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Enhancing Elephant Herding Optimization with Novel Individual Updating Strategies for Large-Scale Optimization Problems

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Abstract: Inspired by the behavior of elephants in nature, elephant herd optimization (EHO) was proposed recently for global optimization. Like most other metaheuristic algorithms, EHO does not use the previous individuals in the later updating process. If the useful information in the previous individuals were fully exploited and used in the later optimization process, the quality of solutions may be improved significantly. In this paper, we propose several new updating strategies for EHO, in which one, two, or three individuals are selected from the previous iterations, and their useful information is incorporated into the updating process. Accordingly, the final individual at this iteration is generated according to the elephant generated by the basic EHO, and the selected previous elephants through a weighted sum. The weights are determined by a random number and the fitness of the elephant individuals at the previous iteration. We incorporated each of the six individual updating strategies individually into the basic EHO, creating six improved variants of EHO. We benchmarked these proposed methods using sixteen test functions. Our experimental results demonstrated that the proposed improved methods significantly outperformed the basic EHO.

Keywords: elephant herding optimization; EHO; swarm intelligence; individual updating strategy; large-scale; benchmark

1. Introduction

Inspired by nature, a large variety of metaheuristic algorithms [1] have been proposed that provide optimal or near-optimal solutions to various complex large-scale problems that are difficult to solve using traditional techniques. Some of the many successful metaheuristic approaches include particle swarm optimization (PSO) [2,3], cooperative coevolution [4–6], seagull optimization algorithm [7], GRASP [8], clustering algorithm [9], and differential evolution (DE) [10,11], among others.

In 2015, Wang et al. [12,13] proposed a new metaheuristic algorithm called elephant herd optimization (EHO), for finding the optimal or near-optimal function values. Although EHO exhibits a good performance on benchmark evaluations [12,13], like most other metaheuristic methods, it does not utilize the best information from the previous elephant individuals to guide current and future searches. This gap will be addressed, because previous individuals can provide a variety of useful information. If such information could be fully exploited and applied in the later updating process, the performance of EHO may be improved significantly, without adding unnecessary operations and fitness evaluations.

In the research presented in this paper, we extended and improved the performance of the original EHO (which we call “the basic EHO”) by fully investigating the information in the previous elephant individuals. Then, we designed six updating strategies to update the individuals. For each

of the six individual updating strategies, first, we selected a certain number of elephants from the previous iterations. This selection could be made in either a fixed or random way, with one, two, or three individuals selected from previous iterations. Next, we used the information from these selected previous individual elephants to update the individuals. In this way, the information from the previous individuals could be reused fully. The final elephant individual at this iteration was generated according to the elephant individual generated by the basic EHO at the current iteration, along with the selected previous elephants using a weighted sum. While there are many ways to determine the weights, in our current work, they were determined by random numbers. Last, by combining the six individual updating strategies with EHO, we developed the improved variants of EHO. To verify our work, we benchmarked these variants using sixteen cases involving large-scale complex functions. Our experimental results showed that the proposed variants of EHO significantly outperformed the originally described EHO.

The organization of the remainder of this paper is as follows. Section 2 reviews the main steps of the basic EHO. In Section 3, we describe the proposed method for incorporating useful information from previous elephants into the EHO. Section 4 provides details of our various experiments on sixteen large-scale functions. Lastly, Section 5 offers our conclusion and suggestions for future work.

2. Related Work

As EHO [12,13] is a newly-proposed swarm intelligence-based algorithm, in this section, some of the most representative work regarding swarm intelligence, including EHO, are summarized and reviewed.

Meena et al. [14] proposed an improved EHO algorithm, which is used to solve the multi-objective distributed energy resources (DER) accommodation problem of distribution systems by combining a technique for order of preference by similarity to ideal solution (TOPSIS). The proposed technique is productively implemented on three small- to large-scale benchmark test distribution systems of 33-bus, 118-bus, and 880-bus.

When the spectral resolution of the satellite imagery is increased, the higher within-class variability reduces the statistical separability between the LU/LC classes in spectral space and tends to continue diminishing the classification accuracy of the traditional classifiers. These are mostly per pixel and parametric in nature. Jayanth et al. [15] used EHO to solve the problems. The experimental results revealed that EHO shows an improvement of 10.7% on Arsikere Taluk and 6.63% on the NITK campus over the support vector machine.

Rashwan et al. [16] carried out a series of experiments on a standard test bench, as well as engineering problems and real-world problems, in order to understand the impact of the control parameters. On top of that, the main aim of this paper is to propose different approaches to enhance the performance of EHO. Case studies ranging from the recent test bench problems of Congress on Evolutionary Computation (CEC) 2016, to the popular engineering problems of the gear train, welded beam, three-bar truss design problem, continuous stirred tank reactor, and fed-batch fermentor, are used to validate and test the performances of the proposed EHOs against existing techniques.

Correia et al. [17] firstly used a metaheuristic algorithm, namely EHO, to address the energy-based source localization problem in wireless sensor networks. Through extensive simulations, the key parameters of the EHO algorithm are optimized, such that they match the energy decay model between two sensor nodes. The simulation results show that the new approach significantly outperforms the existing solutions in noisy environments, encouraging further improvement and testing of metaheuristic methods.

Jafari et al. [18] proposed a new hybrid algorithm that was based on EHO and cultural algorithm (CA), known as the elephant herding optimization cultural (EHOC) algorithm. In this process, the belief space defined by the cultural algorithm was used to improve the standard EHO. In EHOC, based on the belief space, the separating operator is defined, which can create new local optimums in the search space, so as to improve the algorithm search ability and to create an algorithm with an optimal

exploration–exploitation balance. The CA, EHO, and EHOC algorithms are applied to eight mathematical optimization problems and four truss weight minimization problems, and to assess the performance of the proposed algorithm, the results are compared. The results clearly indicate that EHOC can accelerate the convergence rate effectively and can develop better solutions compared with CA and EHO.

Hassanien et al. [19] combined support vector regression (SVR) with EHO in order to predict the values of the three emotional scales as continuous variables. Multiple experiments are applied to evaluate the prediction performance. EHO was applied in two stages of the optimization. Firstly, to fine-tune the regression parameters of the SVR. Secondly, to select the most relevant features extracted from all 40 EEG channels, and to eliminate the ineffective and redundant features. To verify the proposed approach, the results proved EHO-SVR's ability to gain a relatively enhanced performance, measured by a regression accuracy of 98.64%.

Besides EHO, many other swarm intelligence-based algorithms have been proposed, and some of the most representative ones are summarized and reviewed as follows.

Monarch butterfly optimization (MBO) [20] is proposed for global optimization problems, inspired by the migration behavior of monarch butterflies. Yi et al. [21] proposed a novel quantum inspired MBO methodology, called QMBO, by incorporating quantum computation into MBO, which is further used to solve uninhabited combat air vehicles (UCAV) path planning navigation problem [22,23]. In addition, Feng et al. proposed various variants of MBO algorithms to solve the knapsack problem [24–28]. In addition, Wang et al. also improved on the performance of the MBO algorithm from various aspects [29–32]; a variant of the MBO method in combination with two optimization strategies, namely GCMBO, was also put forward.

Inspired by the phototaxis and Lévy flights of the moths, Wang developed a new kind of metaheuristic algorithm, called the moth search (MS) algorithm [33]. Feng et al. [34] divided twelve transfer functions into three families, and combined them with MS, and then twelve discrete versions of MS algorithms are proposed for solving the set-union knapsack problem (SUKP). Based on the improvement of the moth search algorithm (MSA) using differential evolution (DE), Elaziz et al. [35] proposed a new method for the cloud task scheduling problem. In addition, Feng and Wang [36] verified the influence of the Lévy flights operator and fly straightly operator in MS. Nine types of new mutation operators based on the global harmony search have been specially devised to replace the Lévy flights operator.

Inspired by the herding behavior of krill, Gandomi and Alavi proposed a krill herd (KH) [37]. After that, Wang et al. improved the KH algorithms through different optimization strategies [38–46]. More literature regarding the KH algorithm can be found in the literature [47].

The artificial bee colony (ABC) algorithm [48] is a swarm-based meta-heuristic algorithm that was introduced by Karaboga in 2005 for optimizing numerical problems. Wang and Yi [49] presented a robust optimization algorithm, namely KHABC, based on hybridization of KH and ABC methods and the information exchange concept. In addition, Liu et al. [50] presented an ABC algorithm based on the dynamic penalty function and Lévy flight (DPLABC) for constrained optimization problems.

Also, many other researchers have proposed other state-of-the-art metaheuristic algorithms, such as particle swarm optimization (PSO) [51–56], cuckoo search (CS) [57–61], probability-based incremental learning (PBIL) [62], differential evolution (DE) [63–66], evolutionary strategy (ES) [67,68], monarch butterfly optimization (MBO) [20], firefly algorithm (FA) [69–72], earthworm optimization algorithm (EWA) [73], genetic algorithms (GAs) [74–76], ant colony optimization (ACO) [77–79], krill herd (KH) [37,80,81], invasive weed optimization [82–84], stud GA (SGA) [85], biogeography-based optimization (BBO) [86,87], harmony search (HS) [88–90], and bat algorithm (BA) [91,92], among others

Besides benchmark evaluations [93,94], these proposed state-of-the-art metaheuristic algorithms are also used to solve various practical engineering problems, like test-sheet composition [95], scheduling [96,97], clustering [98–100], cyber-physical social systems [101], economic load dispatch [102,103], fault diagnosis [104], flowshop [105], big data optimization [106,107], gesture segmentation [108], target

recognition [109,110], prediction of pupylation sites [111], system identification [112], shape design [113], multi-objective optimization [114], and many-objective optimization [115–117].

3. Elephant Herd Optimization

The basic EHO can be described using the following simplified rules [12]:

- (1) Elephants belonging to different clans live together led by a matriarch. Each clan has a fixed number of elephants. For the purposes of modelling, we assume that each clan consists of an equal, unchanging number of elephants.
- (2) The positions of the elephants in a clan are updated based on their relationship to the matriarch. EHO models this behavior through an updating operator.
- (3) Mature male elephants leave their family groups to live alone. We assume that during each generation, a fixed number of male elephants leave their clans. Accordingly, EHO models the updating process using a separating operator.
- (4) Generally, the matriarch in each clan is the eldest female elephant. For the purposes of modelling and solving the optimization problems, the matriarch is considered the fittest elephant individual in the clan.

As this paper is focused on improving the EHO updating process, in the following subsection, we provide further details of the EHO updating operator as it was originally presented. For details regarding the EHO separating operator, see the literature [12].

3.1. Clan Updating Operator

The following updating strategy of the basic EHO was described by the authors of [12], as follows. Assume that an elephant clan is denoted as ci . The next position of any elephant, j , in the clan is updated using (1), as follows:

$$x_{new,ci,j} = x_{ci,j} + \alpha \times (x_{best,ci} - x_{ci,j}) \times r, \quad (1)$$

where $x_{new,ci,j}$ is the updated position, and $x_{ci,j}$ is the prior position of elephant j in clan ci . $x_{best,ci}$ denotes the matriarch of clan ci ; she is the fittest elephant individual in the clan. The scale factor $\alpha \in [0, 1]$ determines the influence of the matriarch of ci on $x_{ci,j}$. $r \in [0, 1]$, which is a type of stochastic distribution, can provide a significant improvement for the diversity of the population in the later search phase. For the present work, a uniform distribution was used.

It should be noted that $x_{ci,j} = x_{best,ci}$, which means that the matriarch (fittest elephant) in the clan cannot be updated by (1). To avoid this situation, we can update the fittest elephant using the following equation:

$$x_{new,ci,j} = \beta \times x_{center,ci}, \quad (2)$$

where the influence of $x_{center,ci}$ on $x_{new,ci}$ is regulated by $\beta \in [0, 1]$.

In Equation (2), the information from all of the individuals in clan ci is used to create the new individual $x_{new,ci,j}$. The centre of clan ci , $x_{center,ci}$, can be calculated for the d -th dimension through D calculations, where D is the total dimension, as follows:

$$x_{center,ci,d} = \frac{1}{n_{ci}} \times \sum_{j=1}^{n_{ci}} x_{ci,j,d} \quad (3)$$

Here, $1 \leq d \leq D$ represents the d -th dimension, n_{ci} is the number of individuals in ci , and $x_{ci,j,d}$ is the d -th dimension of the individual $x_{ci,j}$.

Algorithm 1 provides the pseudocode for the updating operator.

Algorithm 1: Clan updating operator [12]

```

Begin
  for  $ci = 1$  to  $nClan$  (for all clans in elephant population) do
    for  $j = 1$  to  $n_{ci}$  (for all elephant individuals in clan  $ci$ ) do
      Update  $x_{ci,j}$  and generate  $x_{new,ci,j}$  according to (1).
      if  $x_{ci,j} = x_{best,ci}$  then
        Update  $x_{ci,j}$  and generate  $x_{new,ci,j}$  according to (2).
      end if
    end for  $j$ 
  end for  $ci$ 
End.

```

3.2. Separating Operator

In groups of elephants, male elephants leave their family group and live alone upon reaching puberty. This process of separation can be modeled into a separating operator when solving optimization problems. In order to further improve the search ability of the EHO method, let us assume that the individual elephants with the worst fitness will implement the separating operator for each generation, as shown in (4).

$$x_{worst,ci} = x_{\min} + (x_{\max} - x_{\min} + 1) \times rand \quad (4)$$

where $x_{\max}$ and $x_{\min}$ are the upper and lower bound, respectively, of the position of the individual elephant. $x_{worst,ci}$ is the worst individual elephant in clan ci . $rand \in [0, 1]$ is a kind of stochastic distribution, and the uniform distribution in the range $[0, 1]$ is used in our current work.

Accordingly, the separating operator can be formed, as shown in Algorithm 2.

Algorithm 2: Separating operator

```

Begin
  for  $ci = 1$  to  $nClan$  (all of the clans in the elephant population) do
    Replace the worst elephant individual in clan  $ci$  using (4).
  end for  $ci$ 
End.

```

3.3. Schematic Presentation of the Basic EHO Algorithm

For EHO, like the other metaheuristic algorithms, a kind of elitism strategy is used with the aim of protecting the best elephant individuals from being ruined by the clan updating and separating operators. In the beginning, the best elephant individuals are saved, and the worst ones are replaced by the saved best elephant individuals at the end of the search process. This elitism ensures that the later elephant population is not always worse than the former one. The schematic description can be summarized as shown in Algorithm 3.

As described before, the basic EHO algorithm does not take the best available information in the previous group of individual elephants to guide the current and later searches. This may lead to a slow convergence during the solution of certain complex, large-scale optimization problems. In our current work, some of the information used for the previous individual elephants was reused, with the aim of improving the search ability of the basic EHO algorithm.

Algorithm 3: Elephant Herd Optimization (EHO) [12]**Begin****Step 1: Initialization.**Set the generation counter $t = 1$.Initialize the population P of NP elephant individuals randomly, with uniform distribution in the search space.Set the number of the kept elephants $nKEL$, the maximum generation $MaxGen$, the scale factor α and β , the number of clan $nClan$, and the number of elephants for the ci -th clan n_{ci} .**Step 2: Fitness evaluation.**

Evaluate each elephant individual according to its position.

Step 3: While $t < MaxGen$ do the following:

Sort all of the elephant individuals according to their fitness.

Save the $nKEL$ elephant individuals.

Implement the clan updating operator as shown in Algorithm 1.

Implement the separating operator as shown in Algorithm 2.

Evaluate the population according to the newly updated positions.

Replace the worst elephant individuals with the $nKEL$ saved ones.Update the generation counter, $t = t + 1$.**Step 4: End while****Step 5: Output the best solution.****End.****4. Improving EHO with Individual Updating Strategies**

In this research, we propose six new versions of EHO based on individual updating strategies. In theory, k ($k \geq 1$) previous elephant individuals can be selected, but as more individuals ($k \geq 4$) are chosen, the calculations of the weights become more complex. Therefore, for this paper, we investigate $k \in \{1, 2, 3\}$.

Suppose that x_i^t is the i th individual at iteration t , and x_i and f_i^t are its position and fitness values, respectively. Here, t is the current iteration, $1 \leq i \leq N_p$ is an integer number, and N_p is the population size. y_i^{t+1} is the individual generated by the basic EHO, and f_i^{t+1} is its fitness. The framework of our proposed method is given through the individuals at the $(t - 2)$ th, $(t - 1)$ th, t th, and $(t + 1)$ th iterations.

4.1. Case of $k = 1$

The simplest case is when $k = 1$. The i th individual x_i^{t+1} can be generated as follows:

$$x_i^{t+1} = \theta y_i^{t+1} + \omega x_j^t, \quad (5)$$

where x_j^t is the position for individual j ($j \in \{1, 2, \dots, N_p\}$) at iteration t , and f_j^t is its fitness. θ and ω are weighting factors satisfying $\theta + \omega = 1$. They can be given as follows:

$$\theta = r, \quad \omega = 1 - r \quad (6)$$

Here, r is a random number that is drawn from the uniform distribution in $[0, 1]$. The individual j can be determined in the following ways:

- (1) $\underline{j} = i$;
- (2) $\underline{j} = r_1$, where r_1 is an integer between 1 and N_p that is selected randomly.

The individual generated by the second method has more population diversity than the individual generated the first way. We refer to these updating strategies as R1 and RR1, respectively. Their incorporation into the basic EHO results in EHOR1 and EHORR1, respectively.

4.2. Case of $k = 2$

Two individuals at two previous iterations are collected and used to generate elephant i . For this case, the i th individual x_i^{t+1} can be generated as follows:

$$x_i^{t+1} = \theta y_i^{t+1} + \omega_1 x_{j_1}^t + \omega_2 x_{j_2}^{t-1}, \quad (7)$$

where $x_{j_1}^t$ and $x_{j_2}^{t-1}$ are the positions for individuals j_1 and j_2 ($j_1, j_2 \in \{1, 2, \dots, N_P\}$) at iterations t and $t-1$, and $f_{j_1}^t$ and $f_{j_2}^{t-1}$ are their fitness values, respectively. θ , ω_1 , and ω_2 are weighting factors satisfying $\theta + \omega_1 + \omega_2 = 1$. They can be calculated as follows:

$$\begin{aligned} \theta &= r, \\ \omega_1 &= (1-r) \times \frac{f_{j_2}^{t-1}}{f_{j_2}^{t-1} + f_{j_1}^t}, \\ \omega_2 &= (1-r) \times \frac{f_{j_1}^t}{f_{j_2}^{t-1} + f_{j_1}^t}. \end{aligned} \quad (8)$$

Here, r is a random number that is drawn from the uniform distribution in $[0, 1]$. Individuals j_1 and j_2 in (8) can be determined in several different ways, but in this paper, we focus on the following two approaches:

- (1) $j_1 = j_2 = i$;
- (2) $j_1 = r_1$, and $j_2 = r_2$, where r_1 and r_2 are integers between 1 and N_P selected randomly.

As in the previous case, the individuals generated by the second method have more population diversity than the individuals generated the first way. We refer to these updating strategies as R2 and RR2, respectively. Their incorporation into EHO yields EHOR2 and EHORR2, respectively.

4.3. Case of $k = 3$

Three individuals at three previous iterations are collected and used to generate individual i . For this case, the i th individual x_i^{t+1} can be generated as follows:

$$x_i^{t+1} = \theta y_i^{t+1} + \omega_1 x_{j_1}^t + \omega_2 x_{j_2}^{t-1} + \omega_3 x_{j_3}^{t-2}, \quad (9)$$

where $x_{j_1}^t$, $x_{j_2}^{t-1}$ and $x_{j_3}^{t-2}$ are the positions of individuals j_1 , j_2 , and j_3 ($j_1, j_2, j_3 \in \{1, 2, \dots, N_P\}$) at iterations t , $t-1$, and $t-2$, and $f_{j_1}^t$, $f_{j_2}^{t-1}$, and $f_{j_3}^{t-2}$ are their fitness values, respectively. Their weighting factors are θ , ω_1 , ω_2 , and ω_3 with $\theta + \omega_1 + \omega_2 + \omega_3 = 1$. The calculation can be given as follows:

$$\begin{aligned} \theta &= r, \\ \omega_1 &= \frac{1}{2} \times (1-r) \times \frac{f_{j_2}^{t-1} + f_{j_3}^{t-2}}{f_{j_1}^t + f_{j_2}^{t-1} + f_{j_3}^{t-2}}, \\ \omega_2 &= \frac{1}{2} \times (1-r) \times \frac{f_{j_1}^t + f_{j_3}^{t-2}}{f_{j_1}^t + f_{j_2}^{t-1} + f_{j_3}^{t-2}}, \\ \omega_3 &= \frac{1}{2} \times (1-r) \times \frac{f_{j_1}^t + f_{j_2}^{t-1}}{f_{j_1}^t + f_{j_2}^{t-1} + f_{j_3}^{t-2}}. \end{aligned} \quad (10)$$

Although $j_1 \sim j_3$ can be determined in several ways, in this work, we adopt the following two methods:

- (1) $j_1 = j_2 = j_3 = i$;
- (2) $j_1 = r_1$, $j_2 = r_2$, and $j_3 = r_3$, where $r_1 \sim r_3$ are integer numbers between 1 and N_P selected at random.

As in the previous two cases, the individuals generated using the second method have more population diversity. We refer to these updating strategies as R3 and RR3, respectively. Their incorporation into EHO leads to EHOR3 and EHORR3, respectively.

5. Simulation Results

As discussed in Section 4, in the experimental part of our work, the six individual updating strategies (R1, RR1, R2, RR2, R3, and RR3) were incorporated separately into the basic EHO. Accordingly, we proposed six improved versions of EHO, namely: EHOR1, EHORR1, EHOR2, EHORR2, EHOR3, and EHORR3. For the sake of clarity, the basic EHO can also be identified as EHOR0, and we can call the updating strategies R0, R1, RR1, R2, RR2, R3, and RR3 for short. To provide a full assessment of the performance of each of the proposed individual updating strategies, we compared the six improved EHOs with each other and with the basic EHO. Through this comparison, we could look at the performance of the six updating strategies in order to determine whether these strategies were able to improve the performance of the EHO.

The six variants of the EHO were investigated fully from various respects through a series of experiments, using sixteen large-scale benchmarks with dimensions $D = 50, 100, 200, 500$, and 1000 . These complicated large-scale benchmarks can be found in Table 1. More information about all the benchmarks can be found in the literature [86,118,119].

Table 1. Sixteen benchmark functions.

No.	Name	No.	Name
F01	Ackley	F09	Rastrigin
F02	Alpine	F10	Schwefel 2.26
F03	Brown	F11	Schwefel 1.2
F04	Holzman 2 function	F12	Schwefel 2.22
F05	Levy	F13	Schwefel 2.21
F06	Penalty #1	F14	Sphere
F07	Powell	F15	Sum function
F08	Quartic with noise	F16	Zakharov

As all metaheuristic algorithms are based on a certain distribution, different runs will generate different results. With the aim of getting the most representative statistical results, we performed 30 independent runs under the same conditions, as shown in the literature [120].

For all of the methods studied in this paper, their parameters were set as follows: the scale factor $\alpha = 0.5$, $\beta = 0.1$, the number of the kept elephants $n_{KE} = 2$, and the number of clans $n_{Clan} = 5$. In the simplest form, all of the clans have an equal number of elephants. In our current work, all of the clans have the same number of elephants (i.e., $n_{ci} = 20$). Except for the number of elephants in each clan, the other parameters are the same as in the basic EHO, which can be found in the literature [12,13]. The best function values found by a certain intelligent algorithm are shown in bold font.

5.1. Unconstrained Optimization

5.1.1. $D = 50$

In this section of our work, seven kinds of EHOs (the basic EHO plus the six proposed improved variants) were evaluated using the 16 benchmarks mentioned previously, with dimension $D = 50$. The obtained mean function values and standard values from thirty runs are recorded in Tables 2 and 3.

From Table 2, we can see that in terms of the mean function values, R2 performed the best, at a level far better than the other methods. As for the other methods, R1 and RR1 provided a similar performance to each other, and they could find the smallest fitness values successfully on only one of the complex functions used for benchmarking. From Table 3, obviously, R2 performed in the most stable way, while for the other algorithms, EHO has a significant advantage over the other algorithms.

Table 2. Mean function values obtained by elephant herd optimization (EHO) and six improved methods with $D = 50$.

	EHO	R1	RR1	R2	RR2	R3	RR3
F01	2.57×10^{-4}	7.11×10^{-4}	0.05	8.38×10^{-5}	1.57	1.53×10^{-4}	0.01
F02	1.04×10^{-4}	2.69×10^{-4}	0.01	2.74×10^{-5}	0.23	5.13×10^{-5}	2.38×10^{-3}
F03	4.41×10^{-7}	9.16×10^{-6}	3.37×10^{-3}	6.14×10^{-9}	0.76	4.32×10^{-8}	8.25×10^{-5}
F04	1.50×10^{-15}	4.97×10^{-11}	3.58×10^{-6}	2.27×10^{-16}	0.03	3.38×10^{-16}	1.82×10^{-9}
F05	4.49	4.25	3.95	4.43	4.89	4.44	4.50
F06	1.22	1.06	1.62	1.72	2.01	1.76	1.79
F07	5.13×10^{-7}	2.55×10^{-6}	0.02	3.50×10^{-8}	2.25	1.51×10^{-7}	4.85×10^{-4}
F08	2.57×10^{-16}	1.24×10^{-15}	7.23×10^{-9}	2.21×10^{-16}	1.72×10^{-5}	2.21×10^{-16}	4.03×10^{-13}
F09	2.59×10^{-6}	6.86×10^{-5}	0.03	9.83×10^{-8}	9.28	5.06×10^{-7}	3.92×10^{-3}
F10	1.65×10^4	1.64×10^4	1.63×10^4	1.65×10^4	1.61×10^4	1.64×10^4	1.64×10^4
F11	1.44×10^{-5}	4.47×10^{-4}	0.36	1.02×10^{-6}	49.04	3.52×10^{-6}	2.18
F12	1.07×10^{-3}	3.81×10^{-3}	0.14	2.96×10^{-4}	2.37	5.31×10^{-4}	0.02
F13	6.69×10^{-4}	1.34×10^{-3}	0.07	1.52×10^{-4}	1.21	3.05×10^{-4}	0.01
F14	1.27×10^{-8}	1.63×10^{-7}	3.85×10^{-4}	7.00×10^{-10}	0.04	2.50×10^{-9}	3.02×10^{-6}
F15	6.70×10^{-7}	1.40×10^{-5}	7.03×10^{-3}	4.76×10^{-8}	3.61	1.77×10^{-7}	9.87×10^{-4}
F16	1.28×10^{-3}	0.30	512.90	3.00×10^{-5}	3.87×10^7	1.71×10^{-4}	0.56
TOTAL	0	1	1	14	0	0	0

Table 3. Standard values obtained by EHO and six improved methods with $D = 50$.

	EHO	R1	RR1	R2	RR2	R3	RR3
F01	2.39×10^{-5}	9.89×10^{-4}	0.06	3.60×10^{-5}	0.22	4.89×10^{-5}	0.01
F02	1.37×10^{-5}	3.32×10^{-4}	0.01	1.14×10^{-5}	0.04	1.38×10^{-5}	2.17×10^{-3}
F03	1.55×10^{-8}	3.34×10^{-5}	6.16×10^{-3}	3.72×10^{-9}	0.07	3.52×10^{-8}	7.80×10^{-5}
F04	4.50×10^{-16}	2.49×10^{-10}	1.08×10^{-5}	6.72×10^{-18}	0.01	2.92×10^{-16}	5.64×10^{-9}
F05	0.16	0.26	0.52	0.30	0.20	0.30	0.33
F06	0.23	0.18	0.41	0.28	0.32	0.25	0.27
F07	1.40×10^{-7}	5.25×10^{-6}	0.04	2.71×10^{-8}	0.76	7.78×10^{-8}	1.58×10^{-3}
F08	6.86×10^{-18}	3.99×10^{-15}	1.96×10^{-8}	1.68×10^{-20}	6.10×10^{-6}	1.26×10^{-19}	1.17×10^{-12}
F09	4.61×10^{-7}	1.75×10^{-4}	0.09	6.36×10^{-8}	1.60	3.11×10^{-7}	0.02
F10	444.40	591.40	502.50	506.70	486.40	454.70	349.00
F11	3.73×10^{-6}	1.98×10^{-3}	0.71	8.92×10^{-7}	29.12	2.30×10^{-6}	9.49
F12	1.03×10^{-4}	5.54×10^{-3}	0.15	1.03×10^{-4}	0.28	1.21×10^{-4}	0.01
F13	9.58×10^{-5}	1.97×10^{-3}	0.07	5.39×10^{-5}	0.15	6.84×10^{-5}	6.05×10^{-3}
F14	2.07×10^{-9}	5.02×10^{-7}	1.09×10^{-3}	6.10×10^{-10}	0.01	2.04×10^{-9}	3.90×10^{-6}
F15	1.00×10^{-7}	4.76×10^{-5}	0.02	3.41×10^{-8}	0.73	8.84×10^{-8}	3.40×10^{-3}
F16	8.21×10^{-4}	1.43	1.23×10^3	2.06×10^{-5}	1.92×10^7	1.21×10^{-4}	0.44
TOTAL	3	1	0	11	0	0	1

5.1.2. $D = 100$

As above, the same seven kinds of EHOs were evaluated using the sixteen benchmarks mentioned previously, with dimension $D = 100$. The obtained mean function values and standard values from 30 runs are recorded in Tables 4 and 5.

Regarding the mean function values, Table 4 shows that R2 performed much better than the other algorithms, providing the smallest function values on 13 out of 16 functions. As for the other algorithms, R0, R1, and RR1 gave a similar performance to each other, performing the best only on one function each (F10, F06, and F05, respectively). From Table 5, obviously, R2 performed in the most stable way, while for the other algorithms, EHO has significant advantage over other algorithms.

Table 4. Mean function values obtained by EHO and six improved methods with $D = 100$.

	EHO	R1	RR1	R2	RR2	R3	RR3
F01	3.22×10^{-4}	6.27×10^{-4}	0.12	6.92×10^{-5}	1.70	1.78×10^{-4}	0.01
F02	2.55×10^{-4}	6.86×10^{-4}	0.04	6.28×10^{-5}	0.48	1.08×10^{-4}	4.78×10^{-3}
F03	9.83×10^{-7}	2.65×10^{-5}	1.27×10^{-3}	1.55×10^{-8}	1.50	9.96×10^{-8}	2.01×10^{-4}
F04	1.18×10^{-14}	1.85×10^{-9}	7.43×10^{-6}	2.55×10^{-16}	0.13	7.25×10^{-16}	1.04×10^{-9}
F05	9.19	9.01	8.41	9.22	9.51	9.07	9.26
F06	3.11	2.91	3.89	3.74	4.30	3.80	3.80
F07	2.56×10^{-6}	4.65×10^{-5}	0.02	7.34×10^{-8}	5.33	3.31×10^{-7}	3.35×10^{-3}
F08	4.18×10^{-16}	1.13×10^{-13}	1.38×10^{-7}	2.21×10^{-16}	8.16×10^{-5}	2.24×10^{-16}	3.03×10^{-12}
F09	8.20×10^{-6}	4.58×10^{-5}	0.08	2.47×10^{-7}	19.19	1.05×10^{-6}	1.30×10^{-3}
F10	3.58×10^4	3.56×10^4	3.50×10^4	3.55×10^4	3.59×10^4	3.56×10^4	3.68×10^4
F11	6.15×10^{-5}	4.45×10^{-3}	3.54	5.51×10^{-6}	200.20	1.74×10^{-5}	0.07
F12	2.53×10^{-3}	5.37×10^{-3}	0.26	5.94×10^{-4}	5.27	1.04×10^{-3}	0.05
F13	8.12×10^{-4}	1.50×10^{-3}	0.06	1.72×10^{-4}	1.46	3.52×10^{-4}	0.02
F14	3.99×10^{-8}	1.09×10^{-6}	5.59×10^{-4}	1.23×10^{-9}	0.09	4.99×10^{-9}	8.68×10^{-6}
F15	4.45×10^{-6}	5.90×10^{-4}	0.11	2.52×10^{-7}	15.00	8.77×10^{-7}	1.83×10^{-3}
F16	0.04	2.20	7.85×10^5	7.09×10^{-4}	1.58×10^{10}	3.31×10^{-3}	987.40
TOTAL	1	1	1	13	0	0	0

Table 5. Standard values obtained by EHO and six improved methods with $D = 100$.

	EHO	R1	RR1	R2	RR2	R3	RR3
F01	1.85×10^{-5}	8.00×10^{-4}	0.38	2.47×10^{-5}	0.12	4.83×10^{-5}	0.01
F02	1.24×10^{-5}	1.10×10^{-3}	0.08	2.65×10^{-5}	0.08	3.99×10^{-5}	3.78×10^{-3}
F03	2.22×10^{-8}	9.04×10^{-5}	1.77×10^{-3}	8.47×10^{-9}	0.21	4.81×10^{-8}	1.77×10^{-4}
F04	2.03×10^{-15}	1.02×10^{-8}	2.98×10^{-5}	3.90×10^{-17}	0.05	6.76×10^{-16}	4.03×10^{-9}
F05	0.11	0.29	0.76	0.25	0.10	0.31	0.25
F06	0.31	0.33	0.38	0.29	0.41	0.25	0.29
F07	3.85×10^{-7}	1.44×10^{-4}	0.04	6.16×10^{-8}	0.95	1.70×10^{-7}	0.01
F08	2.21×10^{-17}	5.63×10^{-13}	4.37×10^{-7}	4.23×10^{-20}	3.09×10^{-5}	3.81×10^{-19}	9.87×10^{-12}
F09	8.05×10^{-7}	1.53×10^{-4}	0.18	1.46×10^{-7}	3.21	6.43×10^{-7}	5.79×10^{-4}
F10	733.60	769.20	834.60	606.00	547.10	725.00	621.20
F11	1.18×10^{-5}	0.01	13.78	4.34×10^{-6}	114.20	9.54×10^{-6}	0.25
F12	1.23×10^{-4}	0.01	0.32	2.23×10^{-4}	0.50	2.95×10^{-4}	0.03
F13	6.13×10^{-5}	3.10×10^{-3}	0.08	4.52×10^{-5}	0.17	1.02×10^{-4}	0.01
F14	3.74×10^{-9}	3.58×10^{-6}	1.84×10^{-3}	8.08×10^{-10}	0.01	2.98×10^{-9}	8.97×10^{-6}
F15	5.46×10^{-7}	2.88×10^{-3}	0.23	2.59×10^{-7}	2.76	6.25×10^{-7}	4.04×10^{-3}
F16	3.18×10^{-3}	8.63	3.03×10^6	4.19×10^{-4}	3.52×10^9	1.74×10^{-3}	4.76×10^3
TOTAL	3	0	0	10	2	1	0

5.1.3. $D = 200$

Next, the seven types of EHOs were evaluated using the 16 benchmarks mentioned previously, with dimension $D = 200$. The obtained mean function values and standard values from 30 runs are recorded in Tables 6 and 7.

From Table 6, we can see that in terms of the mean function values, R2 performed much better than the other algorithms, providing the smallest function values on 13 out of 16 of the benchmark functions. As for the other methods, R1 ranked second, having performed the best on two of the benchmark functions. RR1 ranked third, giving the best result on one of the functions. From Table 7, obviously, R2 performed in the most stable way, while for the other algorithms, EHO has a significant advantage over the other algorithms.

Table 6. Mean function values obtained by EHO and six improved methods with $D = 200$.

	EHO	R1	RR1	R2	RR2	R3	RR3
F01	3.68×10^{-4}	8.01×10^{-4}	0.03	8.94×10^{-5}	1.71	1.78×10^{-4}	9.60×10^{-3}
F02	5.65×10^{-4}	7.10×10^{-4}	0.06	1.08×10^{-4}	1.06	2.19×10^{-4}	0.01
F03	2.17×10^{-6}	2.23×10^{-5}	8.24×10^{-3}	2.72×10^{-8}	3.12	1.83×10^{-7}	7.55×10^{-4}
F04	7.43×10^{-14}	1.46×10^{-9}	1.82×10^{-5}	3.25×10^{-16}	0.56	3.46×10^{-15}	2.37×10^{-8}
F05	18.45	18.39	17.57	18.53	18.70	18.50	18.50
F06	6.90	6.89	8.03	7.60	8.67	7.72	7.82
F07	7.24×10^{-6}	2.87×10^{-5}	0.04	1.76×10^{-7}	11.34	6.97×10^{-7}	2.21×10^{-3}
F08	1.20×10^{-15}	2.20×10^{-12}	1.05×10^{-8}	2.22×10^{-16}	3.94×10^{-4}	2.21×10^{-16}	8.55×10^{-11}
F09	2.16×10^{-5}	1.46×10^{-4}	0.14	6.14×10^{-7}	40.87	2.38×10^{-6}	0.01
F10	7.58×10^4	7.43×10^4	7.59×10^4	7.58×10^4	7.50×10^4	7.58×10^4	7.58×10^4
F11	2.41×10^{-4}	2.46×10^{-3}	7.16	2.72×10^{-5}	699.60	7.04×10^{-5}	0.14
F12	5.71×10^{-3}	0.02	0.37	1.29×10^{-3}	11.26	2.41×10^{-3}	0.12
F13	9.65×10^{-4}	2.89×10^{-3}	0.06	1.93×10^{-4}	1.57	3.64×10^{-4}	0.02
F14	1.08×10^{-7}	1.82×10^{-6}	6.24×10^{-4}	2.74×10^{-9}	0.19	1.24×10^{-8}	1.46×10^{-5}
F15	2.25×10^{-5}	1.90×10^{-4}	0.96	1.02×10^{-6}	63.15	4.19×10^{-6}	0.01
F16	1.52	1.53×10^3	4.00×10^8	0.01	5.17×10^{12}	0.08	4.61×10^5
TOTAL	0	2	1	13	0	0	0

Table 7. Standard values obtained by EHO and six improved methods with $D = 200$.

	EHO	R1	RR1	R2	RR2	R3	RR3
F01	1.10×10^{-5}	1.35×10^{-3}	0.04	4.38×10^{-5}	0.13	5.87×10^{-5}	4.78×10^{-3}
F02	2.70×10^{-5}	8.25×10^{-4}	0.10	3.76×10^{-5}	0.12	4.61×10^{-5}	0.01
F03	3.50×10^{-8}	7.44×10^{-5}	0.02	1.65×10^{-8}	0.27	1.10×10^{-7}	1.55×10^{-3}
F04	1.35×10^{-14}	5.94×10^{-9}	4.19×10^{-5}	1.21×10^{-16}	0.14	9.76×10^{-15}	1.22×10^{-7}
F05	0.12	0.16	0.76	0.17	0.05	0.18	0.18
F06	0.34	0.40	0.28	0.30	0.58	0.22	0.29
F07	6.49×10^{-7}	6.58×10^{-5}	0.04	1.11×10^{-7}	2.00	4.21×10^{-7}	5.56×10^{-3}
F08	7.08×10^{-17}	1.13×10^{-11}	1.80×10^{-8}	1.61×10^{-19}	1.28×10^{-4}	1.05×10^{-18}	4.33×10^{-10}
F09	1.44×10^{-6}	3.78×10^{-4}	0.23	5.66×10^{-7}	3.79	1.43×10^{-6}	0.05
F10	897.80	853.40	971.60	1.12×10^3	833.10	1.02×10^3	907.60
F11	4.63×10^{-5}	5.96×10^{-3}	14.39	2.64×10^{-5}	353.00	5.11×10^{-5}	0.27
F12	2.72×10^{-4}	0.03	0.36	4.03×10^{-4}	1.09	8.39×10^{-4}	0.13
F13	7.38×10^{-5}	4.15×10^{-3}	0.08	7.99×10^{-5}	0.16	9.51×10^{-5}	0.01
F14	7.41×10^{-9}	7.42×10^{-6}	1.10×10^{-3}	1.96×10^{-9}	0.02	1.03×10^{-8}	1.03×10^{-5}
F15	1.88×10^{-6}	3.50×10^{-4}	2.13	8.43×10^{-7}	9.40	2.12×10^{-6}	0.04
F16	0.14	4.85×10^3	1.74×10^9	8.45×10^{-3}	7.73×10^{11}	0.04	2.00×10^6
TOTAL	4	0	0	9	2	1	0

5.1.4. $D = 500$

The seven kinds of EHOs also were evaluated using the same 16 benchmarks mentioned previously, with dimension $D = 500$. The obtained mean function values and standard values from 30 runs are recorded in Tables 8 and 9.

In terms of the mean function values, Table 8 shows that R2 performed much better than the other methods, providing the smallest function values on 13 out of 16 functions. In comparison, R0, RR1, and RR3 gave similar performances to each other, performing the best on only one function each. From Table 9, obviously, R2 performed in the most stable way, while for the other algorithms, EHO has a significant advantage over the other algorithms.

Table 8. Mean function values obtained by EHO and six improved methods with $D = 500$.

	EHO	R1	RR1	R2	RR2	R3	RR3
F01	3.92×10^{-4}	7.03×10^{-4}	0.03	8.75×10^{-5}	1.76	1.72×10^{-4}	8.52×10^{-3}
F02	1.53×10^{-3}	3.40×10^{-3}	0.11	2.99×10^{-4}	2.58	5.47×10^{-4}	0.02
F03	5.60×10^{-6}	8.70×10^{-5}	0.04	5.67×10^{-8}	7.91	4.15×10^{-7}	2.14×10^{-3}
F04	6.51×10^{-13}	4.40×10^{-9}	7.69×10^{-3}	1.48×10^{-15}	3.94	1.45×10^{-14}	7.70×10^{-8}
F05	45.80	45.90	45.44	45.92	45.99	45.91	45.93
F06	18.55	19.23	19.93	19.44	21.13	19.43	19.62
F07	2.35×10^{-5}	6.78×10^{-5}	0.24	4.42×10^{-7}	27.03	1.59×10^{-6}	8.63×10^{-3}
F08	8.52×10^{-15}	3.89×10^{-13}	1.60×10^{-7}	2.23×10^{-16}	2.42×10^{-3}	2.39×10^{-16}	1.51×10^{-10}
F09	6.13×10^{-5}	2.15×10^{-3}	0.87	1.31×10^{-6}	107.70	6.53×10^{-6}	0.02
F10	1.96×10^5	1.97×10^5	1.97×10^5	1.99×10^5	1.99×10^5	1.92×10^5	1.95×10^5
F11	1.61×10^{-3}	0.17	10.43	1.40×10^{-4}	5.30×10^3	4.16×10^{-4}	0.71
F12	0.02	0.03	1.30	3.15×10^{-3}	30.31	6.04×10^{-3}	0.57
F13	1.07×10^{-3}	3.01×10^{-3}	0.06	2.07×10^{-4}	1.72	4.22×10^{-4}	0.02
F14	3.12×10^{-7}	4.56×10^{-6}	5.26×10^{-3}	6.15×10^{-9}	0.48	2.74×10^{-8}	6.08×10^{-5}
F15	1.71×10^{-4}	7.41×10^{-4}	1.85	7.59×10^{-6}	427.30	2.17×10^{-5}	0.07
F16	1.60×10^3	3.60×10^7	1.33×10^{12}	1.20	8.83×10^{15}	16.25	2.37×10^9
TOTAL	1	0	1	13	0	0	1

Table 9. Standard function values obtained by EHO and six improved methods with $D = 500$.

	EHO	R1	RR1	R2	RR2	R3	RR3
F01	7.22×10^{-6}	8.01×10^{-4}	0.02	2.66×10^{-5}	0.08	4.32×10^{-5}	4.04×10^{-3}
F02	4.67×10^{-5}	4.09×10^{-3}	0.11	1.19×10^{-4}	0.20	1.49×10^{-4}	0.01
F03	5.65×10^{-8}	2.10×10^{-4}	0.13	2.62×10^{-8}	0.51	2.91×10^{-7}	5.94×10^{-3}
F04	6.17×10^{-14}	1.83×10^{-8}	0.04	3.12×10^{-15}	0.98	2.53×10^{-14}	2.84×10^{-7}
F05	0.09	0.02	0.75	0.03	0.05	0.04	0.02
F06	0.31	0.32	0.18	0.21	0.85	0.15	0.25
F07	1.05×10^{-6}	1.35×10^{-4}	0.78	3.05×10^{-7}	3.86	9.25×10^{-7}	0.02
F08	3.61×10^{-16}	1.72×10^{-12}	4.03×10^{-7}	2.01×10^{-18}	5.40×10^{-4}	1.31×10^{-17}	6.30×10^{-10}
F09	2.15×10^{-6}	0.01	1.71	9.91×10^{-7}	12.30	4.69×10^{-6}	0.04
F10	1.54×10^3	1.43×10^3	1.30×10^3	1.28×10^3	1.38×10^3	1.45×10^3	1.65×10^3
F11	3.44×10^{-4}	0.70	12.21	9.50×10^{-5}	2.00×10^3	2.38×10^{-4}	1.51
F12	3.52×10^{-4}	0.04	1.93	1.23×10^{-3}	2.11	2.09×10^{-3}	1.24
F13	5.90×10^{-5}	4.79×10^{-3}	0.06	6.75×10^{-5}	0.12	1.69×10^{-4}	0.01
F14	1.75×10^{-8}	1.60×10^{-5}	0.02	3.39×10^{-9}	0.05	1.32×10^{-8}	1.16×10^{-4}
F15	8.50×10^{-6}	1.04×10^{-3}	3.96	6.20×10^{-6}	50.28	1.15×10^{-5}	0.19
F16	113.00	1.90×10^8	4.50×10^{12}	2.37	1.61×10^{15}	17.62	1.27×10^{10}
TOTAL	4	0	0	9	0	1	2

5.1.5. $D = 1000$

Finally, the same seven types of EHOs were evaluated using the 16 benchmarks mentioned previously, with dimension $D = 1000$. The obtained mean function values and standard values from 30 runs are recorded in Tables 10 and 11.

In terms of the mean function values, Table 10 shows that R2 had the absolute advantage over the other metaheuristic algorithms, succeeding in finding function values on 12 out of 16 functions. Among the other metaheuristic algorithms, R0 ranked second, having performed the best on three of the benchmark functions. In addition, RR1 was successful in finding the best function value. From Table 11, obviously, R2 performed in the most stable way, while for the other algorithms, EHO has a significant advantage over the other algorithms.

Table 10. Mean function values obtained by EHO and six improved methods with $D = 1000$.

	EHO	R1	RR1	R2	RR2	R3	RR3
F01	4.04×10^{-4}	1.29×10^{-3}	0.03	8.66×10^{-5}	1.79	1.90×10^{-4}	0.01
F02	3.14×10^{-3}	5.68×10^{-3}	0.20	6.20×10^{-4}	5.45	1.29×10^{-3}	0.04
F03	1.11×10^{-5}	5.10×10^{-5}	0.07	1.41×10^{-7}	16.18	8.78×10^{-7}	1.97×10^{-3}
F04	3.05×10^{-12}	1.17×10^{-9}	7.48×10^{-4}	4.48×10^{-15}	16.77	3.01×10^{-14}	4.97×10^{-7}
F05	91.29	91.36	91.19	91.36	91.53	91.36	91.38
F06	38.23	38.90	39.79	39.11	42.71	39.08	39.26
F07	5.15×10^{-5}	1.20×10^{-3}	0.39	1.01×10^{-6}	55.16	3.93×10^{-6}	0.01
F08	3.73×10^{-14}	1.00×10^{-11}	3.77×10^{-6}	2.28×10^{-16}	0.01	2.73×10^{-16}	1.66×10^{-8}
F09	1.33×10^{-4}	1.91×10^{-4}	0.91	3.21×10^{-6}	221.00	1.00×10^{-5}	0.05
F10	3.94×10^5	3.94×10^5	4.01×10^5	4.00×10^5	3.98×10^5	3.97×10^5	3.98×10^5
F11	5.88×10^{-3}	0.39	1.18×10^3	4.79×10^{-4}	2.06×10^4	1.45×10^{-3}	7.41
F12	0.03	0.09	1.94	66.74	72.28	54.85	56.84
F13	1.16×10^{-3}	1.77×10^{-3}	0.12	2.40×10^{-4}	1.87	4.74×10^{-4}	0.02
F14	6.68×10^{-7}	4.59×10^{-6}	0.03	1.79×10^{-8}	1.00	5.22×10^{-8}	1.46×10^{-4}
F15	7.43×10^{-4}	0.02	14.34	2.45×10^{-5}	1.73×10^3	9.52×10^{-5}	0.57
F16	4.71×10^5	8.97×10^8	1.28×10^{14}	77.47	2.51×10^{18}	3.89×10^3	4.76×10^{12}
TOTAL	3	0	1	12	0	0	0

Table 11. Standard function values obtained by EHO and six improved methods with $D = 1000$.

	EHO	R1	RR1	R2	RR2	R3	RR3
F01	8.50×10^{-6}	2.69×10^{-3}	0.04	2.81×10^{-5}	0.07	6.05×10^{-5}	0.01
F02	6.45×10^{-5}	6.76×10^{-3}	0.24	3.09×10^{-4}	0.39	4.28×10^{-4}	0.03
F03	7.82×10^{-8}	1.24×10^{-4}	0.21	1.02×10^{-7}	1.70	4.10×10^{-7}	2.21×10^{-3}
F04	2.22×10^{-13}	3.61×10^{-9}	1.86×10^{-3}	8.75×10^{-15}	2.65	3.01×10^{-14}	1.80×10^{-6}
F05	0.06	0.02	0.49	0.01	0.06	0.01	0.01
F06	0.22	0.39	0.15	0.22	1.08	0.18	0.22
F07	1.06×10^{-6}	4.96×10^{-3}	0.58	6.86×10^{-7}	5.68	1.98×10^{-6}	0.01
F08	9.68×10^{-16}	2.77×10^{-11}	1.55×10^{-5}	3.36×10^{-18}	2.65×10^{-3}	6.47×10^{-17}	9.11×10^{-8}
F09	4.43×10^{-6}	3.58×10^{-4}	1.64	2.11×10^{-6}	20.66	7.28×10^{-6}	0.09
F10	2.19×10^3	2.04×10^3	2.11×10^3	1.98×10^3	1.60×10^3	1.93×10^3	2.75×10^3
F11	9.97×10^{-4}	1.46	4.01×10^3	4.03×10^{-4}	9.58×10^3	8.04×10^{-4}	24.87
F12	5.39×10^{-4}	0.15	1.76	4.90	1.61	2.31	1.48
F13	6.61×10^{-5}	2.68×10^{-3}	0.15	9.41×10^{-5}	0.15	1.45×10^{-4}	0.01
F14	2.21×10^{-8}	1.05×10^{-5}	0.10	1.05×10^{-8}	0.08	2.83×10^{-8}	2.89×10^{-4}
F15	2.62×10^{-5}	0.04	52.24	2.14×10^{-5}	213.10	4.96×10^{-5}	1.43
F16	2.34×10^4	3.39×10^9	4.12×10^{14}	93.37	4.52×10^{17}	3.42×10^3	2.67×10^{13}
TOTAL	5	0	1	8	1	1	0

5.1.6. Summary of Function Values Obtained by Seven Variants of EHOs

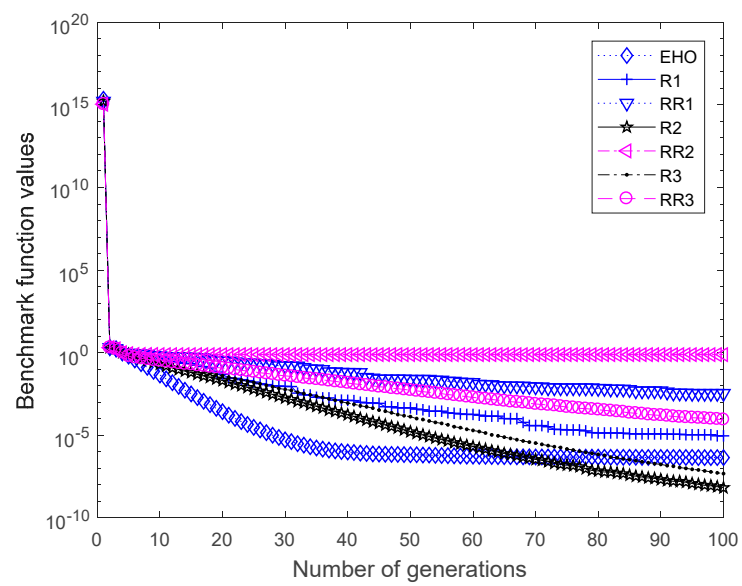
In Section 4.1, the mean function values obtained from 30 runs were collected and analyzed. In addition, the best, mean, worst, and standard (STD) function values obtained from 30 implementations were summarized and are recorded, as shown in Table 12.

From Table 12, we can see that, in general, R2 performed far better than the six other algorithms. R1 and R0 were second and third in performance, respectively, among all of the seven tested methods. Except for R0, R1, and R2, the other metaheuristic algorithms provided similar performances, which were highly inferior to R0, R1, and R2. Looking carefully at Table 12 for the best function values, R1 provided the best performance among the seven metaheuristic algorithms, which was far better than R2. This indicates that finding a means to improve the best performance of R2 further is a challenging question in EHO studies.

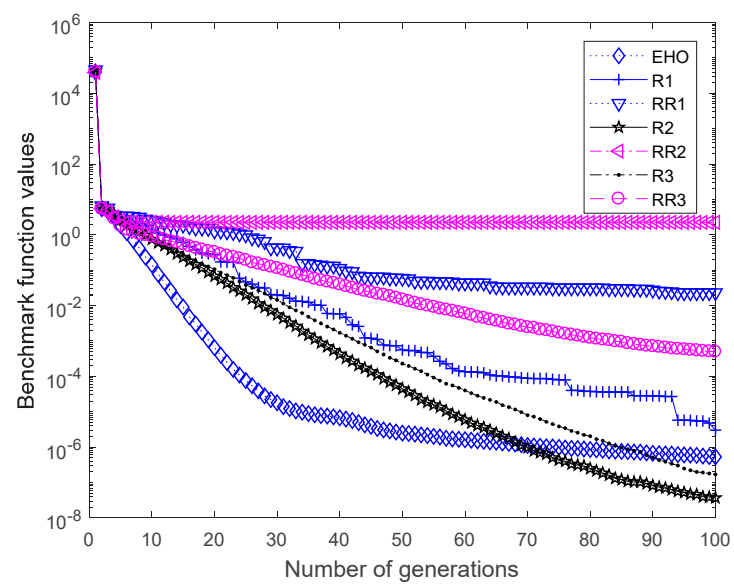
Table 12. Optimization results values obtained by EHO and six improved methods for 16 benchmark functions. STD—standard.

<i>D</i>		EHO	R1	RR1	R2	RR2	R3	RR3
50	BEST	0	13	2	1	0	0	0
	MEAN	0	1	1	14	0	0	0
	WORST	0	2	0	13	0	0	1
	STD	3	1	0	11	0	0	1
	TOTAL	3	17	3	39	0	0	2
100	BEST	0	13	1	2	0	0	0
	MEAN	1	1	1	13	0	0	0
	WORST	1	2	0	13	0	0	0
	STD	3	0	0	10	2	1	0
	TOTAL	5	16	2	38	2	1	0
200	BEST	1	10	1	3	0	1	0
	MEAN	0	2	1	13	0	0	0
	WORST	1	2	0	13	0	0	0
	STD	4	0	0	9	2	1	0
	TOTAL	6	14	2	38	2	2	0
500	BEST	1	11	1	2	0	0	1
	MEAN	1	0	1	13	0	0	1
	WORST	2	0	0	14	0	0	0
	STD	4	0	0	9	0	1	2
	TOTAL	8	11	2	38	0	1	4
1000	BEST	0	11	1	3	0	0	1
	MEAN	3	0	1	12	0	0	0
	WORST	3	1	0	12	0	0	0
	STD	5	0	1	8	1	1	0
	TOTAL	11	12	3	35	1	1	1

To provide a clear demonstration of the effectiveness of the different individual updating strategies, in this part of our work, we selected five functions randomly from the 16 large-scale complex functions, and their convergence histories with dimension $D = 50, 100, 200, 500$, and 1000 . From Figures 1–5, we can see that of the seven metaheuristic algorithms, R2 succeeded in finding the best function values at the end of the search in each of these five large-scale complicated functions. This trend coincided with our previous analysis.

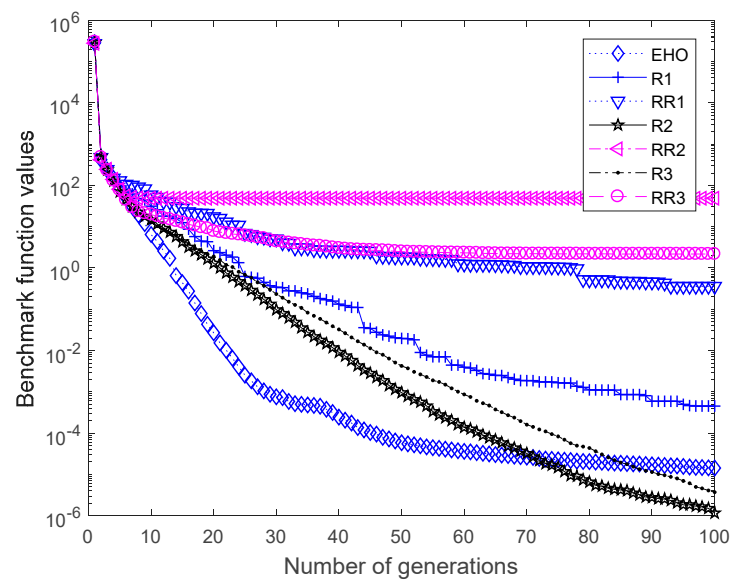


(a) F03

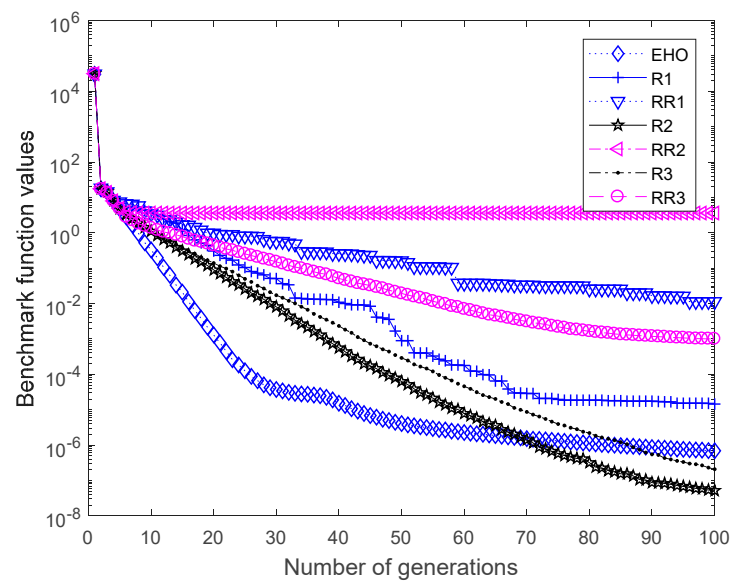


(b) F07

Figure 1. Cont.

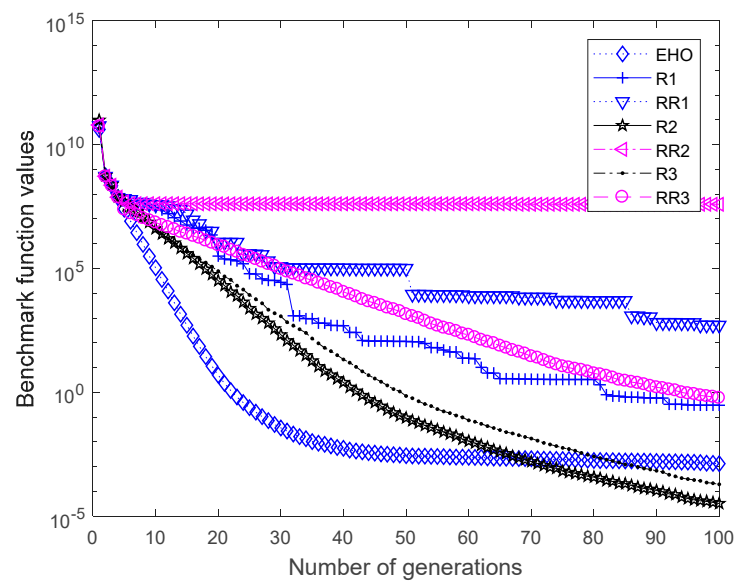


(c) F11



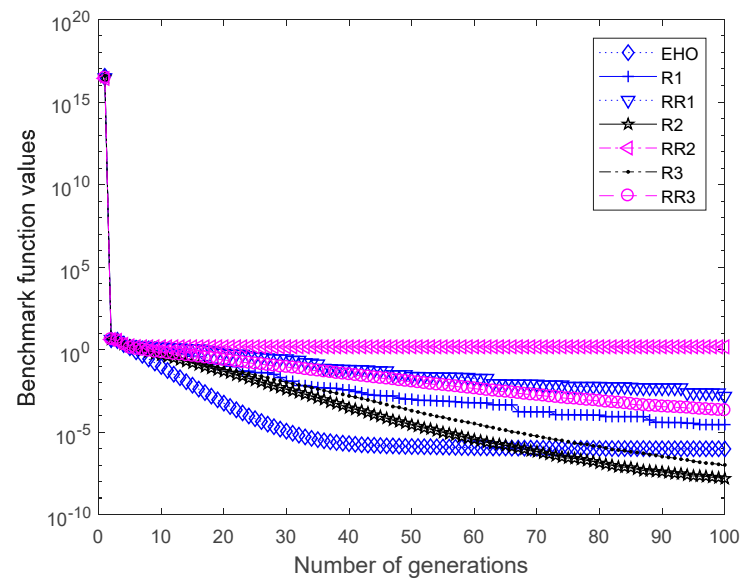
(d) F15

Figure 1. Cont.



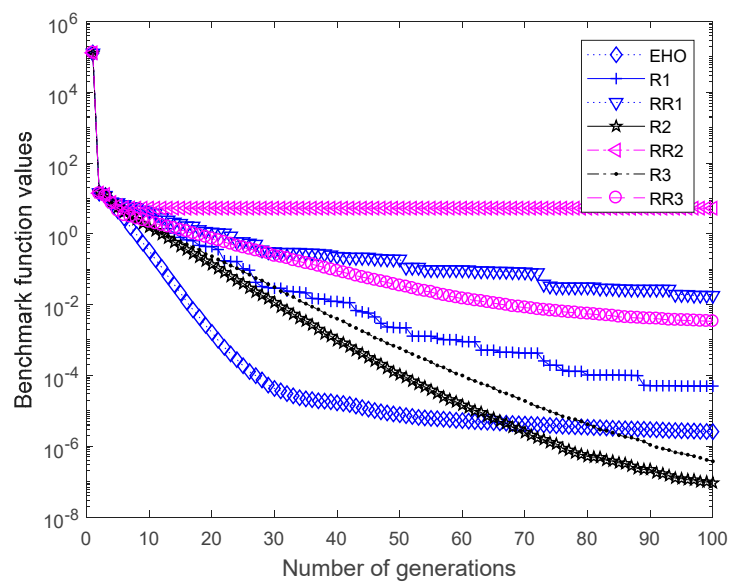
(e) F16

Figure 1. Optimization process of seven algorithms on five functions with $D = 50$. EHO—elephant herd optimization.

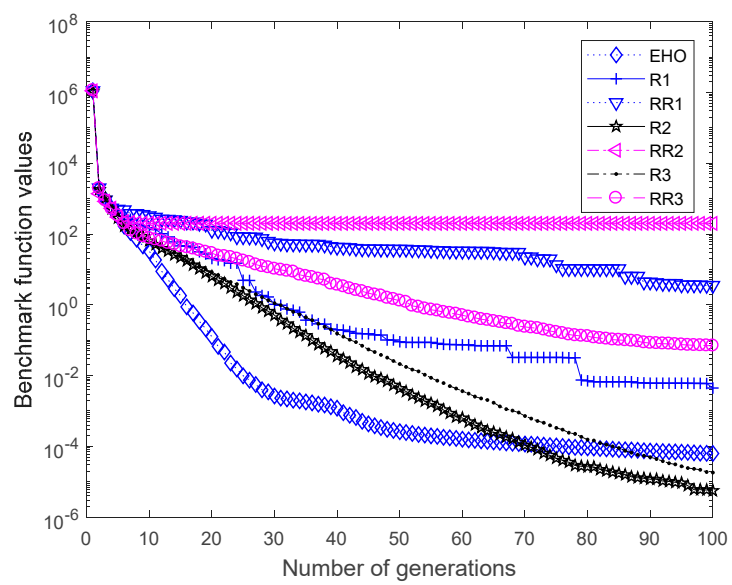


(a) F03

Figure 2. Cont.

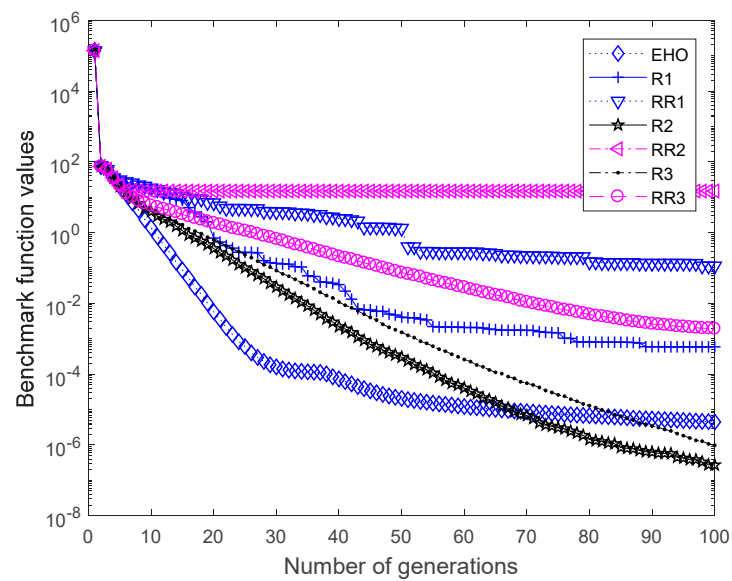


(b) F07

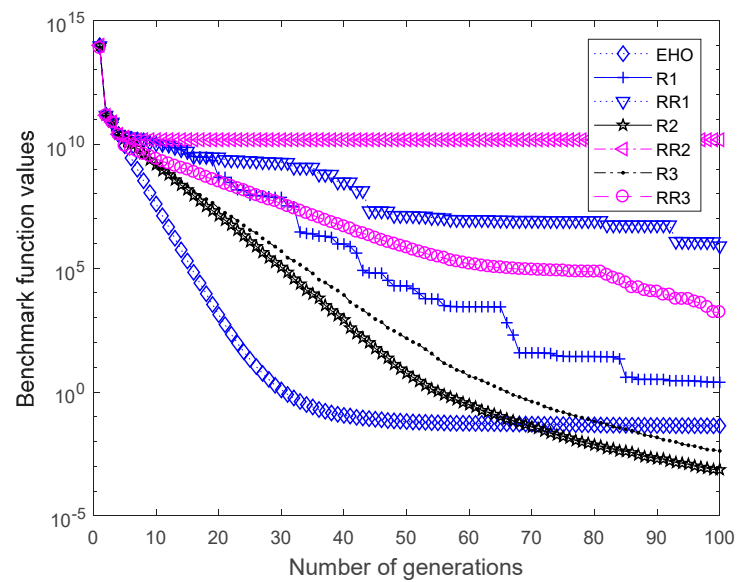


(c) F11

Figure 2. Cont.

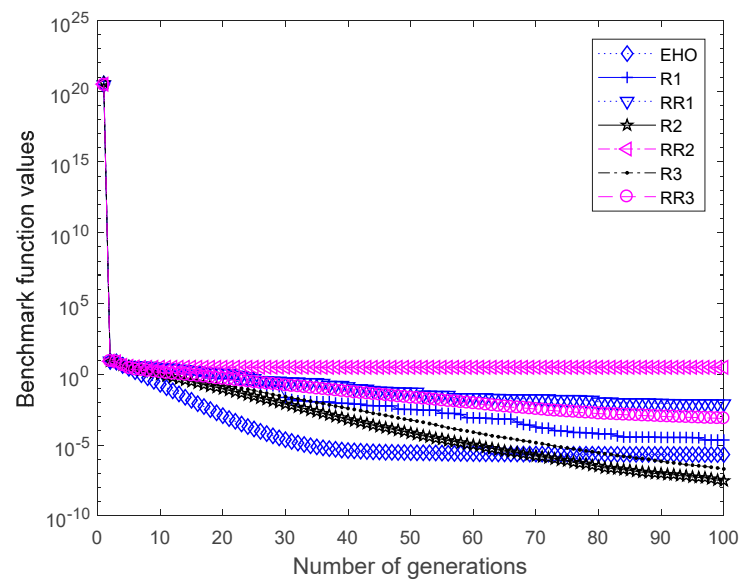


(d) F15

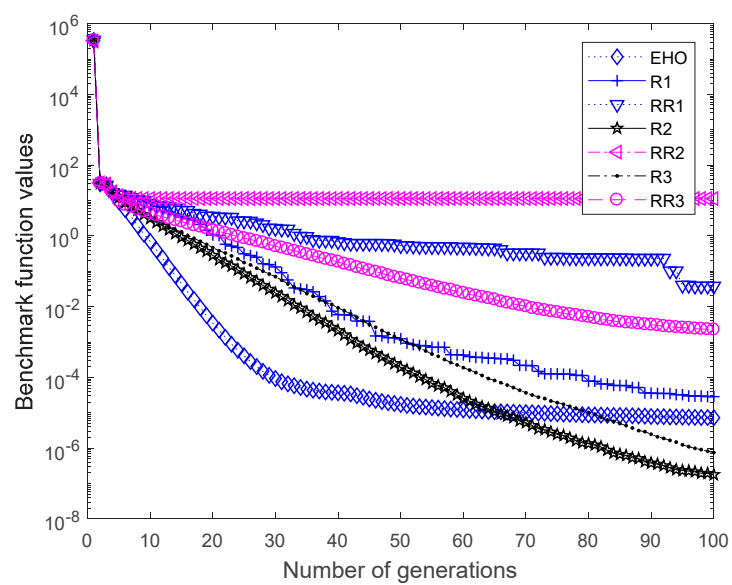


(e) F16

Figure 2. Optimization process of seven algorithms on five functions with $D = 100$.

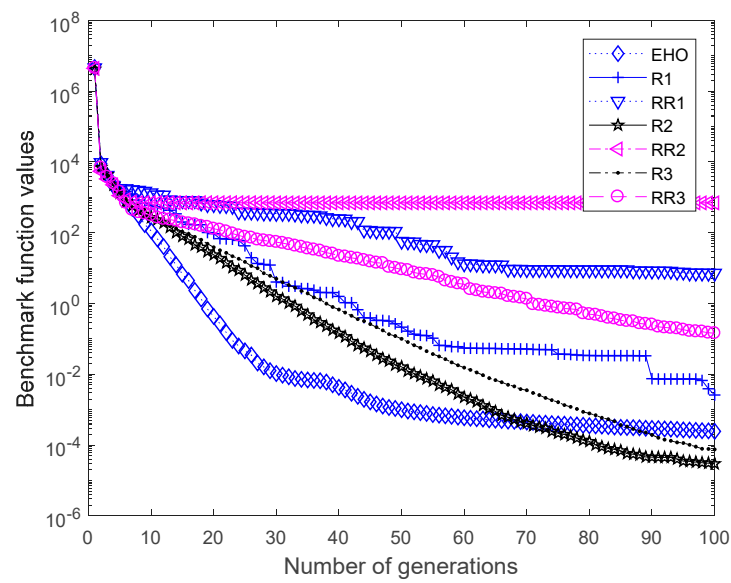


(a) F03

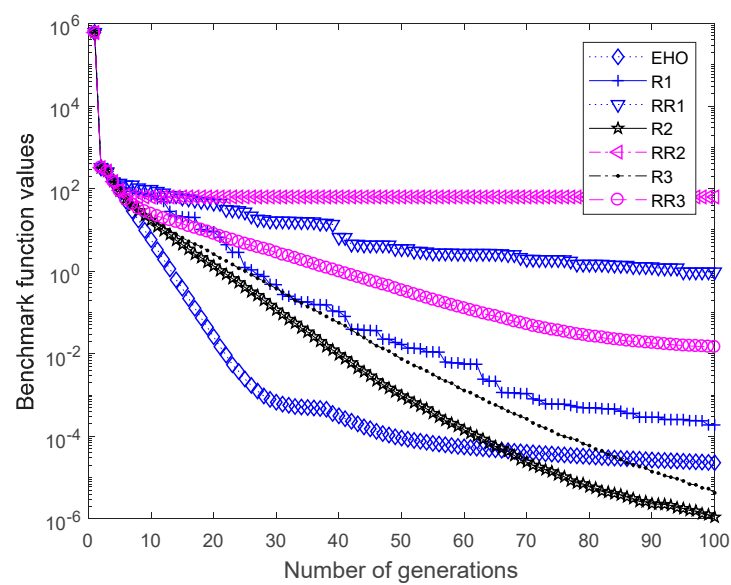


(b) F07

Figure 3. Cont.

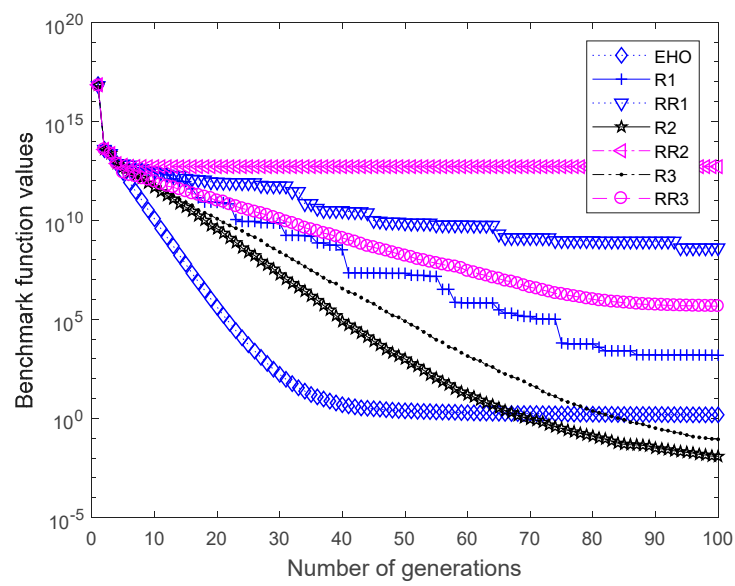


(c) F11

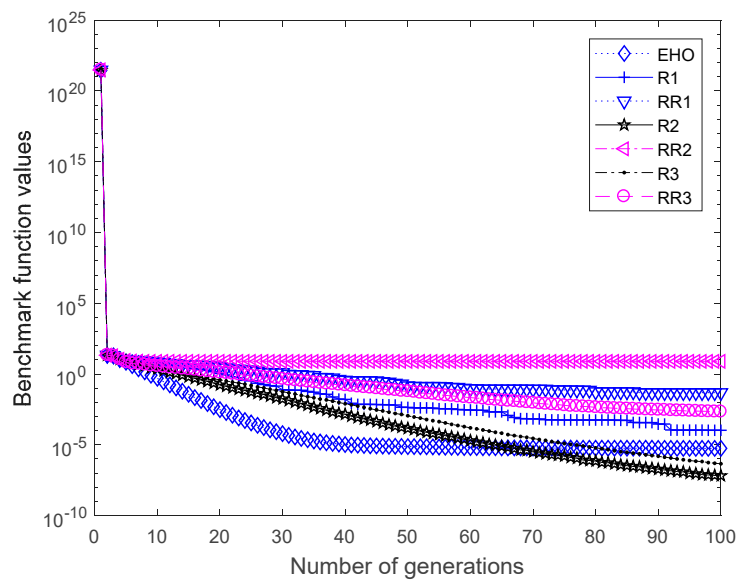


(d) F15

Figure 3. Cont.

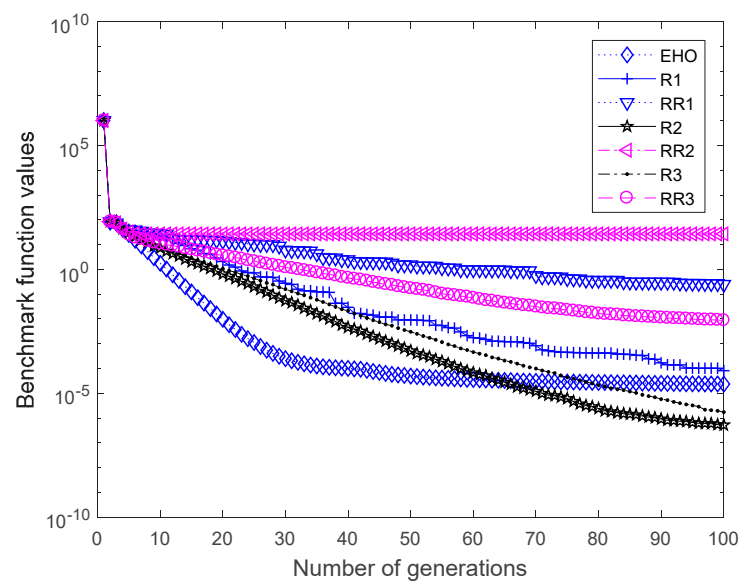


(e) F16

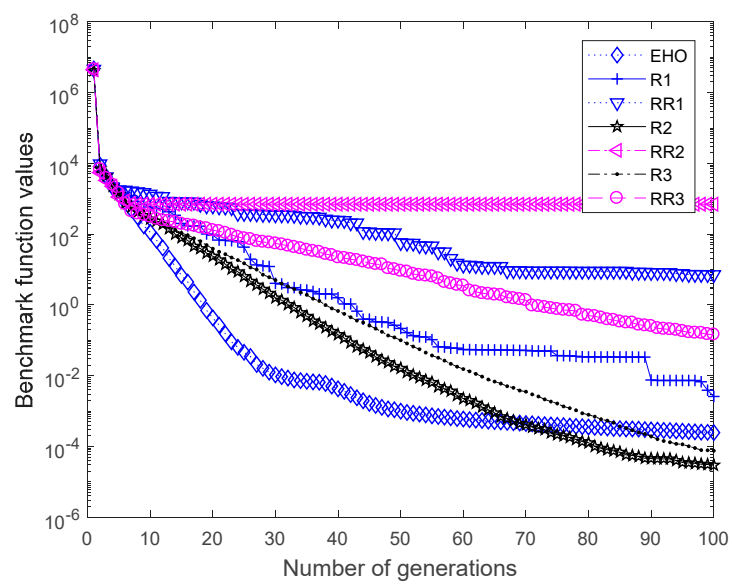
Figure 3. Optimization process of seven algorithms on five functions with $D = 200$.

(a) F03

Figure 4. Cont.

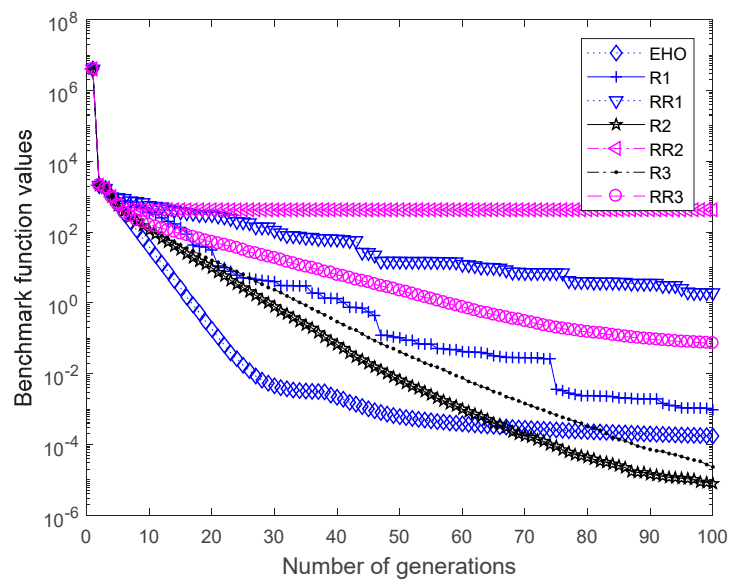


(b) F07

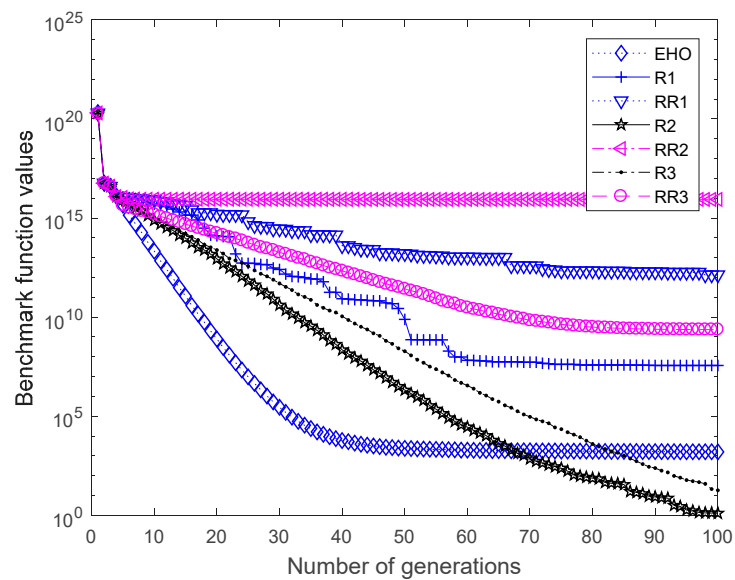


(c) F11

Figure 4. Cont.

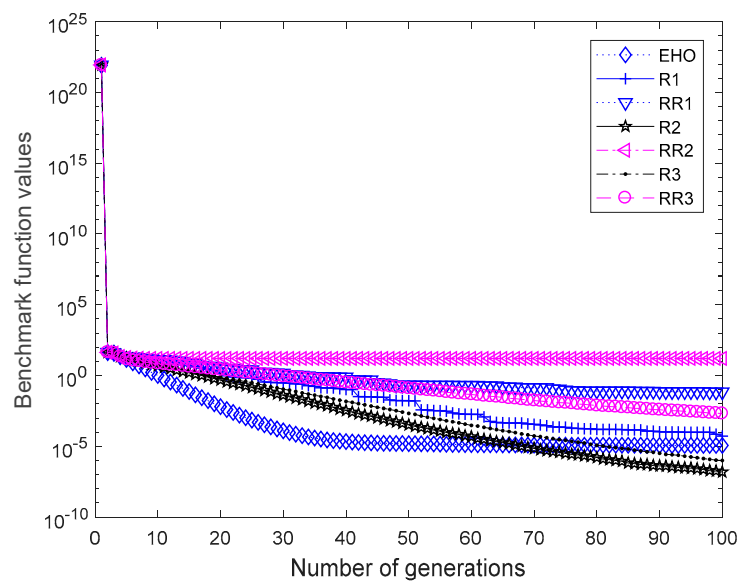


(d) F15

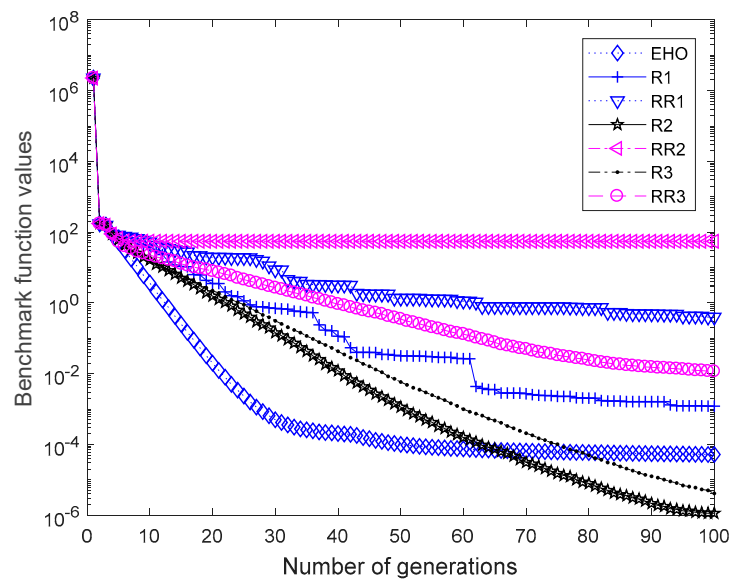


(e) F16

Figure 4. Optimization process of seven algorithms on five functions with $D = 500$.

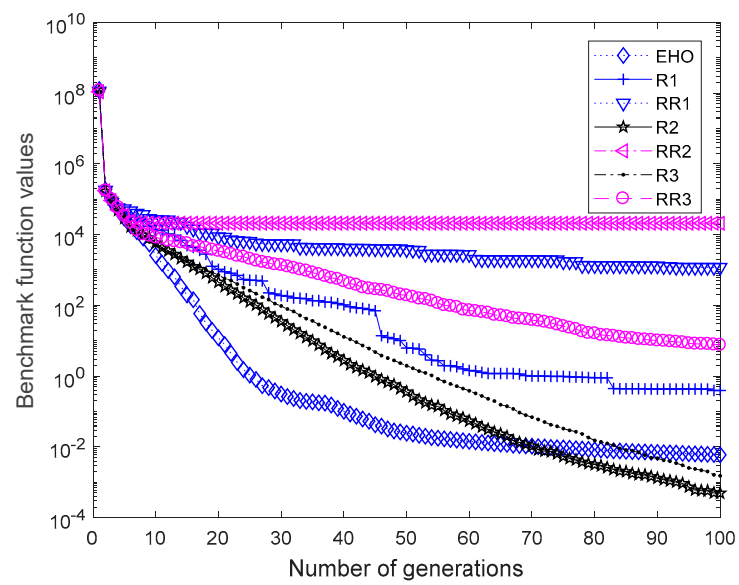


(a) F03

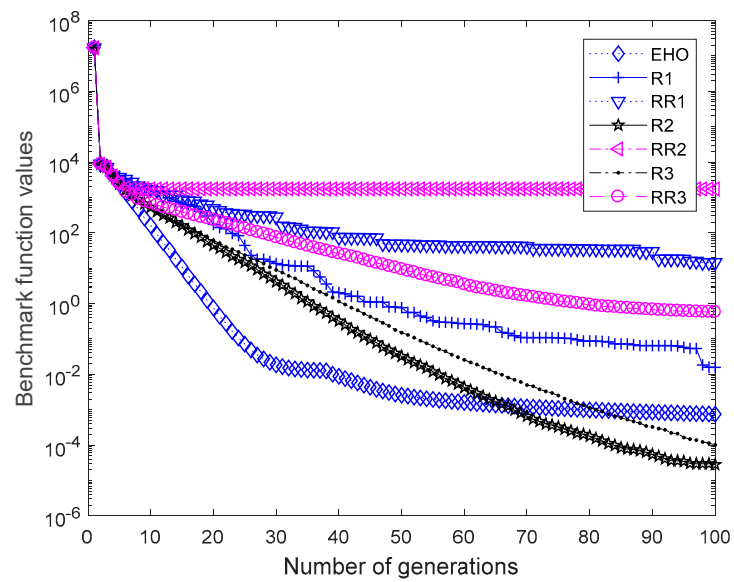


(b) F07

Figure 5. Cont.



(c) F11



(d) F15

Figure 5. Cont.

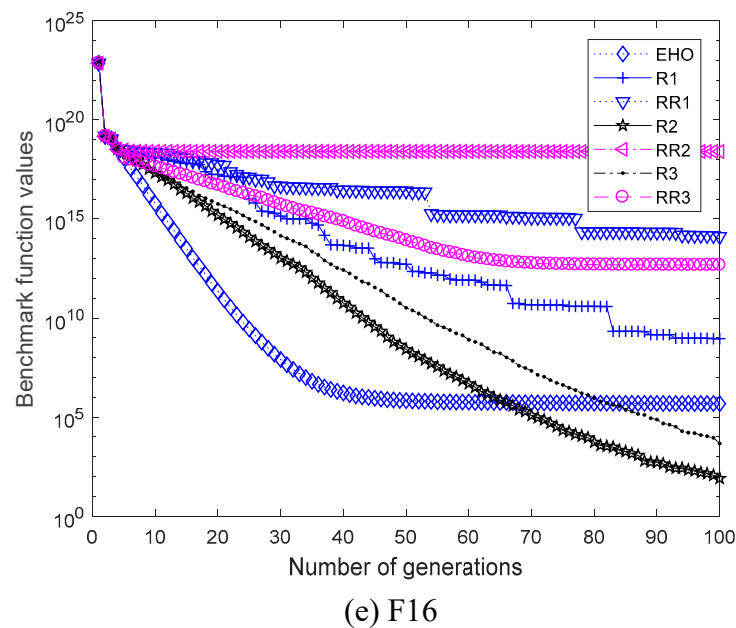


Figure 5. Optimization process of seven algorithms on five functions with $D = 1000$.

5.2. Constrained Optimization

Besides the standard benchmark evaluation, in this section, fourteen constrained optimization problems originated from CEC 2017 [121] are selected in order to further verify the performance of six improved versions of EHO, namely: EHOR1, EHORR1, EHOR2, EHORR2, EHOR3, and EHORR3. The six variants of the EHO were investigated fully through a series of experiments, using fourteen large-scale constrained benchmarks with dimensions $D = 50$, and 100 . These complicated large-scale benchmarks can be found in Table 13. More information about all of the benchmarks can be found in [86,118,119]. As before, we performed 30 independent runs under the same conditions as shown in the literature [120]. For all of the parameters, they are the same as before.

Table 13. Details of 14 Congress on Evolutionary Computation (CEC) 2017 constrained functions. D is the number of decision variables, I is the number of inequality constraints, and E is the number of equality constraints.

No.	Problem	Search Range	Type of Objective	Number of Constraints	
				E	I
F01	C05	$[-10, 10]^D$	Non-Separable	0	2 Non-Separable, Rotated
F02	C06	$[-20, 20]^D$	Separable	6	0 Separable
F03	C07	$[-50, 50]^D$	Separable	2 Separable	0
F04	C08	$[-100, 100]^D$	Separable	2 Non-Separable	0
F05	C09	$[-10, 10]^D$	Separable	2 Non-Separable	0
F06	C10	$[-100, 100]^D$	Separable	2 Non-Separable	0
F07	C12	$[-100, 100]^D$	Separable	0	2 Separable
F08	C13	$[-100, 100]^D$	Non-Separable	0	3 Separable

Table 13. Cont.

No.	Problem	Search Range	Type of Objective	Number of Constraints	
				<i>E</i>	<i>I</i>
F09	C15	$[-100, 100]^D$	Separable	1	1
F10	C16	$[-100, 100]^D$	Separable	1	1
F11	C17	$[-100, 100]^D$	Non-Separable	Non-Separable	Separable
				1	1
F12	C18	$[-100, 100]^D$	Separable	1	2
F13	C25	$[-100, 100]^D$	Rotated	1	Non-Separable
				Rotated	1
F14	C26	$[-100, 100]^D$	Rotated	1	1
				Rotated	Rotated

5.2.1. $D = 50$

In this section of our work, seven kinds of EHOs were evaluated using the 14 constrained benchmarks mentioned previously, with dimension $D = 50$. The obtained mean function values and standard values from thirty runs are recorded in Tables 14 and 15.

Table 14. Mean function values obtained by EHO and six improved methods on fourteen CEC 2017 constrained optimization functions with $D = 50$.

	EHO	R1	RR1	R2	RR2	R3	RR3
F01	8.50×10^{-6}	2.69×10^{-3}	0.04	2.81×10^{-5}	0.07	6.05×10^{-5}	0.01
F02	6.45×10^{-5}	6.76×10^{-3}	0.24	3.09×10^{-4}	0.39	4.28×10^{-4}	0.03
F03	7.82×10^{-8}	1.24×10^{-4}	0.21	1.02×10^{-7}	1.70	4.10×10^{-7}	2.21×10^{-3}
F04	2.22×10^{-13}	3.61×10^{-9}	1.86×10^{-3}	8.75×10^{-15}	2.65	3.01×10^{-14}	1.80×10^{-6}
F05	0.06	0.02	0.49	0.01	0.06	0.01	0.01
F06	0.22	0.39	0.15	0.22	1.08	0.18	0.22
F07	1.06×10^{-6}	4.96×10^{-3}	0.58	6.86×10^{-7}	5.68	1.98×10^{-6}	0.01
F08	9.68×10^{-16}	2.77×10^{-11}	1.55×10^{-5}	3.36×10^{-18}	2.65×10^{-3}	6.47×10^{-17}	9.11×10^{-8}
F09	4.43×10^{-6}	3.58×10^{-4}	1.64	2.11×10^{-6}	20.66	7.28×10^{-6}	0.09
F10	2.19×10^3	2.04×10^3	2.11×10^3	1.98×10^3	1.60×10^3	1.93×10^3	2.75×10^3
F11	9.97×10^{-4}	1.46	4.01×10^3	4.03×10^{-4}	9.58×10^3	8.04×10^{-4}	24.87
F12	5.39×10^{-4}	0.15	1.76	4.90	1.61	2.31	1.48
F13	6.61×10^{-5}	2.68×10^{-3}	0.15	9.41×10^{-5}	0.15	1.45×10^{-4}	0.01
F14	2.21×10^{-8}	1.05×10^{-5}	0.10	1.05×10^{-8}	0.08	2.83×10^{-8}	2.89×10^{-4}
TOTAL	1	2	9	2	0	0	0

Table 15. Standard values obtained by EHO and six improved methods on fourteen CEC 2017 constrained optimization functions with $D = 50$.

	EHO	R1	RR1	R2	RR2	R3	RR3
F01	4.06×10^4	3.58×10^4	4.08×10^4	5.75×10^4	6.26×10^4	5.65×10^4	4.16×10^4
F02	127.50	123.10	100.70	86.83	102.00	122.10	59.11
F03	65.68	46.15	48.94	50.84	38.43	42.04	51.71
F04	0.37	0.61	0.72	0.72	0.34	0.70	0.76
F05	0.36	0.35	0.31	0.37	0.27	0.29	0.22
F06	2.75	2.56	1.91	2.73	2.66	2.15	1.19
F07	1.73×10^3	1.42×10^3	1.44×10^3	1.79×10^3	1.67×10^3	1.61×10^3	1.37×10^3
F08	2.51×10^8	2.07×10^8	2.54×10^8	2.49×10^8	3.87×10^8	2.38×10^8	2.63×10^8
F09	1.39	1.77	1.44	2.32	2.31	2.01	1.44
F10	41.32	40.76	33.77	39.29	41.69	36.24	34.50
F11	0.45	0.37	0.49	0.36	0.52	0.43	0.37
F12	1.81×10^3	1.48×10^3	1.65×10^3	1.84×10^3	1.85×10^3	1.75×10^3	1.63×10^3
F13	80.20	67.86	75.43	75.82	72.32	69.12	61.46
F14	1.39	1.54	2.28	1.66	2.11	1.88	1.76
TOTAL	2	3	1	1	2	0	5

From Table 14, we can see that in terms of the mean function values, RR1 performed the best, at a level far better than the other methods. As for the other methods, R1 and R2 provided a similar performance to each other, and they could find the smallest fitness values successfully on two constrained functions. From Table 15, it can be observed that RR3 performed in the most stable way, while for the other algorithms, they have a similar stable performance.

5.2.2. $D = 100$

As above, the same seven kinds of EHOs were evaluated using the fourteen constrained benchmarks mentioned previously, with dimension $D = 100$. The obtained mean function values and standard values from 30 runs are recorded in Tables 16 and 17.

Table 16. Mean function values obtained by EHO and six improved methods on fourteen CEC 2017 constrained optimization functions with $D = 100$.

	EHO	R1	RR1	R2	RR2	R3	RR3
F01	1.09×10^6	9.71×10^5	9.48×10^5	9.49×10^5	1.13×10^6	9.71×10^5	1.01×10^6
F02	3.88×10^3	3.76×10^3	3.80×10^3	3.77×10^3	4.00×10^3	3.77×10^3	3.88×10^3
F03	9.35×10^3	9.35×10^3	9.35×10^3	9.35×10^3	9.34×10^3	9.35×10^3	9.37×10^3
F04	1.01×10^3	1.01×10^3	1.01×10^3	1.01×10^3	1.01×10^3	1.01×10^3	1.01×10^3
F05	9.39	9.37	9.37	9.36	9.76	9.35	9.43
F06	1.04×10^3	1.04×10^3	1.04×10^3	1.04×10^3	1.05×10^3	1.04×10^3	1.04×10^3
F07	6.99×10^4	6.79×10^4	6.58×10^4	6.50×10^4	7.20×10^4	6.68×10^4	6.96×10^4
F08	8.77×10^9	8.02×10^9	7.80×10^9	8.19×10^9	8.95×10^9	8.14×10^9	8.62×10^9
F09	47.52	47.20	47.73	47.98	49.02	47.56	46.86
F10	2.18×10^3	2.16×10^3	2.13×10^3	2.14×10^3	2.22×10^3	2.17×10^3	2.16×10^3
F11	17.99	17.63	17.18	17.22	18.60	17.51	17.96
F12	6.82×10^4	6.60×10^4	6.50×10^4	6.52×10^4	7.01×10^4	6.63×10^4	6.87×10^4
F13	3.60×10^3	3.55×10^3	3.58×10^3	3.57×10^3	3.78×10^3	3.59×10^3	3.66×10^3
F14	55.89	55.19	53.99	53.88	60.16	54.61	56.37
TOTAL	1	2	4	4	1	1	1

Table 17. Standard values obtained by EHO and six improved methods on fourteen CEC 2017 constrained optimization functions with $D = 100$.

	EHO	R1	RR1	R2	RR2	R3	RR3
F01	7.04×10^4	6.49×10^4	7.85×10^4	8.34×10^4	9.27×10^4	6.51×10^4	7.03×10^4
F02	122.00	125.20	106.10	111.90	137.30	113.80	102.50
F03	73.82	74.87	72.61	66.75	96.82	80.98	65.38
F04	0.25	0.23	0.39	0.25	0.31	0.24	0.36
F05	0.14	0.14	0.14	0.19	0.13	0.14	0.09
F06	1.20	1.23	1.39	1.46	1.73	1.22	1.27
F07	2.93×10^3	2.42×10^3	3.14×10^3	3.02×10^3	3.25×10^3	2.73×10^3	2.52×10^3
F08	5.94×10^8	6.11×10^8	7.07×10^8	7.24×10^8	1.01×10^9	6.43×10^8	5.77×10^8
F09	0.77	0.64	1.25	0.93	0.57	0.95	1.06
F10	52.32	60.00	53.11	63.71	52.42	56.61	51.64
F11	0.67	0.72	0.73	0.72	0.84	0.57	0.59
F12	2.35×10^3	2.77×10^3	3.13×10^3	2.71×10^3	3.00×10^3	3.00×10^3	2.34×10^3
F13	80.54	77.54	125.50	116.70	119.80	108.50	72.75
F14	3.02	2.93	3.44	3.27	3.05	3.38	2.56
TOTAL	1	3	0	0	1	1	8

Regarding the mean function values, Table 16 shows that RR1 and R2 have the same performance, which performed much better than the other algorithms, providing the smallest function values on 4 out of 14 functions. As for the other algorithms, R1 ranks R2, EHO, RR2, R3, and RR3 gave a similar performance to each other, performing the best only on one function each. From Table 17, it can be

observed that RR3 performed in the most stable way, while for the other algorithms, R1 has a significant advantage over the other algorithms.

6. Conclusions

In optimization research, few metaheuristic algorithms reuse previous information to guide the later updating process. In our proposed improvement for basic elephant herd optimization, the previous information in the population is extracted to guide the later search process. We select one, two, or three elephant individuals from the previous iterations in either a fixed or random manner. Using the information from the selected previous elephant individuals, we offer six individual updating strategies (R1, RR1, R2, RR2, R3, and RR3) that are then incorporated into the basic EHO in order to generate six variants of EHO. The final EHO individual at this iteration is generated according to the individual generated by the basic EHO at the current iteration, along with the selected previous individuals using a weighted sum. The weights are determined by a random number and the fitness of the elephant individuals at the previous iteration.

We tested our six proposed algorithms against 16 large-scale test cases. Among the six individual updating strategies, R2 performed much better than the others on most benchmarks. The experimental results demonstrated that the proposed EHO variations significantly outperformed the basic EHO.

In future research, we will propose more individual updating strategies to further improve the performance of EHO. In addition, the proposed individual updating strategies will be incorporated into other metaheuristic algorithms. We believe they will generate promising results on large-scale test functions and practical engineering cases.

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