An attitude tracking method for star sensor under dynamic conditions*

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The tracking performance of star sensor degrades seriously under dynamic conditions. To improve the tracking accuracy and efficiency, an attitude tracking method based on unscented Kalman filter (UKF) and singular value decomposition (SVD) is proposed in this paper. The star sensor is modeled as a nonlinear stochastic system, the state of which is attitude quaternion. The quaternion can be estimated by UKF, then the predicted attitude and corresponding star positions are obtained. To ensure the stability of attitude tracking, SVD is applied to obtain the sigma points in UKF continuously. The experimental results indicate that the proposed method yields high accuracy and efficiency in attitude tracking. This method provides a practical approach to ensure the tracking performance of star sensor under dynamic conditions.

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Star sensor is one of the most promising attitude measurement instruments for its high accuracy[1]. Star sensor has two operating modes: lost-in-space (LIS) mode and tracking mode, and the tracking mode is the primary operating mode of star sensor. Generally, the star position in the next frame can be searched around the current star position, but this method is effective only when star sensor works under nearly static condition. Under dynamic conditions, the pixels required to search in the star image will increase drastically, leading to lower accuracy and update rate of star sensor.

To ensure the tracking accuracy and efficiency of star sensor under dynamic conditions, several tracking methods based on attitude estimation have been proposed. Based on Kalman filter (KF)[2], Lu et al[3] proposed a hybrid method using a star sensor and gyroscopes. Yu et al[4] proposed a star tracking method based on multi-exposure imaging for intensified star sensor. Ye and Zhou[5] proposed an autonomous space target recognition and tracking approach. Jin et al[6] proposed a star tracking algorithm under highly dynamic conditions. KF algorithm has been widely used for prediction in linear stochastic system, but modeling the dynamic star sensor as an ideal linear system is inaccurate, thus the KF algorithm may lead to a serious decrease of tracking accuracy in practice. Extended Kalman filter (EKF) proposed by Bucy and Senne[7] as well as Sunahara and Ohsumi[8] is a widely used method for estimation and tracking in practical engineering, especially in some nonlinear systems. Based on EKF algorithm, Li et al[9] proposed an attitude tracking method for star sensors. Sun et al[10] proposed a star tracking method based on the combination of EKF algorithm and optical flow analysis. Chang et al[11] and Qin et al[12] proposed attitude estimation methods using iterated multiplicative extend Kalman filter (IMEKF) and sequential multiplicative extended Kalman filter (SMEKF). There is no doubt that EKF algorithm is effective in star tracking under dynamic conditions, but it also has several deficiencies. Different from EKF algorithm, UKF algorithm proposed by Julier and Uhlmann[13] applied unscented transform (UT) instead of Taylor expansion to the nonlinear system, thus it is more accurate in estimation and tracking. Based on UKF algorithm, Li et al[14] proposed a gyro-less attitude estimation method of star sensor and Guo et al[15] proposed an on-orbit attitude estimation method of star sensors. However, UKF algorithm is based on UT method, the robustness of which is sensitive to the error covariance matrix. Once the error covariance matrix is non-positive definite, UKF algorithm will stop running and the attitude will not be estimated continually.

An effective attitude tracking method based on UKF

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algorithm and SVD method is proposed to improve the tracking accuracy and efficiency of star sensor under dynamic conditions. Taking into account the characteristic of dynamic star sensor, we model it as a nonlinear stochastic system, the state of which is attitude quaternion. The quaternion can be estimated by UKF algorithm, then the predicted attitude and corresponding star positions can be obtained. To ensure the stability of attitude tracking, SVD is applied to obtain the sigma points in UKF continuously.

The attitude measurement method of star sensor is introduced firstly. Starlight is imaged on the focal plane of image detector through the lens of star sensor, then the coordinates of stars are used to calculate the attitude based on the attitude measurement model of star sensor. As shown in Fig.1, $o_c-x_cy_cz_c$ is the celestial coordinate system and $o_s-x_sy_sz_s$ is the star sensor coordinate system. $\boldsymbol{u}_c(t)$ represents the reference vector of the navigation star in celestial coordinate system at time t, and $\boldsymbol{u}_c(t)$ is given by

$$\boldsymbol{u}_c(t)=[\cos\alpha\cos\delta\ \ \sin\alpha\cos\delta\ \ \sin\delta]^T , \quad (1)$$

where α and δ represent the right ascension and declination of the navigation star, respectively.

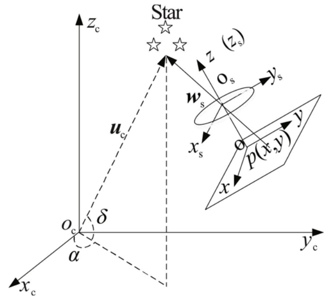


Fig.1 The attitude measurement model of star sensor

$\boldsymbol{w}_s(t)$ represents the measurement vector of the navigation star in star sensor coordinate system at time t, and $\boldsymbol{w}_s(t)$ is given by

$$\boldsymbol{w}_s(t)=\frac{1}{\sqrt{x^2(t)+y^2(t)+f^2}}[-x(t)\ \ -y(t)\ \ f]^T , \quad (2)$$

where f represents the focal length of star sensor, $x(t)$ and $y(t)$ are the star coordinates in the focal plane of star sensor at time t.

The reference vector $\boldsymbol{u}_c(t)$ and the measurement vector $\boldsymbol{w}_s(t)$ satisfy

$$\boldsymbol{w}_s(t)=\boldsymbol{A}(t)\boldsymbol{u}_c(t), \quad (3)$$

where $\boldsymbol{A}(t)$ is the attitude matrix of star sensor.

Generally, the attitude matrix $\boldsymbol{A}(t)$ can be expressed by Euler angles and quaternion in space mission. The quaternion is defined to express the rotation of rigid body relative to reference axis, and the definition of quaternion is given by[16]

$$\boldsymbol{q}=[q_0\ \ q_1\ \ q_2\ \ q_3]^T=[\cos\frac{\gamma}{2}\ \ (\sin\frac{\gamma}{2})\boldsymbol{e}]^T , \quad (4)$$

where $\boldsymbol{e}$ is the axis of rotation and γ is the angle of rotation about $\boldsymbol{e}$. The quaternion is defined as an unit vector and satisfies the constraint of $\|\boldsymbol{q}\|=1$. Generally, to ensure the uniqueness of quaternion, q_0 is limited to $q_0\geq 0$.

A reasonable system model is the prerequisite of accurate attitude estimation. Based on the attitude measurement model, a nonlinear stochastic system is established by considering the characteristic of dynamic star sensor. The quaternion is selected as the state which is given by

$$\boldsymbol{s}=\begin{bmatrix}q_0 & q_1 & q_2 & q_3\end{bmatrix}^T . \quad (5)$$

A model required to transfer the state from current time to the next can be defined as

$$\boldsymbol{s}_k=\phi_{k-1}(\boldsymbol{s}_{k-1})+\boldsymbol{\delta}_{k-1} , \quad (6)$$

where ϕ_{k-1} is the state transition function which extrapolates the state from time t_{k-1} to t_k, δ_{k-1} is assumed to be additive Gaussian white noise (AGWN) with zero mean and noise covariance matrix $\boldsymbol{Q}$.

The quaternion updates in real time under dynamic conditions and the relation between quaternion $\boldsymbol{q}$ and angular velocity $\boldsymbol{w}$ can be established by

$$\dot{\boldsymbol{q}}(t)=\frac{1}{2}\begin{bmatrix}0\\ w_x\\ w_y\\ w_z\end{bmatrix}\otimes\boldsymbol{q}(t)=$$

$$\frac{1}{2}\begin{pmatrix}0 & -w_x & -w_y & -w_z\\ w_x & 0 & -w_z & w_y\\ w_y & w_z & 0 & -w_x\\ w_z & -w_y & w_x & 0\end{pmatrix}\boldsymbol{q}(t)=\boldsymbol{W}\boldsymbol{q}(t) , \quad (7)$$

where the symbol $\otimes$ denotes quaternion multiplication, $\boldsymbol{w}=[w_x\ w_y\ w_z]^T$ represents the angular velocity vector of star sensor and it is assumed to be constant from time t_{k-1} to t_k because the time interval is extremely short.

Solving $\boldsymbol{q}(t)$ in the differential equation Eq.(7) gives

$$\boldsymbol{q}(t)=\boldsymbol{e}^{\boldsymbol{W}\cdot t}. \quad (8)$$

According to Eq.(8), the relation between $\boldsymbol{q}(t_{k-1})$ and $\boldsymbol{q}(t_k)$ is given by

$$\boldsymbol{q}(t_k)=\boldsymbol{e}^{\boldsymbol{W}\cdot\Delta t}\boldsymbol{q}(t_{k-1}) , \quad (9)$$

where $\Delta t=t_k-t_{k-1}$, which represents the time interval between time t_{k-1} and t_k.

Obviously, $\frac{2\boldsymbol{W}}{\|\boldsymbol{w}\|}$ is an orthogonal and skew-symmetric matrix, hence the matrix exponential in Eq.(9) can be updated to

$$\boldsymbol{e}^{\boldsymbol{W}\cdot\Delta t}=\cos(\frac{\|\boldsymbol{w}\|}{2}\cdot\Delta t)\cdot\boldsymbol{I}+\frac{2}{\|\boldsymbol{w}\|}\sin(\frac{\|\boldsymbol{w}\|}{2}\cdot\Delta t)\cdot\boldsymbol{W} . \quad (10)$$

According to Eqs.(7), (9) and (10), the state transition function is given by

$$\phi_k(\boldsymbol{s}_k)=[\cos(\frac{\|\boldsymbol{w}\|}{2}\cdot\Delta t)\cdot\boldsymbol{I}+$$

$$\frac{2}{\|\boldsymbol{w}\|}\sin(\frac{\|\boldsymbol{w}\|}{2}\cdot\Delta t)\cdot\boldsymbol{W}]\boldsymbol{s}_k . \quad (11)$$

The measurement equation of the stochastic system is defined as

$$z_k = h_k(s_k) + \eta_k , \quad (12)$$

where η_k is assumed to be AGWN with zero mean and error covariance matrix R. Measurement function h_k firstly converts the reference vector in celestial coordinate system to the measurement vector in star sensor coordinate system with the attitude matrix formed with the quaternion at time t_k, which is given by

$$A(t_k) = \begin{bmatrix} q_0^2+q_1^2-q_2^2-q_3^2 & 2(q_1q_2+q_0q_3) & 2(q_1q_3-q_0q_2) \\ 2(q_1q_2-q_0q_3) & q_0^2-q_1^2+q_2^2-q_3^2 & 2(q_2q_3+q_0q_1) \\ 2(q_1q_3+q_0q_2) & 2(q_2q_3-q_0q_1) & q_0^2-q_1^2-q_2^2+q_3^2 \end{bmatrix} . \quad (13)$$

Based on Eq.(13), the measurement vector in star sensor coordinate system can be calculated as

$$w_s(t_k) = A(t_k) u_c(t_k) , \quad (14)$$

where $w_s = \left(w_s^x, w_s^y, w_s^z\right)^T$ is an unit vector in star sensor coordinate system.

Finally, the measurement function h_k maps the measurement vector to the star coordinates (x_i, y_i) on focal plane according to Eq.(2), which is computed as

$$x_i = -f\frac{w_s^x(t_k)}{w_s^z(t_k)},\ y_i = -f\frac{w_s^y(t_k)}{w_s^z(t_k)} . \quad (15)$$

The eigenvalues of matrix W are $\pm\frac{i}{2}\sqrt{w_x^2+w_y^2+w_z^2}$, $\pm\frac{i}{2}\sqrt{w_x^2+w_y^2+w_z^2}$, thus the established system of dynamic star sensor is stable according to Lyapunov theory.

UKF algorithm has been widely used in the star tracking of dynamic star sensor because of its high accuracy and efficiency in attitude estimation. Based on the established model of dynamic star sensor, the procedure of UKF algorithm[13] is shown in Tab.1.

Even though UKF algorithm can achieve high accuracy and efficiency in attitude estimation of dynamic star sensor, its robustness is pretty poor. Generally, in the process of attitude estimation based on UKF algorithm, the error covariance matrix P_{k-1} varies randomly. Once P_{k-1} is non-positive definite, the Cholesky decomposition in Tab.1 is disabled and the sigma points would not be obtained[17], which causes the divergence of UKF algorithm and the termination of attitude estimation.

SVD method is one of the most widely used methods for matrix decomposition in linear algebra, by using which the error covariance matrix P_{k-1} can be decomposed as

$$P_{k-1} = U_{k-1}\cdot \Lambda_{k-1}\cdot V_{k-1}^T = U_{k-1}\cdot\begin{bmatrix} S_{k-1} & 0 \\ 0 & 0\end{bmatrix}\cdot V_{k-1}^T , \quad (16)$$

where $U_{k-1}\in R^{n\times n}$, $V_{k-1}\in R^{n\times n}$ are composed by eigenvectors of matrix $P_{k-1}P_{k-1}^T$ and $P_{k-1}^T P_{k-1}$ respectively, $S_{k-1} = diag(a_1, a_2, \ldots a_r)$, $a_1, a_2, \ldots a_r$ are the eigenvalues of matrix $P_{k-1}^T P_{k-1}$ as well as the singular values of matrix P_{k-1}, which satisfy $a_1 \geq a_2 \geq \ldots \geq a_r \geq 0$.

Tab.1 The procedure of UKF algorithm

1. Initialization
   $\hat{s}_0 = E[s_0]$, $P_0 = E[(s_0-\hat{s}_0)(s_0-\hat{s}_0)^T]$
2. Decompose the matrix P_{k-1} using Cholesky decomposition
   $P_{k-1} = C_{k-1}\cdot C_{k-1}^T$
3. Compute the sigma points of the state
   $s_{i,k-1} = \left[\hat{s}_{k-1}\ \ \hat{s}_{k-1} + (\sqrt{(n+\lambda)}C_{k-1})_i\ \ \hat{s}_{k-1} - (\sqrt{(n+\lambda)}C_{k-1})_i\right]$
4. Compute the propagated sigma points of state
   $s_{i,k|k-1} = \phi_{k-1}(s_{i,k-1})$
5. Compute the priori predicted state and error variance matrix
   $\hat{s}_{k|k-1} = \sum_{i=0}^{2n} W_i^{(m)} s_{i,k|k-1}$
   $P_{k|k-1} = \sum_{i=0}^{2n} W_i^{(c)}[(s_{i,k|k-1}-\hat{s}_{k|k-1})(s_{i,k|k-1}-\hat{s}_{k|k-1})^T] + Q_k$
6. Compute the priori predicted measurement
   $z_{i,k|k-1} = h(s_{i,k|k-1})$, $\hat{z}_{k|k-1} = \sum_{i=0}^{2n} W_i^{(m)} z_{i,k|k-1}$
7. Compute the filter gain
   $P_{z_k,z_k} = \sum_{i=0}^{2n} W_i^{(c)}[(z_{i,k|k-1}-\hat{z}_{k|k-1})(z_{i,k|k-1}-\hat{z}_{k|k-1})^T] + R_k$
   $P_{s_k,z_k} = \sum_{i=0}^{2n} W_i^{(c)}[(s_{i,k|k-1}-\hat{s}_{k|k-1})(z_{i,k|k-1}-\hat{z}_{k|k-1})^T]$
   $K_k = P_{s_k,z_k} P_{z_k,z_k}^{-1}$
8. Compute the posteriori predicted state and error variance matrix
   $\hat{s}_k = \hat{s}_{k|k-1} + K_k(z_k - \hat{z}_{k|k-1})$, $P_k = P_{k|k-1} - K_k P_{z_k,z_k} K_k^T$

Since U_{k-1}, V_{k-1} and Λ_{k-1} are real matrices, thus for any error covariance matrix P_{k-1}, the decomposition result is existing and unique, hence the SVD method is reasonably robust in matrix decomposition. Given the advantage of SVD method, a SVD-UKF based method is proposed to conduct the attitude tracking of dynamic star sensor, which greatly enhances the robustness of UKF algorithm. By using SVD-UKF algorithm, the process of attitude tracking would be more stable. The algorithm procedure of SVD-UKF is similar to that of UKF algorithm and the acquisition of sigma points is the major difference. Based on SVD method, the computation of the sigma points is updated to

$$\begin{cases} D_{k-1} = U_{k-1}\sqrt{\Lambda_{k-1}} \\ s_{i,k-1} = [\hat{s}_{k-1}\ \ \hat{s}_{k-1} + (\sqrt{(n+\lambda)}D_{k-1})\ \ \hat{s}_{k-1} - (\sqrt{(n+\lambda)}D_{k-1})]^T \end{cases} . \quad (17)$$

Several simulations are performed to evaluate the accuracy and efficiency of the proposed attitude tracking method[18,19]. The simulations are implemented with MATLAB in Windows operating system on a Core VII computer with 2.5 GHz basic frequency. The detailed parameters of the star sensor are listed in Tab.2.

Tab.2 Parameters of the star sensor

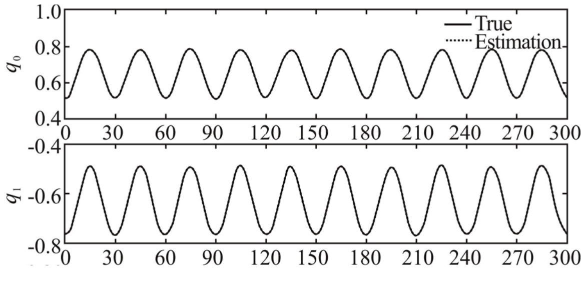
Parameter	Value
FOV	25°
Focal length	30.1 mm
Pixel array	2 048×2 048 pixels
Pixel size	6.5 μm
Detectable magnitude	6.5 MV

Before the simulations are performed, initial conditions should be specified, which include the initial state estimation $\hat{s}_0$ and its corresponding error covariance matrix $\boldsymbol{P}_0$, initial process noise covariance matrix $\boldsymbol{Q}$, initial measurement noise covariance matrix $\boldsymbol{R}$ and the dynamic condition of star sensor.

Obtained from the initial attitude of star sensor, the initial state estimation of the nonlinear stochastic system is (0.512 763, −0.761 644, −0.222 448, 0.327 855), and the diagonal terms of its corresponding error covariance matrix are $\{3\times10^{-7}, 3\times10^{-7}, 3\times10^{-7}, 3\times10^{-7}\}$ while the off-diagonal terms are zero. Taking into account the actual working conditions of star sensor, the diagonal terms of the process noise covariance matrix are $\{1\times10^{-9}, 1\times10^{-9}, 1\times10^{-9}, 1\times10^{-9}\}$ and the off-diagonal terms are zero, which are assumed to be constant throughout the simulations. Generally, the measurement noise satisfies Gaussian distribution, whose standard deviation ranges from 0.04 to 0.18 pixel[20], hence the measurement error covariance matrix is a diagonal matrix whose diagonal terms are the noise variances and the off-diagonal terms are zero. Different from the process noise covariance matrix, the dimension of the measurement noise covariance matrix is not constant because the stars in the FOV are changing continuously and the dimension of the measurement vector is variable under dynamic conditions. According to the actual working conditions of dynamic star sensor, the angular velocity and the angular acceleration of it are defined as

$$w_x = 5\sin(\frac{\pi}{15}t),\ a_x = \frac{\pi}{3}\cos(\frac{\pi}{15}t) \quad . \tag{18}$$

The simulation is run for 300 s and the time interval is set as 0.1 s based on the update rate of star sensor. The estimated states obtained from the proposed method along with the true states are both shown in Fig.2. Intuitively, the estimated quaternion and the true quaternion are quite close, hence the attitude tracking is fairly stable in the process of the simulation.



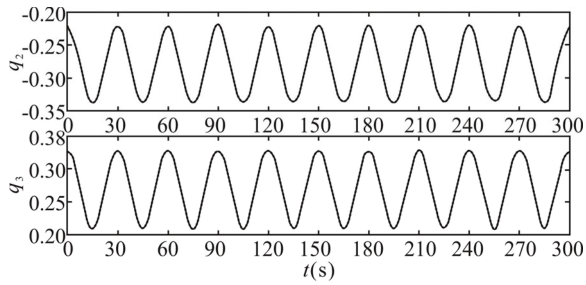


Fig.2 Attitude tracking results in simulation

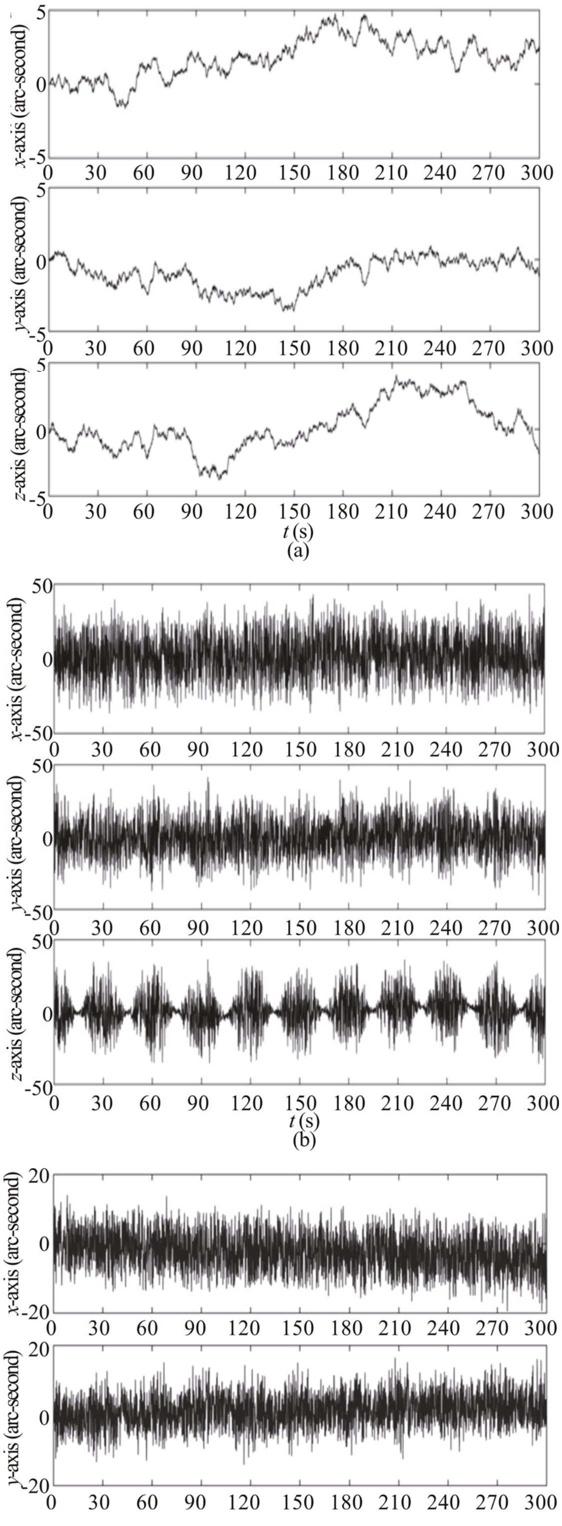
To further evaluate the tracking accuracy of the proposed method, a series of contrast simulations are performed between the proposed method, EKF[7,8] and SMEKF[12] under the same experimental conditions above. Given that the attitude tracking of UKF algorithm is not continuous, thus it is not added into the contrast simulations.

The estimated Euler angles of SVD-UKF, EKF and SMEKF which are transformed from the estimated quaternion are compared with the true Euler angles respectively. The Euler angles estimation errors are shown in Fig.3. Intuitively, the Euler angles estimation errors of SVD-UKF, EKF and SMEKF are within 5 arc-seconds, 50 arc-seconds and 20 arc-seconds respectively for the three axes. In addition, obtained from the simulation results, the standard deviations of the Euler angles estimation errors of SVD-UKF are 1.337 7, 1.048 3 and 1.762 3 arc-seconds respectively, besides, the results of EKF are 15.287 3, 13.392 8 and 11.233 3 arc-seconds and the results of SMEKF are 5.803 3, 4.979 7 and 4.112 1. Obviously, the Euler angles estimation errors and the corresponding standard deviations of SVD-UKF are both less than that of EKF and SMEKF. The results indicate that the attitude tracking of SVD-UKF is more accurate and stable compared with EKF and SMEKF under dynamic condition, which is rather intuitive through the comparison of Fig.3(a), (b) and (c).

The predicted star coordinates in a star image can be obtained from the estimated attitude quaternion by using Eqs.(13) —(15). In the process of simulation, the maximum prediction error of star coordinates in every prediction can be obtained through the comparison with the true star coordinates, and the maximum prediction errors of SVD-UKF, EKF and SMEKF throughout the simulation are shown in Fig.4. Obviously, the maximum prediction errors of SVD-UKF, EKF and SMEKF are within 0.1 pixel, 0.8 pixel and 0.6 pixel, respectively. Generally, the prediction error of star coordinate directly affects the size of tracking window under tracking mode, the smaller the prediction error is, the smaller the size of tracking window is and the more stable the tracking is.

In addition, to evaluate the efficiency of the proposed method, a simulation is performed to compare the consuming time of SVD-UKF, EKF and SMEKF. The consuming time of SVD-UKF, EKF and SMEKF are recorded every 30 s during the simulation, then the curves of the results are shown in Fig.5. Obviously, the curves of SVD-UKF, EKF and SMEKF nearly satisfy positive linear growth, and the consuming time of SVD-UKF is consistently shorter than that of EKF and SMEKF in each record, which indicates SVD-UKF is more efficient in the attitude tracking of dynamic star sensor.

A series of simulations are implemented above to evaluate the accuracy and efficiency of the proposed method. However, all the simulations are based on a single run and are not necessarily typical. Therefore, repeated simulations are performed to further evaluate the robustness of the proposed method.



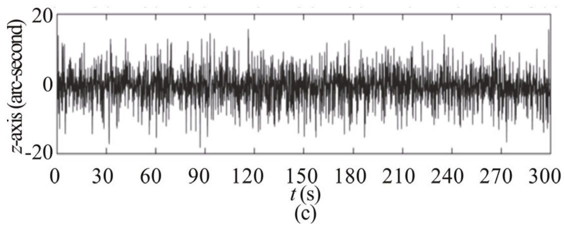


Fig.3 Attitude tracking errors of (a) SVD-UKF, (b) EKF and (c) SMEKF

Following the same experimental conditions above, the simulation is repeated for 100 times and the root mean square (*RMS*) estimation errors of the Euler angles for the three axes are shown in Fig.6. We note that the initial *RMS* estimation errors are the maximum, while the *RMS* errors get smaller from the next frame, because the initial attitude estimation is unstable. Besides, the *RMS* estimation errors of the *x*-axis are larger than that of the *y*-axis and *z*-axis, probably because the angular velocity of the *x*-axis changes as sinusoidal distribution, which increases the instability of attitude tracking. In general, the maximum *RMS* estimation errors in the repeated simulations are about 0.1 arc-second, which indicates the SVD-UKF based attitude tracking is fairly stable.

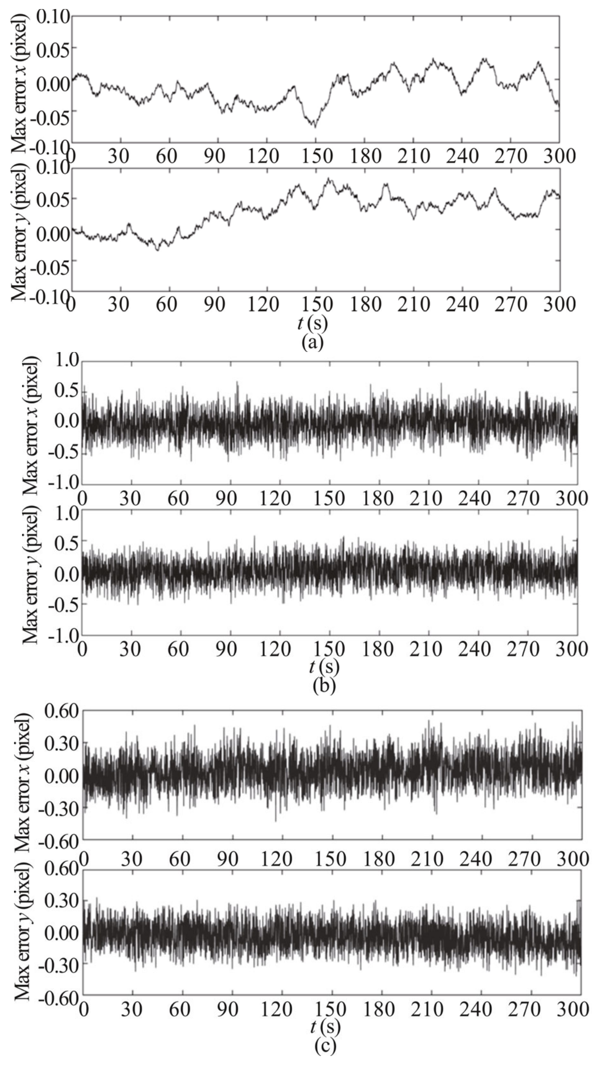


Fig.4 Maximum prediction errors of (a) SVD-UKF, (b) EKF and (c) SMEKF

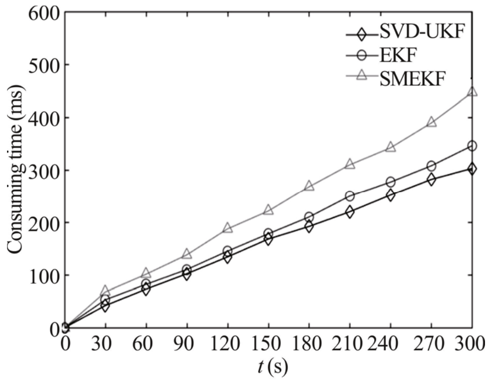


Fig.5 Consuming time of SVD-UKF, EKF and SMEKF

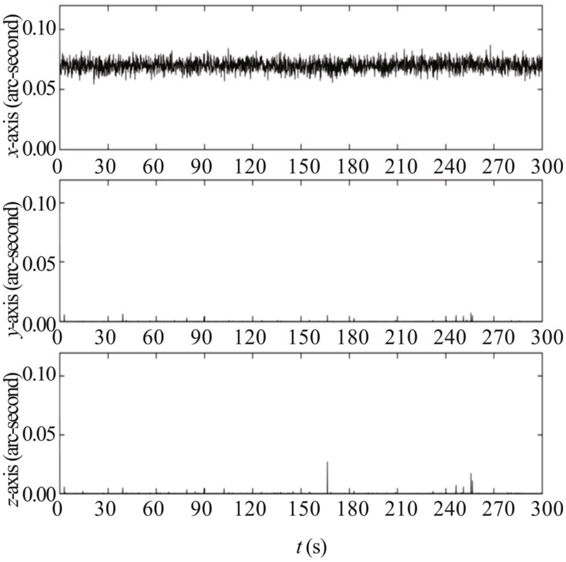


Fig.6 Root mean square estimation errors of SVD-UKF

To improve the tracking accuracy and efficiency of star sensor under dynamic conditions, an effective attitude tracking method based on UKF algorithm and SVD method is proposed in this paper. The experimental results indicate that the proposed method can achieve accurate and efficient attitude tracking for star sensor under dynamic conditions. The attitude tracking can converge throughout the simulations, during which the Euler angles estimation errors are within 5 arc-seconds and the prediction errors of star coordinates are within 0.1 pixel, which are both less than that of EKF and SMEKF. In addition, the consuming time of SVD-UKF is consistently shorter than that of EKF and SMEKF in the process of simulation. Besides, in the repeated simulations, the maximum *RMS* estimation errors of Euler angles are about 0.1 arc-second, based on which the stability of the SVD-UKF is further verified. In general, the proposed method can ensure the tracking performance of star sensor under dynamic conditions.

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