMS = 48.4 nm).



Fig. 7. The calculated result by LEF & LSI PSA. (a) the iterative curve, (b) and (c) the calculated phase and phase shift with different iterative times.

5. Conclusion

In this paper, we present a random two step phase shifting algorithm based on Lissajous ellipse fitting and least squares technologies, the initial phase distribution and unknown phase shift are calculated by LEF PSA firstly, then more accurate phase distribution and phase shift are extracted by LSI PSA after several iterations. The proposed algorithm can achieve higher accuracy than LEF PSA, and it can be used in different situations, such as the different phase shift values, fluctuations and non-uniform of the background intensity and modulation amplitude. We have demonstrated the proposed method with the simulated data and experimental data of a diamond turned optical surface. This method has the potential applications for the high accurate phase extraction in phase-shifting interferometry.

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