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Original research article

# Coherent beam combining based on the SPGD algorithm with a momentum term

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## ABSTRACT

The stochastic parallel gradient descent with a momentum term (named MomSPGD) algorithm is innovatively presented and applied for coherent beam combining to substitute for the traditional SPGD algorithm. The feasibility of coherent synthesis system using the MomSPGD algorithm is validated through numerical simulations and experiments. The results of the simulations and the experiment indicate that the proposed algorithm can improve convergence speed and immunity of the synthesis system compared with the traditional SPGD algorithm.

## 1. Introduction

Fiber lasers that have high conversion efficiency and excellent beam quality are widely applied in industry and defense [1–4]. However, the ultimate output power of single fiber laser is limited due to medium nonlinear effect, heat damage and optical element. Coherent beam combining of multiple fiber laser modules, a promising technology for achieving high density while maintaining the beam quality simultaneously, can effectively solve the limitations [5–10]. So far, the highest-power demonstrations of coherent beam combining have involved active phase controlling with the master oscillator power amplifier (MOPA) configuration [1,11]. There are many demonstrations of coherent synthesis system using active phase control methods, such as the heterodyne phase detection technique [12,13], multi-dither technique [14], and stochastic parallel gradient descent (SPGD) algorithm technique [15–18]. The SPGD algorithm that is widely used in many demonstrations on coherent beam combining due to the advantage of less complexity [19–23]. Nevertheless, the SPGD algorithm has the issue of poor immunity, which has greatly limited the actual application of it especially under the circumstance of dynamic disturbance.

The momentum method which can increase the rate of convergence dramatically is proposed for training artificial neural networks in deep learning at first [24]. Ning mathematically analyzes the effect of the momentum term on the speed of learning. To our knowledge, there are few articles that report application of the momentum method in coherent beam combining. In this paper, the momentum method is adopted to work with the SPGD algorithm and applied for coherent beam combining. This paper is organized as follows: In Section 2, the basic principle of the momentum method is described; Section 3 details the feasibility of coherent beam combining using the MomSPGD algorithm by simulation of correcting phase distortions. Moreover, in Section 4, the experiments of two laser beam coherent combining are carried out to testify the proposed algorithm; Conclusions are given in Section 5.

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## 2. Algorithm principle

The SPGD algorithm is a special gradient descent algorithm that adopts random parallel perturbation for gradient estimation. The metric function  $J = J(\mathbf{u})$  that is obtained from the detector, is a function of the control voltages  $\mathbf{u} = \{u_1, u_2, \dots, u_N\}$  which is applied to the phase modulators. Slight perturbations  $\delta\mathbf{u} = \{\delta u_1, \delta u_2, \dots, \delta u_N\}$  typically chosen as statistically independent variables which have zero mean and equal variance, are applied simultaneously to the system control voltages. The corresponding perturbed metric values are evaluated as follows:

$$J_{\pm} = J(u_1 \pm \delta u_1, u_2 \pm \delta u_2, \dots, u_N \pm \delta u_N) \quad (1)$$

The metric perturbation is calculated as the difference of the two perturbed values:

$$\delta J = J_+ - J_- \quad (2)$$

In the SPGD algorithm, the control voltages are updated according to the rule:

$$\mathbf{u}^{(k)} = \mathbf{u}^{(k-1)} + \gamma \delta J^{(k)} \delta \mathbf{u}^{(k)} \quad (3)$$

Where the superscript  $k$  ( $k = 1, 2, \dots$ ) represents the  $k$ -th iteration.  $\gamma$  is the update gain. When the system performance index needs to be optimized to the maximum value,  $\gamma$  is a positive value, and when the system performance index needs to be optimized to a minimum value,  $\gamma$  is a negative value.

The momentum method is derived from the Newtonian equation. The motion equation for a point mass  $m$  moving in a viscous medium with friction coefficient  $\mu$  under the influence of a conservative force field with potential energy  $E(\mathbf{u})$  can be expressed as:

$$m \frac{d^2 \mathbf{u}}{dt^2} + \mu \frac{d\mathbf{u}}{dt} = -\nabla_{\mathbf{u}} E(\mathbf{u}) \quad (4)$$

where  $\mathbf{u}$  is the coordinate vector of the particle.

Eq. (4) can be rewritten in a discrete form as showed below:

$$m \frac{\mathbf{u}_{t+\Delta t} + \mathbf{u}_{t-\Delta t} - 2\mathbf{u}_t}{(\Delta t)^2} + \mu \frac{\mathbf{u}_{t+\Delta t} - \mathbf{u}_t}{\Delta t} = -\nabla_{\mathbf{u}} E(\mathbf{u}) \quad (5)$$

After rearrangements, Eq. (5) can be written as:

$$\mathbf{u}_{t+\Delta t} - \mathbf{u}_t = -\frac{(\Delta t)^2}{m + \mu \Delta t} \nabla_{\mathbf{u}} E(\mathbf{u}) + \frac{m}{m + \mu \Delta t} (\mathbf{u}_t - \mathbf{u}_{t-\Delta t}) \quad (6)$$

we let the update gain  $\gamma$  and the momentum  $p$  be related to the friction coefficient  $\mu$  and mass  $m$  according to:

$$\gamma = \frac{(\Delta t)^2}{m + \mu \Delta t} \quad (7)$$

$$p = \frac{m}{m + \mu \Delta t} \quad (8)$$

Therefore, Eq. (6) can be written as :

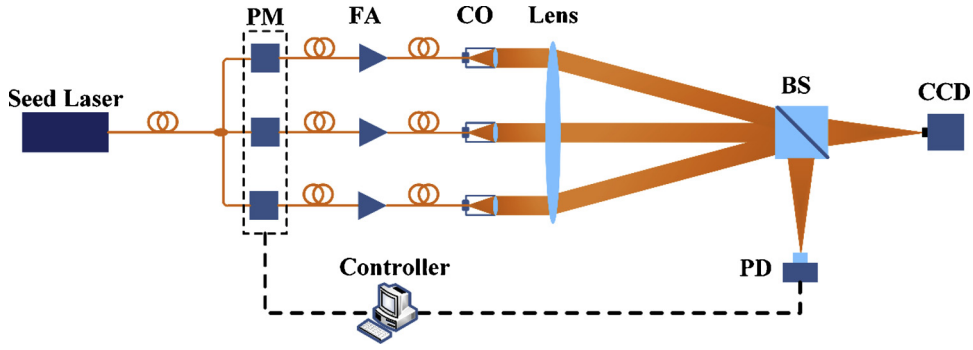
$$\mathbf{u}_{t+\Delta t} - \mathbf{u}_t = -\gamma \nabla_{\mathbf{u}} E(\mathbf{u}) + p(\mathbf{u}_t - \mathbf{u}_{t-\Delta t}) \quad (9)$$

It is clear that Eq. (3) can be viewed as the special case of Eq. (9) for a massless particle. Therefore, the stochastic parallel gradient descent algorithm with a momentum term is equivalent to Newtonian equation of a particle moving through a viscous medium under the influence of a conservative force field. The MomSPGD algorithm that is employed in coherent beam combining can be indicated as following flowchart:

- 1) Set initial control parameters  $\mathbf{u}^{(0)}$ .
- 2) Generate slight perturbations  $\delta\mathbf{u}$  that satisfy the Bernoulli probability distribution with zero mean.
- 3) Apply the perturbations on the system control voltages to get the metric values  $J(\mathbf{u}^{(k-1)} + \delta\mathbf{u})$  and  $J(\mathbf{u}^{(k-1)} - \delta\mathbf{u})$ .
- 4) Evaluate the gradient using Eq. (2).
- 5) Update the control parameters using Eq. (9).
- 6) If the control parameters don't satisfy the requirement, repeat the step 2 to step 6.

## 3. Simulation and analysis

The notional schematic of coherent beam combining system is illustrated by Fig. 1. It could be noted that the system consists of three subsystems, that is, the laser source, the phase control and the beam combining. The seed laser is split into  $N$  channels and each is connected to a phase modulator. The laser beams from the phase modulator are delivered to the fiber amplifier and then sent into free-space via the collimator. The  $N$  channels of collimated beam are tiled via beam combining subsystem and then irradiated at a beam splitting mirror with high transmittance. The reflected laser irradiates a photo-electronics detector which transfers the optical



**Fig. 1.** The notional scheme of coherent beam combining system. PM: Phase Modulator; FA: Fiber Amplifier; CO: Collimator; BS: Beam Splitting Mirror; PD: Photodetector.

signal into the electrical signal that is defined as the metric function  $J$ . Then the metric function  $J$  is sent to the phase controller. Initially, the phase controller is not implemented and the beam combining system is an opened loop, the uncontrolled optical phase of each laser channel fluctuates randomly due to the outside environmental noise and fiber amplifiers, which make the far-field intensity distribution changes randomly. In this situation, the phase controller is implemented and generates the control signal according to the electrical signal from the detector that contains the information of phase fluctuation in each laser channel. The control signal is added to the phase modulator, so that the whole system is in a closed loop. The numerical simulation in this paper was performed based on this notional schematic.

In this section, the numerical simulation of the MomSPGD algorithm is carried out to verify the effectiveness of it for coherent beam synthesis. The system parameters are illustrated in Table 1. The beam waist of the single beam is 5 mm. The center distance of the beam is 20 mm. The laser wavelength is 1064 nm. The propagation distance is 200 m. In practice, due to the thermal effect of the fiber gain medium and the influence of the external environment, the phase of the beam of each laser unit is inconsistent. When performing numerical simulation, it is assumed that the rms value of the phase error for the laser array is  $2\pi$ .

Fig. 2 shows the evaluation curves of beam combining system against iteration number. It is evidently expressed that the MomSPGD algorithm can converge to the extremum of the metric and realize the phase lock output of the coherent beam combining system. In addition, the numerical results strongly demonstrate that the MomSPGD algorithm has faster convergence speed compared with the SPGD algorithm. Therefore, the program of coherent beam summation using the MomSPGD algorithm is feasible, which can accelerate the correction process of coherent beam combining system.

In practical engineering applications, external environmental changes, mechanical vibrations, noise, etc., can be the source for phase distortions, which affect the convergence speed and effectiveness of the beam combining system. We perform dynamic simulations for coherent beam combining using the MomSPGD algorithm. A random sequence of Gaussian distributions is used to simulate dynamic phase noise. It is assumed that the control processor can perform 100 KHz.

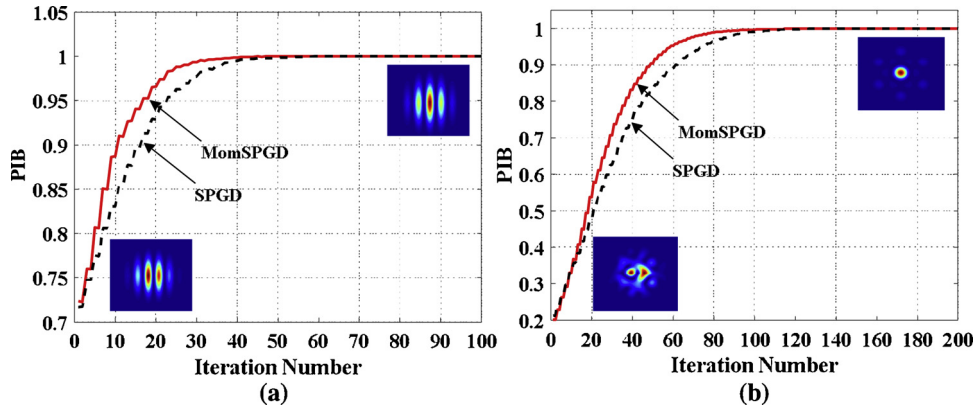
The evaluation curves of 2-laser and 7-laser array synthesis system against amplitude of dynamic phase stochastic disturbance at different frequencies are displayed in Fig. 3. It can be obtained that the convergence effect of the coherent beam synthesis system becomes worse with the amplitude or frequency of dynamic phase stochastic disturbance increasing, and the 7-laser array system is more susceptible to phase noise than the 2-laser array. Evidently, the convergence effect of the synthesis system using MomSPGD algorithm is superior to that using SPGD algorithm under the same phase noise. Essentially, when using momentum, a ball is pushed down a hill. The ball accumulates momentum as it rolls downhill, becoming faster and faster on the way. The same thing happens to the beam synthesis system controlling: the momentum term can increase for dimensions whose gradients point in the same directions and reduce updates for dimensions whose gradients change directions. As a result, the MomSPGD algorithm can improve convergence speed and disturbance immunity of the coherent beam combining compared to the conventional SPGD algorithm.

#### 4. Experimental results

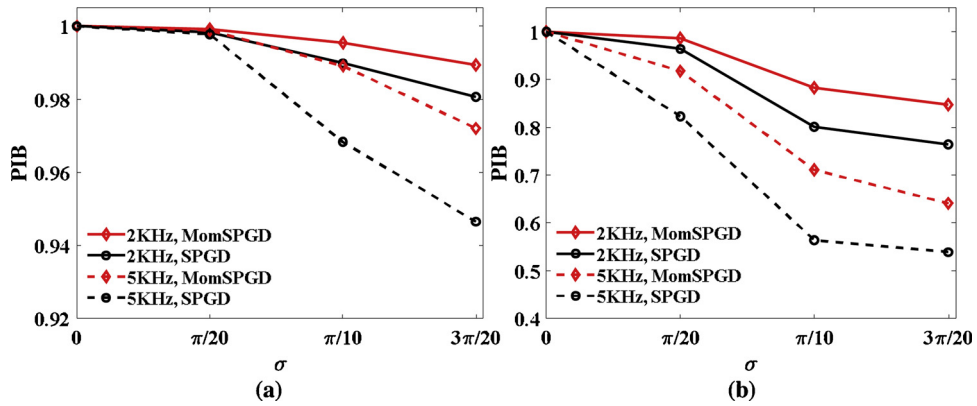
In this section, the experiments of two laser beam coherent combining are carried out to testify the proposed algorithm. The experimental setup is the same as shown in Fig. 1. The dependence of metric function (the energy encircled of photodetector) on time in open loop and closed loop is presented in Fig. 4. When the MomSPGD algorithm controller is not implemented and the coherent

**Table 1**  
The system parameters in the simulation.

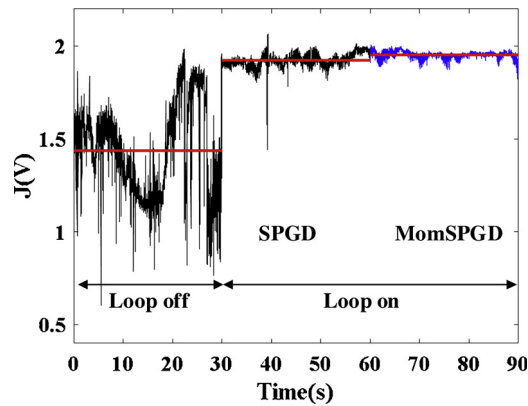
Parameters	Values
Distance: $L$	200 m
Wavelength: $\lambda$	$1064 \times 10^{-9}$ m
Beam waist: $w_0$	$5 \times 10^{-3}$ m
Sampling number: $N$	256



**Fig. 2.** The curves evaluation of beam combining system against iteration number (the average result of 200 times numerical simulations). (a) 2-laser array; (b) 7-laser array. The insets show the examples of the far-field intensity distribution before (on the left) and after (on the right) application of the controlling algorithm.



**Fig. 3.** Evaluation curves of coherent beam synthesis system against amplitude of dynamic phase stochastic disturbance at different frequencies. (a) 2-laser array; (b) 7-laser array.



**Fig. 4.** Dependence of metric function (the energy encircled in photodetector) on time in loop off and loop on.

beam combining system is in an open loop, the power encircled of photodetector fluctuate fast between 0 and 2 V randomly, and the mean value was 1.4 V. The intensity pattern at the observing plane keeps shifting due to phase fluctuations in each fiber amplifier. The long-exposure (30 s) of the far field intensity distribution is shown in Fig. 5(a). Once the MomSPGD algorithm is implemented, the system enters in to a closed loop. The metric function obtained by photodetector can be locked steadily with the calculated mean value 1.96 V, which is 1.4 times bigger than the one when the system is in an open loop. The intensity pattern at the observing plane is clear and steady, and the long-exposure of the far-field intensity distribution is shown in Fig. 5(c). The experimental results demonstrate that the metric fluctuating can be controlled by the MomSPGD algorithm effectively.

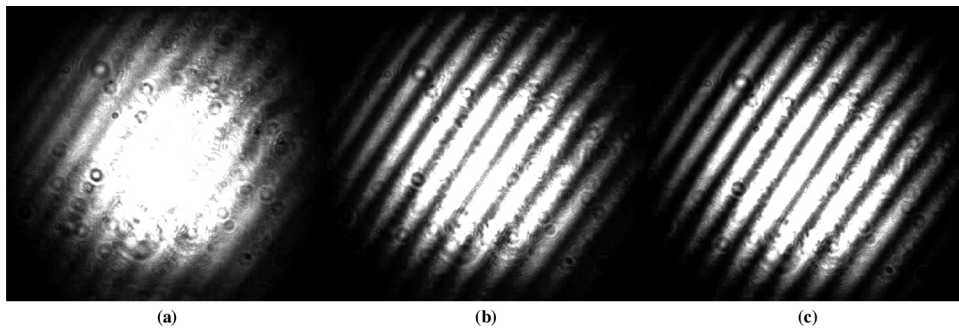


Fig. 5. Long-exposure far-field intensity pattern of the combined laser: (a) loop off, (b) loop on with SPGD algorithm, (c) loop on with MomSPGD algorithm.

In addition, as a reference for evaluating the effectiveness of the MomSPGD algorithm in the close loop, the experiments of beam coherent combining using traditional SPGD algorithm are implemented. It is found from Fig. 4 that the jitter of metric function of system using MomSPGD algorithm is smaller than that using SPGD algorithm, and the mean value of metric function is greater than that using SPGD algorithm. When the phase fluctuations in each amplifier are being controlled, the long-exposure of the far-field intensity distribution of the synthesis system using traditional SPGD is shown in Fig. 5(b). The fringe contrast is calculated to be about 58.88%, which is far less than 73.42% of that using MomSPGD algorithm, where the fringe visibility is defined by the formula  $(I_{max} - I_{min}) / (I_{max} + I_{min})$ ,  $I_{max}$  and  $I_{min}$  are the maximum intensity and the adjacent minimum of the intensity pattern respectively. Therefore, it can be concluded that the MomSPGD algorithm can enhance the system capability in real application.

## 5. Conclusion

In summary, the SPGD algorithm with a momentum term (MomSPGD algorithm) is innovatively applied for coherent beam combining to substitute for the traditional SPGD algorithm. Firstly, the concept of the momentum method is analyzed in detail, and the MomSPGD algorithm that is used in coherent beam synthesis system is introduced. Secondly, the synthesis system using MomSPGD algorithm is implemented by numerical simulation. The numerical results indicate that the MomSPGD algorithm can converge to the extremum of the metric and realize the phase lock output of the combining system, which is feasible for the field of coherent beam combining. Considering the time-varying phase distortions in practical engineering, the dynamic simulation for coherent beam combining using the MomSPGD algorithm are performed. Compared with the traditional SPGD algorithm, the MomSPGD algorithm can improve convergence speed and disturbance immunity of the coherent beam combining. Furthermore, the experiment of two laser beam coherent combining is carried out. The metric function obtained by photodetector can be locked steadily in a closed loop, which is 1.4 times bigger than the one when the system is in an open loop. The fringe contrast of the long-exposure of the far-field intensity distribution is calculated to be more than 73.42%, which is much higher than 58.88% of that using SPGD algorithm. The experimental results demonstrate that the MomSPGD algorithm can enhance the system capability in real applications.

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