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ARTICLE

Motion modeling of a non-holonomic wheeled mobile robot based on trajectory tracking control

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Abstract: Trajectory tracking is a problem of emphasis for the mobile robot. In this study, a coordinate transformation method was used to build a kinematic model of the wheeled mobile robot. A traditional proportional-integral-derivative control method was researched and improved by combining it with a neural network. A neural network proportional-integral-derivative trajectory tracking control method was thus designed, and a simulation experiment was performed using Simulink. The results show that in circular trajectory tracking control, the maximum errors of the X axis, Y axis, and θ were approximately 2.1 m, 2.3 m, and 0.4 rad, respectively, and that the system remained stable after running for 10 s. In straight-line trajectory tracking control, the maximum errors of the X axis, Y axis, and θ were approximately −0.8 m, 1.3 m, and 0.3 rad, respectively, and the system remained stable after running for 8 s. The error was relatively small, and the effect of trajectory tracking control was good. The studied method had good performance in terms of wheeled mobile robot trajectory tracking control and is worthy of further promotion and application.

Key words: wheeled mobile robot, non-holonomic, trajectory tracking, motion modeling, neural network.

Résumé : Le problème de suivi de trajectoire est l'une des priorités du robot mobile. Dans cette étude, la méthode de transformation de coordonnées a d'abord été utilisée pour construire le modèle cinématique du robot mobile à roues. La méthode de contrôle traditionnelle à dérivée proportionnelle-intégrale a été étudiée et améliorée en la combinant avec un réseau neuronal. Une méthode de contrôle de suivi de trajectoire dérivée proportionnelle-intégrale de réseau neuronal a été conçue et une expérience de simulation préformée dans l'environnement de Simulink. Les résultats ont montré qu'en contrôle de suivi de trajectoire circulaire, les erreurs maximales des axes X, Y et θ étaient 2,1 m, 2,3 m et 0,4 rad, et que le système restait stable après 10 s de fonctionnement ; dans le contrôle de suivi de trajectoire en ligne droite, les erreurs maximales des axes X, Y et θ étaient −0,8 m, 1,3 m et 0,3 rad, et le système restait stable après une course de 8 secondes. L'erreur était relativement petite et l'effet du contrôle de suivi de trajectoire était bon. La méthode mentionnée dans cette étude avait de bonnes performances en matière de contrôle de suivi de trajectoire du robot mobile à roues et méritait une promotion et une application plus poussées. [Traduit par la Rédaction]

Mots-clés : robot mobile sur roues, non holonomique, suivi de trajectoire, modélisation du mouvement, réseau de neurones.

1. Introduction

Robots are widely used in various fields of daily production and life (Abdelmoula et al. 2014). Robots can accomplish many tasks that are difficult or impossible for humans to complete (Iagnemma et al. 2016), which is of great value (Kretzschmar et al. 2016). A wheeled mobile robot (WMR) is a common mobile robot with simple structure, high flexibility, and good operating performance. The WMR is widely used in real life. The WMR is a non-holonomic system (Gao et al. 2014), and problems with its trajectory tracking control have received wide attention from researchers. Asif et al. (2014) proposed an adaptive sliding mode dynamics controller especially for the WMR. The kinematic input of the WMR was obtained

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by a motion controller, and then used as the input of the dynamic controller. The simulation results showed that the method had good stability, fast error convergence, and robustness. Liang et al. (2015) proposed a method based on adaptive images, and they analyzed the error convergence of the method through Lyapunov stability. It was found that the method had good control effect. Li et al. (2016) proposed a hybrid intelligent algorithm for trajectory tracking control with good antijamming capability, which combined kinematics and Takagi-Sugeno-Kang (TSK)-fuzzy control, and the path was determined by improved D* Lite algorithm. The reliability was verified by the experiment of an eight-shaped reference trajectory. Zhai and Song (2019) proposed a unconstrained non-singular fast terminal sliding mode control method, which not only effectively improved the disadvantages of the singularity, but also remained the advantages of the sliding mode control. The current research of WMR trajectory tracking control has made good progress, but there are still some problems such as poor stability and low precision in practical applications. Therefore, based on the PID controller, this study analyzed the trajectory tracking control method with stronger adaptability and more stable control effect. First, a WMR kinematic model was built, and then, the trajectory tracking control method was studied based on the model. Further, a traditional proportional-integral-derivative (PID) control method was combined with a neural network. The reliabilty of the method was verified, providing a contribution to the research of robot trajectory tracking control method.

2. Non-holonomic wheeled mobile robot

When studying motion control problems, the constraint conditions of the system need to be analyzed. In a constraint equation, n is the number of particles and s the number of constraints. If there is no particle speed that can be converted to finite form, which is $f_\alpha(y_i) = 0$, $i = 1, 2, \ldots, n$, $\alpha = 1, 2, \ldots, s$, then it will be a holonomial constraint. If there is particle speed that cannot be converted, i.e., $f_\alpha(y_i, \dot{y}_i) = 0$, $i = 1, 2, \ldots, n$, $\alpha = 1, 2, \ldots, s$, then it will be a non-holonomial constraint.

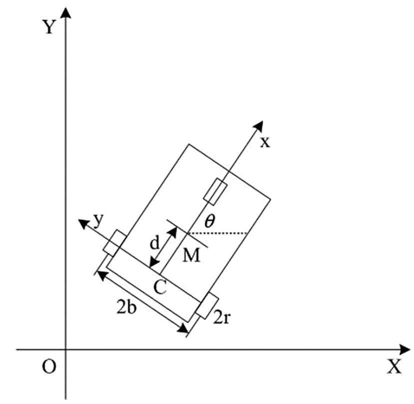
Non-holonomial constraint means that the spatial position and kinematic velocity of an object are both restricted in the movement process. Non-holonomial constraint of the WMR only shows in the pure rolling motion of wheels and the ground, and can be described by displacement, velocity, and acceleration. In the control of non-holonomic systems, WMR motion control has great difficulty (Roy et al. 2015).

3. WMR motion modeling

3.1. WMR physical model

The research object in this study was a three-wheeled mobile robot. The physical model of the WMR is shown in Fig. 1.

Fig. 1. Physical model of the studied three-wheeled mobile robot.



The global coordinate system is expressed by $\{O, X, Y\}$; the local coordinate system is expressed by $\{c, x, y\}$; the midpoint of the two back wheels of the WMR is expressed by C; (x, y) is the coordinate in $\{O, X, Y\}$; θ is the angle between two coordinate systems; r is the radius of the back wheels; $2b$ is the distance between the two back wheels, M is the centroid of the WMR; d is the distance between C and M.

3.2. WMR kinematic model

Kinematic modeling of the WMR was carried out by a coordinate transformation method, and the kinematic velocity equations of the robot system were combined to obtain a simultaneous equation

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{r}{2}\cos\theta + \frac{r}{2b} d\sin\theta & \frac{r}{2}\cos\theta - \frac{r}{2b} d\sin\theta \\ \frac{r}{2}\sin\theta - \frac{r}{2b} d\cos\theta & \frac{r}{2}\sin\theta + \frac{r}{2b} d\cos\theta \\ \frac{r}{2b} & -\frac{r}{2b} \end{bmatrix} \begin{bmatrix} \omega_L \\ \omega_R \end{bmatrix}$$

where ω_L and ω_R are the steering angular velocity of the left and right wheels of the WMR, respectively.

The non-holonomic constraint of the WMR system is represented as $\dot{x}\sin\theta - \dot{y}\cos\theta - d\dot{\theta} = 0$, where q is the pose of the WMR. The non-holonomic constraint can be rewritten into matrix form as

$$A(q) = [\sin\theta \; -\cos\theta \; -d]$$

According to

$$\begin{bmatrix} \omega_R \\ \omega_L \end{bmatrix} = \begin{bmatrix} \frac{1}{r} & \frac{b}{r} \\ \frac{1}{r} & -\frac{b}{r} \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

where v in the equation is the linear velocity of motion of robots, and then the equation of the kinematic model of the WMR is obtained

$$\dot{q} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & d\sin\theta \\ \sin\theta & -d\cos\theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

The trajectory tracking error of the WMR can be represented as

$$\begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - x \\ \theta_r - \theta \end{bmatrix}$$

4. PID trajectory tracking control algorithm based on neural network

4.1. PID control method

PID control method is a typical trajectory tracking control method (Huynh et al. 2017). It is assumed that $x_{e'}$ and $y_{e'}$ are input signals as the expected lateral and longitudinal distance difference, x_e and y_e are output signals as the actual lateral and longitudinal distance difference, and the error ex can be represented as $ex = x_{e'} - x_e$. According to the control law of PID, $U_x = P_x ex + I_x ex + D_x ex$ is obtained, where P_x, I_x, and D_x are the parameters of the PID controller on the X axis. In the same way, the control law of the state error feedback can be obtained

$$U_y = P_y ex + I_y ex + D_y ex$$

4.2. PID control method of neural network

To improve the control effects of the PID controller, the controller was combined with a neural network to obtain a new trajectory tracking control algorithm.

First, the parameters P_x, I_x, and D_x of the PID controller on the X axis are set as the output of the output layer of the neural network, and O is the output of the network input layer. The following can be obtained:

$$\begin{cases} O_j^1(x) = e(x-j),\ j = 1,2,\ldots,M-1 \\ O_M^1(x) = 1 \end{cases}$$

Then, the input and output of the network hidden layer can be represented as

$$\begin{cases} \mathrm{net}_i^2(x) = \sum_{j=0}^{M} w_{ij}^2(x) O_j^1(x) \\ O_i^2(k) = f\left[\mathrm{net}_i^2(x)\right],\ i = 1,2,\ldots,N-1 \\ O_N^2(x) = 1 \end{cases}$$

where w_{ij}^2 is the weight of the hidden layer. The activation function of the hidden layer is $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$, and the input and output of the network output layer can be represented as

$$\begin{cases} \mathrm{net}_i^3(x) = \sum_{i=0}^{N} w_{ij}^3 O_i^2(x) \\ O_i^3(x) = g\left[\mathrm{net}_i^3(x)\right] \end{cases}$$

where

$$g(x) = \frac{1}{2}\left[1 + \frac{e^x - e^{-x}}{e^x + e^{-x}}\right]$$

The output of the neural network is

$$\begin{cases} O_1^3 = P_x \\ O_2^3 = I_x \\ O_3^3 = D_x \end{cases}$$

The specific process of the PID control method of the neural network is that the parameters are first initialized, $x_{e'}$ and $y_{e'}$ are sampled, the neural network is used to calculate P_x, I_x, and D_x, and then neural network learning is performed till the performance can satisfy the needs.

4.3. WMR trajectory tracking control

According to the kinematics equation of the WMR,

$$\dot{q} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & d\sin\theta \\ \sin\theta & -d\cos\theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

When k is set as the time of sampling, the following is obtained

$$\begin{cases} x_i(k+1) = x_i(k) + v_i(k)\cos(\theta_i(k))T \\ y_i(k+1) = y_i(k) + v_i(k)\cos(\theta_i(k))T \\ \theta_i(k+1) = \theta_i(k) + \omega_i(k)T \end{cases}$$

where k is the time of sampling; T is the sampling period; (x_i, y_i, θ_i) is the actual pose coordinate of the WMR.

The tracking error of k can be represented as

$$\begin{cases} e_x(k) = x_r(k) - x_i(k) \\ e_y(k) = y_r(k) - y_i(k) \\ e_\theta(k) = \theta_r(k) - \theta_i(k) \end{cases}$$

and (x_r, y_r, θ_r) is the expected pose coordinate.

According to the PID control method of a neural network, the controlled quantity, $u_x(k)$, $u_y(k)$, and $u_\theta(k)$, of the WMR system can be obtained, where every pose variate of the WMR can be represented as

$$\begin{cases} x_i(k+1) = u_x(k) + x_i(k) \\ y_i(k+1) = u_y(k) + y_i(k) \\ \theta_i(k+1) = u_\theta(k) + \theta_i(k) \end{cases}$$

Then, each control input quantity is substituted into the kinematics model of the WMR, and the actual pose variate of the WMR is obtained, which is inversely pushed to the controller, thereby realizing the trajectory tracking control of the WMR.

5. Simulation experimental results

The method in this study was verified using Simulink, and two trajectories, circle and linear, were analyzed. It is assumed that the angular velocity and linear velocity of the robots were both uniform. The distance between the centroid and the midpoint of the two wheels was set as $d = 5$, the wheel radius as $r = 6$, and the distance between the two wheels as $2b = 28$.

5.1. Circular trajectory tracking control result

In the environment of Simulink, it is assumed that the master robot and slave robot start moving from the same place and that the parameters of the two robots are the same. $(P, I, D) = (1.5, 0.6, 0.8)$, the sampling time was 25 ms, the expected lateral and longitudinal distance difference was $(x_{e'}, y_{e'})$ and the speed of the master robot was $(v, \omega) = (1, 0.2)$. The robot performed circular path motion. The trajectory tracking control results are shown in Fig. 2.

The red color in Fig. 2 represents the trajectory of the master robot, and the blue color represents the trajectory of the slave robot. According to Fig. 2, the trajectory tracking of the master robot and slave robot was good. Under the action of the controller, the short-time change of the motor speed will often cause overshoot, but the overshoot caused by the improved neural network PID controller in this study was small, so that the controller could stabilize the system in a short time. To further analyze the circular trajectory tracking effect, the experiment was repeated 100 times, and the average error was calculated. The tracking error curves of the WMR are shown in Fig. 3.

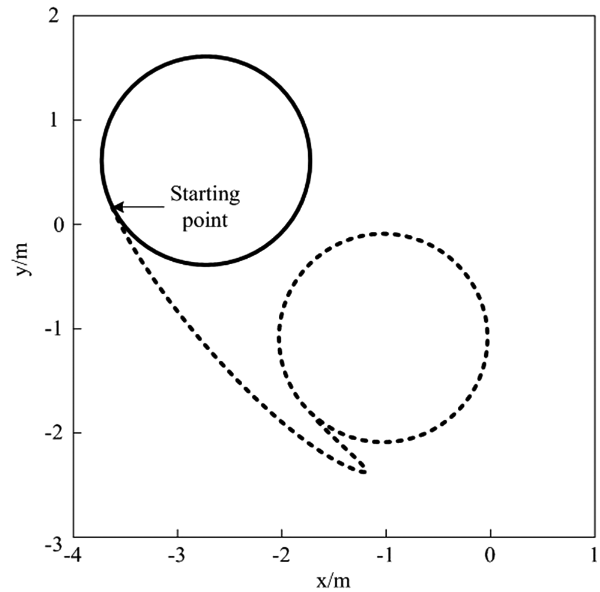
It is seen from Fig. 3 that during the trajectory tracking process, the errors of the X axis, the Y axis, and θ are all small. The maximum error in the X axis direction was approximately 2.1 m, the maximum error in the Y axis direction approximately 2.3 m, and the maximum error of θ approximately 0.4 rad. After running for approximately 10 s, the three errors were all stable at 0, without oscillation, indicating that the trajectory tracking control method could control the error within a small range and had a good control effect.

5.2. Straight-line trajectory tracking control result

The trajectory tracking control method of this study was also used to track the linear trajectory. The result is shown in Fig. 4.

The red color in Fig. 4 represents the trajectory of the master robot, and the blue color represents the trajectory of the slave robot. Using the method of trajectory tracking in this study, the WMR also had a good tracking effect in linear motion. Although there was some overshoot, the stability speed of the system was faster. To further analyze the circular trajectory tracking effect, the experiment was repeated 100 times, and the average error was calculated. The linear tracking error is shown in Fig. 5.

Fig. 2. Circular trajectory tracking result. The red color represents the trajectory of the master robot, and the blue color represents the trajectory of the slave robot.



It is seen from Fig. 5 that the error of the WMR in linear motion was smaller than that of circular trajectory. The maximum error in the X axis direction was approximately −0.8 m, the maximum error in the Y axis direction approximately 1.3 m, and the maximum error of θ approximately 0.3 rad. After running for approximately 8 s, the system was stable, without oscillation, indicating the reliability of the method in this study.

6. Discussion

With the development of society and the improvement of people's needs, robots are constantly developing higher intelligence and more functions. The advancement of robot technology not only facilitates the work in various fields, but also reflects the national science and technology level, which has extremely high practical significance. The WMR is one category of mobile robots. The WMR has fast moving speed and high flexibility, and can be applied in several different fields, such as military, medical, family, and agriculture (Ko et al. 2015). Owing to its own structural problems, the WMR is subject to purely rolling non-holonomic constraints during the motor process (Esmaeili et al. 2017), so it is controlled by non-holonomic systems during motion. The problem of motion control of the robot is one of the main focuses of robot research. Trajectory tracking is an important part of motion control (Pan et al. 2018). In this study, the trajectory tracking control of the WMR was researched through motion modeling.

After the modeling was completed, the PID control method was researched. PID is a classic trajectory tracking control method. To improve its performance,

Fig. 3. Circular trajectory tracking error.

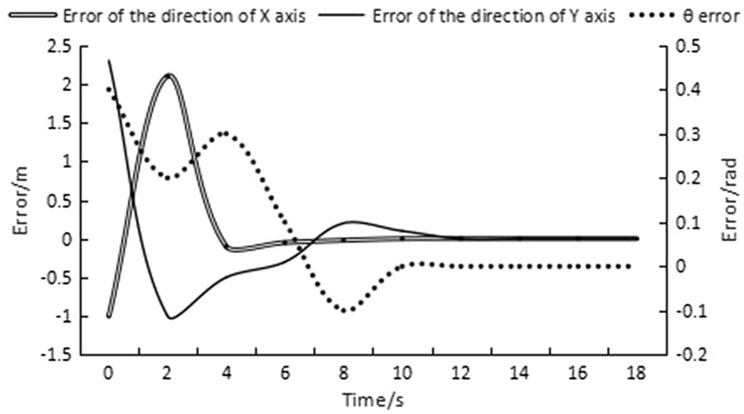
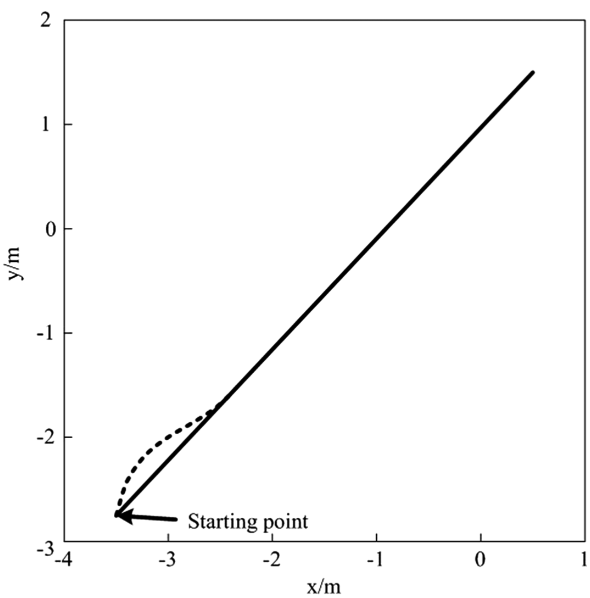


Fig. 4. Straight-line trajectory tracking result. The red color represents the trajectory of the master robot, and the blue color represents the trajectory of the slave robot.



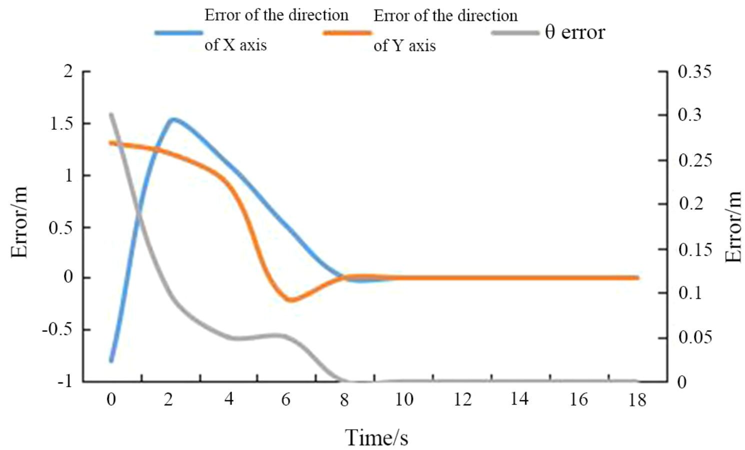
this study proposed a method that combines a neural network algorithm and PID control method that helps adjust the PID control parameters, so that the error of the trajectory tracking was controlled within a small range. Using Simulink, simulation experiments of the circular and linear trajectories were carried out. According to the experimental results, the neural network trajectory tracking control method of this study had a good performance in both circular trajectory tracking and linear trajectory tracking (Figs. 2 and 4). The trajectory of the main robot can be tracked smoothly and quickly. According to the error curve, it can be found that under the control of the trajectory tracking method, the errors of the X axis direction, the Y axis direction, and θ were small, and the system had good stability. In the circular trajectory tracking, the system was stable after 10 s, and the linear trajectory could be stabilized after 8 s, indicating the reliability of the method. The findings suggest that the neural network PID control method is more effective than the traditional control method and has higher stability and smaller tracking error compared with the extensively used sliding mode control method and adaptive control method.

Trajectory tracking control is a key and difficult problem in WMR research. Owing to the great application value of the WMR in daily life, research on its trajectory tracking control method is very important. Owing to the limitations of conditions and time, there are still some shortcomings in the design of this study, such as the modeling of the WMR in an unknown environment and trajectory tracking between multiple robots. In future studies, the following points need to be further studied: the effect of the neural network PID control method in tracking trajectories other than circular and straight line trajectories, the application effect of the method in a live scenario, and multi-robot tracking and robot trajectory tracking control in unknown dynamic environments. Moreover, the application of a multi-robot system in real life is more extensive, and studying the trajectory tracking control problem is beneficial to solving more complicated realistic problems; therefore, it is worth further study.

7. Conclusion

In this study, the trajectory tracking control method of a WMR was researched. First, the kinematics model of the WMR was established. Then, based on the model, the traditional PID controller was improved by adding a neural network algorithm to obtain a neural network PID trajectory. From simulation experiments involving circular trajectory and linear trajectory, it was found that the tracking control method could greatly improve the trajectory tracking of the WMR, control the error of the master robot and slave robot within a small range, and make the system stable in a shorter time. These findings

Fig. 5. Straight-line trajectory tracking error. [Colour online.]



verify the reliability of the proposed method and provide some theoretical basis for the further study of WMR trajectory tracking control.

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