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Effect of lateral error on the coupling efficiency and beam quality of gaussian beam launched into large-core fiber

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ABSTRACT

We study the effect of lateral error on the coupling efficiency and beam quality of a Gaussian beam coupled into a large-core multimode fiber. The formula for the effect of lateral error on coupling efficiency is derived in detail and verified with both simulations and experiments. The effect of lateral error on beam quality is also analyzed with the use of ZEMAX and verified with our experimental setup. The results show that lateral error can change the path of the light coupled into the fiber to such a large extent that it influences the power distribution of the Gaussian beam.

1. Introduction

High-power lasers have unique advantages in power density and are thus widely used in material processing, laser surgery, and optoelectronic countermeasures. The use of optical fiber to transmit high-power laser has significant advantages, including flexible transmission, high transmission efficiency, small size, light weight and good environmental adaptability.

The smaller the diameter of the fiber core is, the greater the internal power density, and the more susceptible the fiber is to damage. Compared with single-mode fiber, large-core multimode fiber is more suitable for laser power transmission, mainly because of the large diameter. The effect of modal dispersion inherent in multimode fiber can almost be neglected in applications involving laser power transmission. In addition, the energy transfer capacity of large-core multimode optical fiber has increased greatly due to the continuous advancement of related technologies.

The factors that affect the performance of a Gaussian beam launched into large-core multimode fiber mainly include alignment error, Fresnel reflection, and the condition of the incident end face of the fiber. Among them, the alignment error, which consists of lateral error, longitudinal error and angular error, affects not only the coupling efficiency but also the beam quality [1,2]. Furthermore, lateral error has the greatest effect on coupling efficiency and beam quality [3,4].

In fact, the coupling to single-mode fiber has been extensively investigated. J. Sakai et al discussed misalignment effects on coupling efficiency between a semiconductor laser and a single-mode fiber [5]. Wagner et al provided a compact description of coupling efficiency of optics in single-mode fiber components that includes the effects of aberrations, fiber misalignments, and fiber-mode mismatch [6]. Winzer et al derived a general expression for the efficiency with which quasi-monochromatic random light can be coupled to an optical fiber by means of a lens by employing the van Cittert–Zernike theorem of classical coherence theory [7]. Tang et al. developed a theoretical model for the optical coupling between a wedged single-mode fiber and a highly elliptical laser diode to determining the effects of various angular misalignment on the coupling efficiency and alignment process [8]. Chao et al.

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reported a method to calculate the fiber coupling efficiency in non-Kolmogorov satellite links [9]. Mandal et al present a theoretical investigation of coupling efficiency of a laser diode to a single-mode circular core triangular index fiber coupling via upside-down tapered hemispherical microlens on the tip of the fiber in presence of possible transverse and angular offsets [10]. All of them used the similar method of the overlap integral of the field distribution of receiving-fiber mode pattern and the incident optical field distribution to calculate coupling efficiency. However, the conclusions obtained above are not applicable for large-core multimode fiber. Because the field distributions of single-mode fiber and large-core multimode fiber are quite different, a large-core multimode fiber requires less precise alignment in power coupling from light source than a single-mode fiber to keep high coupling efficiency.

In this paper, a new method to calculate the coupling efficiency of a Gaussian beam coupled into a large-core multimode fiber is presented, which is able to well evaluate the effect of lateral error on coupling efficiency. In addition, the effect of lateral error on beam quality is analyzed macroscopically from the perspective of geometric optics.

2. Preparation

2.1. Basic coupling conditions

The basic conditions for the laser to be fully coupled into the fiber are [11]:

$$d_{in} < d_{core} \tag{1}$$

$$\theta_{in} < \theta_{\max} = 2\arcsin(NA) \tag{2}$$

where d_{in} is the spot diameter of the laser beam at the incident end face of the fiber, d_{core} is the core diameter of the fiber, θ_{in} is the divergence angle of the laser beam, and NA is the numerical aperture of the optical fiber.

2.2. Simplification

To perform theoretical calculations and simulations, the following simplifications are made:

1) The effect of Fresnel reflection is ignored so that we can concentrate on the influence of lateral error on coupling efficiency [12]. In practice, when the effect of Fresnel reflection needs to be taken into consideration, the coupling efficiency can simply be multiplied by the reflectivity.

2) The incident end face of the fiber is perfect, and there are no contaminants, scratches or other defects which can lead to the degradation of optical interface performance [13].

3) Because the diameter of the multimode fiber core is several orders of magnitude larger than the laser wavelength, it is feasible to use the knowledge of geometric optics to analyze the effect of lateral error on coupling efficiency and beam quality [14].

4) The light entering the fiber cladding is completely absorbed and does not contribute to the coupling efficiency. Although fiber cladding has the capability of transmitting light, the process will increase the risk of optical fiber damage. At the same time, beam quality will deteriorate after the beam is transmitted through the fiber cladding [15,16]. Therefore, it is necessary to filter the light in the fiber cladding, especially in the case of high-power laser transmission.

To illustrate the effect of light in the fiber cladding on the power distribution of the beam, we conducted a simulation using ZEMAX [17]. The simulation model is established with the conventional method of NSC with ports. Simulation parameters are set as follows: the lateral error d is 0.45 mm; the fiber core radius a is 0.5 mm; the radius of the fiber cladding R is 3 mm (to more clearly observe the power distribution of the beam in the fiber cladding); and the spot radius r is 0.2 mm.

The simulation results (Fig. 1) show that when the lateral error d is 0.45 mm, approximately 66.6% of the light can be coupled into the fiber core, and nearly 33% of the light will enter the fiber cladding. When the fiber length L is small, the light distribution in the fiber cladding is very uneven, which seriously affects the beam quality. With increasing L, the distribution of light in the fiber cladding gradually becomes uniform, but the beam spot radius and divergence angle still increase, resulting in poor beam quality.

3. Calculation and analysis

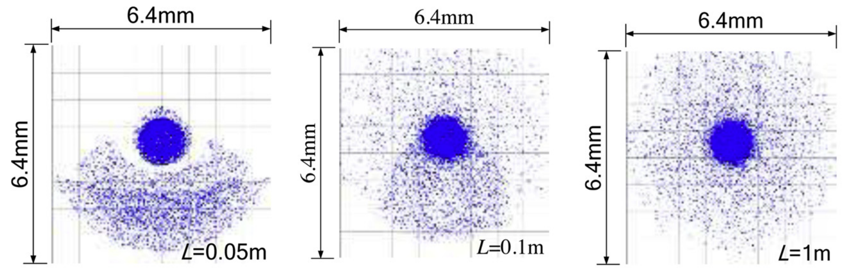
3.1. Derivation of the formula for lateral error and coupling efficiency

Because Gaussian approximation is not applicable to describe the field distribution of a large-core multimode fiber, we use the overlap integral of power distribution to calculate the power coupled into fiber core directly, and the coupling efficiency is defined as the ratio of the power coupled into fiber core to the power in the receiver aperture plane in this paper. To ensure accuracy in the formula for coupling efficiency, we will proceed from the Gaussian distribution and make a strict derivation without any approximation.

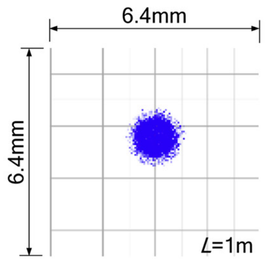
The field vector of a Gaussian beam propagating along the z-axis can be expressed by the Kirchhoff formula:

$$E(x, y, z) = \frac{A_0}{\omega(z)}\exp\left[-\frac{(x^2+y^2)}{\omega^2(z)}\right]\bullet\exp\left\{-i\left[k(z+\frac{x^2+y^2}{2R(z)}) - \phi(z)\right]\right\} \tag{3}$$

where A_0 is the center light amplitude at the origin ($z = 0$), k is the wavenumber, $k = \frac{2\pi n}{\lambda}$, n is the refractive index, $\omega(z)$ is the radius of the spot at z, and $R(z)$ is the radius of the wavefront at z.



(a) Power distribution of the beam with light in the fiber cladding for different fiber lengths



(b) Power distribution of the beam with the light in the fiber cladding absorbed

Fig. 1. Effect of light in the fiber cladding on the beam quality.

The amplitude of the field vector E at any distance z in the beam propagation direction is:

$$E = \frac{A_0}{\omega(z)} \exp\left[-\frac{r^2}{\omega^2(z)}\right] \tag{4}$$

The relationship between power distribution I and E is $I \propto E^2$ [18]. Thus, the power distribution I can be expressed as:

$$I(r) = \varepsilon E^2 = \varepsilon\left\{\frac{A_0}{\omega(z)} \exp\left[-\frac{r^2}{\omega^2(z)}\right]\right\}^2 = \varepsilon\frac{A_0^2}{\omega^2(z)} \exp\left[-\frac{2r^2}{\omega^2(z)}\right] \tag{5}$$

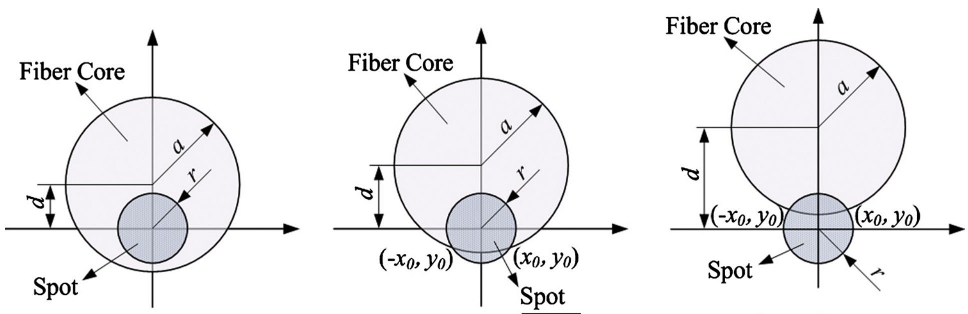
where ε is a scale coefficient.

The Gaussian beam power with semi-aperture ρ is:

$$P(\rho) = \iint_s I(r)ds = \frac{\varepsilon\pi A_0^2}{2}\left[1 - \exp(\frac{-2\rho^2}{\omega^2(z)})\right] \tag{6}$$

where s is the overlapping area of the Gaussian beam and the fiber core at the incident end face of the fiber.

The influence of the lateral error d on the coupling efficiency is directly reflected in the offset of the optical axis of the beam from the axis of the optical fiber, which causes a change in the overlapping area of the beam spot and the optical fiber core, as shown in Fig. 2 below. When the offset reaches a certain value, a portion of the light cannot enter the fiber core, resulting in a decrease in coupling efficiency.



(a) $0 \le d < a - r$ (b) $a - r \le d < \sqrt{a^2 - r^2}$ (c) $\sqrt{a^2 - r^2} \le d \le a + r$

Fig. 2. Schematic diagram of the effect of lateral error on coupling efficiency.

From Eq. (6) above, we see that in the absence of lateral error, the power of a beam with spot radius r coupled into a fiber with core radius a ($a > r$) is [19]:

$$P_0 = \iint_s I(x, y)ds = \frac{\varepsilon\pi A_0^2}{2}[1 - \exp(-2)] \tag{7}$$

As indicated in Fig. 7(a), when $0 \le d < a - r$ and the reflection loss at the fiber end face is ignored, the beam can be completely coupled into the fiber core, and the lateral error has no influence on the coupling efficiency. Thus,

$$P(d) = P_0 \tag{8}$$

When $a - r \le d \le a + r$, the outline of the spot and fiber core can be expressed as:

$$x^2 + y^2 = r^2 \tag{9}$$

$$x^2 + (y - d)^2 = a^2 \tag{10}$$

The two circles intersect at two points ($-x_0$, y_0) and (x_0, y_0). The values of x_0 and y_0 are:

$$x_0 = \frac{\sqrt{2a^2d^2 + 2r^2d^2 + 2a^2r^2 - a^4 - d^4 - r^4}}{2d} \tag{11}$$

$$y_0 = \frac{r^2 + d^2 - a^2}{2d} \tag{12}$$

When $a - r \le d < \sqrt{a^2 - r^2}$, the integrand $I(x,y)$ is not integrable in the region $s(d)$. To integrate $I(x,y)$, the region $s(d)$ is divided into three parts. Then, we can obtain the following equation:

$$\begin{aligned} P(d) &= \iint_{s(d)} I(x, y)dxdy = \varepsilon\frac{A_0^2}{r^2}\iint_{s(d)} \exp\left[-\frac{2(x^2 + y^2)}{r^2}\right]dxdy \\ &= \varepsilon\frac{A_0^2}{r^2}\int_{-x_0}^{x_0}\int_{d-\sqrt{a^2-x^2}}^{y_0} \exp\left[-\frac{2(x^2 + y^2)}{r^2}\right]dxdy + \varepsilon\frac{A_0^2}{r^2}\int_{-\sqrt{r^2-y^2}}^{\sqrt{r^2-y^2}}\int_{y_0}^{0} \exp\left[-\frac{2(x^2 + y^2)}{r^2}\right]dxdy \\ &+ \varepsilon\frac{A_0^2}{r^2}\int_{-r}^{r}\int_{0}^{\sqrt{r^2-x^2}} \exp\left[-\frac{2(x^2 + y^2)}{r^2}\right]dxdy \end{aligned} \tag{13}$$

Since the integrand $I(x,y)$ is rotationally symmetric, we can perform the transformation shown in Eq. (14). This transformation can make the terms in Eq. (13) more consistent.

$$\varepsilon\frac{A_0^2}{r^2}\int_{-\sqrt{r^2-y^2}}^{\sqrt{r^2-y^2}}\int_{y_0}^{0} \exp\left[-\frac{2(x^2 + y^2)}{r^2}\right]dxdy = \varepsilon\frac{A_0^2}{r^2}\int_{y_0}^{0}\int_{-\sqrt{r^2-x^2}}^{\sqrt{r^2-x^2}} \exp\left[-\frac{2(x^2 + y^2)}{r^2}\right]dxdy \tag{14}$$

When $\sqrt{a^2 - r^2} \le d \le a + r$, $I(x,y)$ can be integrated directly on the region $s(d)$:

$$\begin{aligned} P(d) &= \iint_{s(d)} I(x, y)dxdy = \varepsilon\frac{A_0^2}{R^2}\iint_{s(d)} \exp\left[-\frac{2(x^2 + y^2)}{r^2}\right]dxdy \\ &= \varepsilon\frac{A_0^2}{R^2}\int_{-x_0}^{x_0}\int_{d-\sqrt{a^2-x^2}}^{\sqrt{r^2-x^2}} \exp\left[-\frac{2(x^2 + y^2)}{r^2}\right]dxdy \end{aligned} \tag{15}$$

Using Eqs. (8),(13), and (15) derived above, we can obtain the following equation for the effect of lateral error on coupling efficiency:

$$\eta_d = P(d)/P_0 \tag{16}$$

Although we cannot obtain the analytic solutions of the double integral equation (Eqs. 13 and 15) derived above, the results can be easily obtained by numerical calculations with the method of integral area subdivision.

3.2. Effect of lateral error on beam quality

Any plane in the fiber that passes through the axis of the fiber is called a meridional plane; accordingly, the light in the meridional plane is called meridional light. According to the reflection theorem, the incident light and the reflected light are always in the same plane. Therefore, meridional light is still in the original plane after total reflection. After each reflection, it intersects with the optical fiber axis. There is also other light called skew light in the optical fiber that is neither parallel to nor intersects with the fiber axis. Its total reflection is performed in three-dimensional space. Since the skew light and the fiber axis are not in one plane, the orientation of the skew light changes after each reflection. Its optical path is a spiral line that is equidistant from the fiber axis. The trajectory of

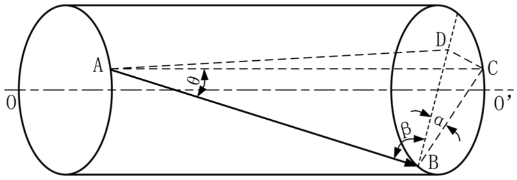


Fig. 3. The trajectory of skew light.

skew light is determined by two parameters: the axial angle θ and the azimuthal angle α. The axial angle θ is the angle between the light ray and the fiber axis. The azimuthal angle α is the angle between the projection of the light ray onto the fiber cross section and the line connecting the reflection point to the cross section center, as shown in Fig. 3.

During the process of a Gaussian beam transmitting through an optical fiber, meridional light can hardly cause a change in the beam quality, while skew light has a great influence on the beam quality. Taking skew light with an azimuthal angle $\alpha = 30^{\circ}$ and a fiber core radius $a = 0.5$ mm as an example, the cross-sectional projection of the light path and ZEMAX simulation results of skew light are shown in Fig. 4 and Fig. 5, respectively, where L is the length of the fiber.

The contrast between Fig. 4 and Fig. 5(a) shows that the contours of the two plots are very similar, which is consistent with our understanding. By comparing the power distributions with different L in Fig. 5, it is found that with increasing L, the power distribution of the skew light gradually changes from a triangle to a ring. This is because the azimuthal angle α of the skew light set in the simulation is not strictly equal to 30°. Only a small part of the skew light whose azimuthal angles strictly equal to 30° will remain on the 'triangle' path. The power distribution of other skew light will gradually become a ring, which is quite different from a Gaussian distribution.

For a Gaussian beam, lateral error d will change most of the meridional light into skew light, causing the power concentrated in the center to gradually extend outwards as the lateral error increases, forming a ring distribution. In addition, the increase in lateral error will directly lead to an increase in the azimuthal angle α of the skew light, increasing the diameter of the ring.

4. Results and discussion

4.1. Setup

To verify the calculation and analysis mentioned above, we perform simulations and experiments. First, we use ZEMAX to simulate the effect of lateral error on coupling efficiency and beam quality. The simulation method has the following advantages: 1) the values of the fiber core diameter, spot diameter and lateral error can be precisely controlled; 2) the effects of Fresnel reflection and transmission loss can be completely eliminated; 3) the model in ZEMAX is ideal without any defects; and 4) the effects of the light in the fiber cladding can be completely eliminated. During the simulation, we set the fiber core radius a to be 0.5 mm, and the spot radius r to be 0.2 mm.

The experimental setup is used for further verification. The setup consists of a laser, a fiber mount, a large-core multimode fiber, a power meter (NOVA II, *OPHIR*), and a beam-profiling camera (PYROCAM III, *OPHIR*), as shown in Fig. 6. The fiber mount allows precise control of the position of the fiber relative to the laser's optical axis, which is necessary to measure the effect of lateral error on

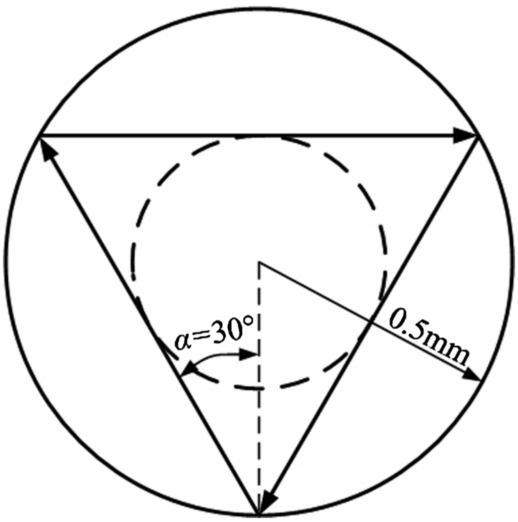


Fig. 4. The cross-sectional projection of the skew light path.

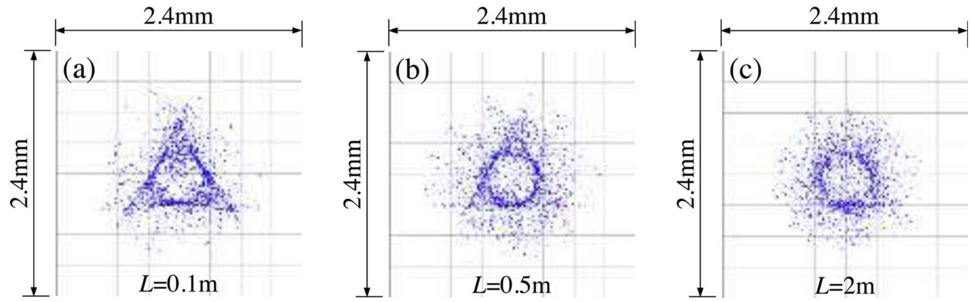
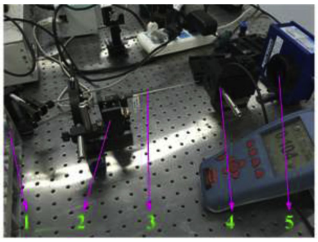


Fig. 5. Power distribution of the skew light with different fiber lengths: (a) $L = 0.1$ m, (b) $L = 0.5$ m and (c) $L = 2$ m.



1. Laser, 2. Fiber Mount, 3. Large-core Multimode Fiber, 4. Power Meter, 5. Beam-profiling Camera

Fig. 6. Experimental setup for measuring the effect of lateral error on coupling efficiency and beam quality.

coupling efficiency and beam quality. The fiber we use has a core diameter of 1 mm and a length of 25 cm.

4.2. Effect of lateral error on coupling efficiency

Using the parameters ($a = 0.5$ mm, $r = 0.2$ mm), we obtain the results of the calculation, simulation and experiment for the relationship between lateral error and coupling efficiency, as shown in Fig. 7 below.

From Fig. 7, it can be seen that the calculated result is in good agreement with the simulation data, thus proving to some extent the accuracy of our formula. Additionally, the overall trend of the experimental result remains consistent with the other two results. In addition, the experimental coupling efficiencies are lower than the calculated and simulated results due to reflection and transmission losses; other types of alignment errors besides lateral error; and the fact that the spot radius r at the incident end face of the fiber does not strictly equal 0.2 mm.

It is also seen that the coupling efficiency decreases nonlinearly with increasing lateral error, with the efficiency decreasing most rapidly, when the lateral error is equal to the fiber core radius a.

4.3. Effect of lateral error on beam quality

To verify the analysis mentioned above, we use ZEMAX to simulate the spot energy distribution in the case of lateral error $d = 0$ mm, $d = 0.1$ mm, $d = 0.2$ mm and $d = 0.3$ mm. The results are shown in Fig. 8. Similarly, we set $a = 0.5$ mm, $r = 0.2$ mm, and the fiber length L to be 2 m. Then, using the experimental setup shown in Fig. 6, the spot energy distribution under the condition of lateral error $d = 0$ mm, $d = 0.2$ mm, $d = 0.5$ mm, and $d = 0.6$ mm is measured, as shown in Fig. 9 below.

As the lateral error increases, the energy distribution of the laser gradually changes from a spot to a ring, and the diameter of the ring increases. This behavior is completely consistent with the results we analyzed above.

The above analysis results are more instructive for short fiber lengths in actual situations. This is because microscopic bends, irregularity of the core-cladding boundary and refractive index distribution fluctuations will change the mode distribution of the laser

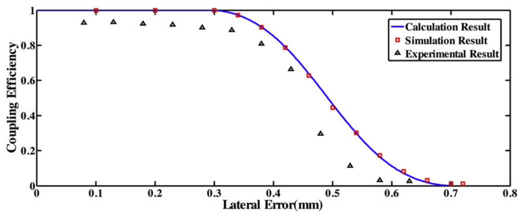


Fig. 7. Results of the effect of lateral error on coupling efficiency.

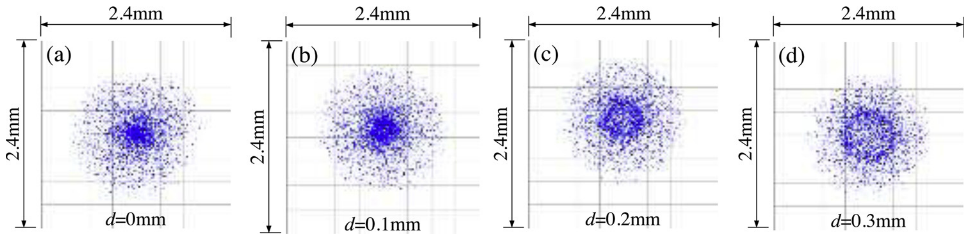


Fig. 8. Simulation results of the power distribution with different lateral errors: (a) $d = 0$ mm, (b) $d = 0.1$ mm, (c) $d = 0.2$ mm, and (d) $d = 0.3$ mm.

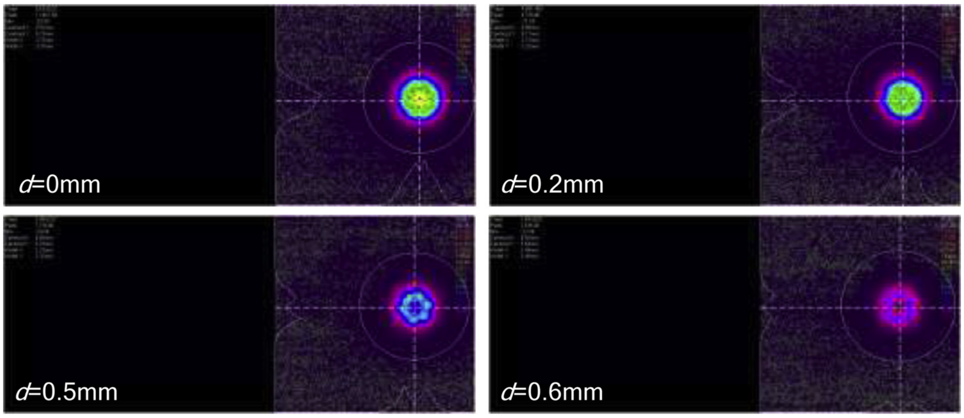


Fig. 9. Experimental results of the power distribution with different lateral errors: (a) $d = 0$ mm, (b) $d = 0.2$ mm, (c) $d = 0.5$ mm, and (d) $d = 0.6$ mm (the scale of the four figures is the same).

in the fiber core [20], and the influence increases with the increase of transmission distance.

5. Conclusion

In this paper, we have derived in detail a formula for calculating the effect of lateral error on the coupling efficiency of a Gaussian beam launched into a large-core multimode fiber. The formula has been verified with simulations and experiments, thus proving the formula to be accurate and reliable. In addition, this method of calculation has general applicability; as long as we know the power distribution function, we can derive a formula for the dependence of coupling efficiency on lateral error in the same way.

With the use of ZEMAX, we analyzed the two forms of light (meridional and skew light) transmitted through the fiber core and determined the effect of lateral error on the energy distribution of the Gaussian beam. When the optical axis of the beam is precisely aligned with the axis of the fiber, the meridional light dominates the fiber, and the power distribution remains the same. The lateral error can affect the ratio and distribution of the meridional light and the skew light in the beam. When the amount of skew light in the beam increases, the spot will gradually become a ring, with the radius of the ring increasing with increasing azimuthal angle.

The conclusions we have obtained can not only guide the design of a large-core multimode fiber coupler but also serve as a basis for the performance evaluation and fault diagnosis of the coupler.

Declarations of interest

None.

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