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Implementation of high-precision inertial reference for Taiji-1 satellite and its ground evaluation based on torsion pendulum system

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As the key measurement load of Taiji-1 satellite, inertial sensor detects the acceleration disturbance of test mass (TM) under nonconservative force in line with the basic principle of capacitive sensing, while keeping the TM in equilibrium position through electrostatic drive. In order to ensure the smooth progress of the mission, it is necessary to test and evaluate the performance of inertial sensor on the ground. In this paper, a torsion pendulum system is designed to eliminate the influence of the Earth's gravity so as to meet the requirements of ground test. The experimental results show that the inertial sensor in closed-loop control mode can stably keep the TM at equilibrium position. At the same time, the ground detection of acceleration resolution of inertial sensor is greatly affected by ground vibration noise. If the inertial sensor operates normally in space, its acceleration resolution can reach $3.96 \times 10^{-9} \mathrm{m/s^2/\sqrt{Hz}}$, thus meeting the requirement of Taiji-1.

Keywords: Inertial sensor; sensitive structure; torsion pendulum; acceleration resolution.

1. Introduction

As China's first satellite for verifying the key technologies of spatial gravitational wave detection, Taiji-1 satellite was successfully launched on August 31, 2019. Taiji program is a gravitational-wave detection mission initiated by the Chinese Academy of Sciences. It is expected to launch three satellites by 2030, which will form an equilateral triangle with a side length of 3 million km to detect gravitational waves in the frequency band of 1 mHz to 1 Hz.[1–3] Taiji program is a three-step program. Taiji-1 is just the first step. Its main task is to verify key technologies such as inertial sensing and laser interference ranging.

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[¶]For more details, please refer to article 2102002 of this Special Issue.

Inertial sensor is one of the key loads for detecting spatial gravitational waves.[4] The test mass (TM) in the inertial sensor is the measurement benchmark for gravitational wave detection. One of its surfaces is used to reflect the laser, and is defined as sensitive axis direction. The inertial sensor measures the displacement and acceleration of the TM in real time in line with the basic principle of capacitive displacement sensing. In addition, it controls the position and attitude of the TM with the aid of electrostatic drive control system to minimize the disturbance of space environment to the TM and ensure the free suspension of the TM in the direction of sensitive axis.

The detection of gravitational waves in mHz frequency band requires a spaceborne inertial reference sensor with unprecedented residual acceleration and a laser interferometer system that is able to detect exceptionally small deviations between two reference sensors several million kilometers apart in space. The performance of inertial sensor is determined by the level of residual TM acceleration at mHz frequency band. A heavier TM, a larger gap between the TM and its housing, and a quieter thermal, gravitational and electromagnetic spacecraft environment will lead to a lower acceleration noise level. In order to evaluate and test the performance of spaceborne inertial sensor on the ground, specific methods such as high-pressure suspension, tower crane test and torsion pendulum measurement are needed to overcome the gravity of the Earth and avoid the influence of ground vibration as far as possible.[5–7]

Considering the sub-nanometer resolution of Taiji-1 inertial sensor at the frequency of <1 Hz in the ground environment, a set of torsion pendulum measurement system was designed and developed to test and evaluate it on the ground. The measurement result shows that the acceleration resolution of the inertial sensor can reach $2.6 \times 10^{-7} \mathrm{m/s^2/\sqrt{Hz}}$ in the ground test. The measurement process is greatly affected by ground vibration. In Sec. 2 of this paper, the composition of torsion pendulum system and the specific design of each component are introduced. In Sec. 3, the stability and acceleration resolution of the inertial sensor are measured and evaluated by using the torsion pendulum system, and the experimental results are analyzed. In Sec. 4, a conclusion is drawn.

2. System Composition

The torsion pendulum system used in the ground test and evaluation of inertial sensor mainly includes a sensitive structure, a front-end electronic unit, a torsion pendulum system and a vacuum maintenance system. The sensing structure and front-end electronics (FEE) constitute the inertial sensor. The sensitive structure is mainly composed of TM and electrode housing. The TM and the electrode plates arranged in the electrode housing constitute multiple groups of differential capacitors, which transform the acceleration and displacement signals of the TM into differential capacitance signals. The FEE mainly includes a capacitance sensing measurement circuit and an electrostatic driving circuit. The two circuits can detect the capacitance signal and drive the TM to ensure that the TM is at the center of the electrode housing. The torsion pendulum system enables the TM to overcome the influence of the Earth's gravity, reduces the air damping and weakens the noise influence caused by air flow.

2.1. *Sensitive structure*

The sensitive structure of inertial sensor is a device that transforms displacement signal into capacitance signal. It is mainly composed of TM and electrode housing. The TM and the opposite electrodes on both sides constitute a pair of differential capacitors. When the position of the TM changes, the capacitance will change because of the change in the distance between the two electrodes in a single capacitor. In this case, the displacement of the TM can be obtained by measuring the capacitance change. Then, according to the measured displacement, an appropriate voltage is applied to the electrode plate to drive the TM back to the central position. The basic principle in which two groups of differential capacitors are arranged around the TM is shown in Fig. 1. When the TM is completely in the middle of the electrode plate, the four capacitances are equal, as follows:

$$C_1 = C_2 = C_3 = C_4 = C_0 = \frac{\varepsilon_0 S}{d_0}, \tag{1}$$

where S is the area of electrode plate; $d_0 d_0$ is the distance between the TM and the electrode when the TM is at the center of the electrode plate; ε_0 is the permittivity of vacuum.

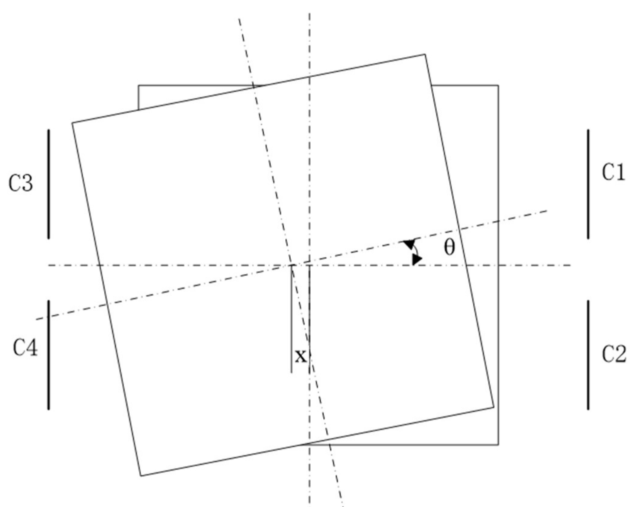

Fig. 1. Basic principle of capacitance sensing.

When the translational displacement x and the rotation angle θ happen to the TM, the relationships between the capacitance difference (between the opposite capacitor plates) and the translational displacement /rotation angle can be expressed as follows:

$$\begin{cases} \Delta C_1 = C_1 - C_3 = \frac{2C_0 x}{d_0} - \frac{C_0 b\theta}{d_0} \\ \Delta C_2 = C_2 - C_4 = \frac{2C_0 x}{d_0} + \frac{C_0 b\theta}{d_0} \end{cases}, \tag{2}$$

where b is the width of the electrode plate and d_0 is the initial spacing. When the relative capacitance difference is measured, the displacement and rotation angle of the TM can be calculated as follows:

$$\begin{cases} x = \frac{(\Delta C_1 + \Delta C_2)d_0}{4C_0} \\ \theta = \frac{(\Delta C_1 + \Delta C_2)d_0}{2C_0 b} \end{cases}. \tag{3}$$

The above formula shows the relationships between capacitance difference and translation displacement/rotation angle when two pairs of electrodes are arranged on both sides of the TM. When the TM only moves or rotates, the displacement or rotation angle can be measured by a single pair of differential capacitors. The number of electrode pairs is equal to the number of degrees of freedom of TM.

The sensitive structure of inertial sensor in Taiji-1 is shown in Fig. 2. The electrode housing is mainly composed of three parts, namely upper, middle and lower electrode plates. The middle electrode plate is a hollow structure in which the TM is in the middle.

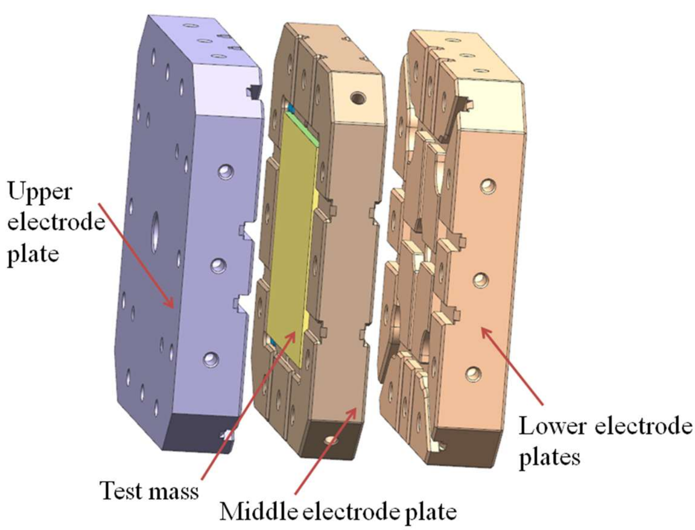

Fig. 2. Sensitive structure of inertial sensor.

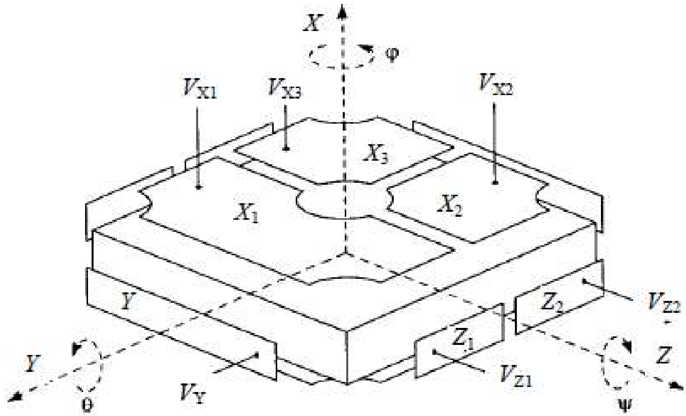

Fig. 3. Layout of electrode pairs.

Six electrode pairs should be provided to realize the measurement and control of six degrees of freedom. In order to fully ensure the rationality of structural layout, three electrode pairs, one electrode pair and two electrode pairs are arranged in *X*-axis, *Y*-axis and *Z*-axis, respectively. The layout of electrode pairs is shown in Fig. 3. The three electrode pairs in the *X*-axis direction are used to measure and control the translation of the TM in *X*-axis and its rotation around *Y*-axis and *Z*-axis. The one electrode pair in the *Y*-axis direction is used to measure and control the translation of the TM in *Y*-axis. The two electrode pairs in the *Z*-axis direction are used to measure and control the translation of the TM in *Z*-axis and its rotation around *X*-axis.

In the fabrication process of electrode housing, a piece of glass ceramic (zerodur®) with ultra-low expansion coefficient is machined initially, and then the whole surface of electrode plate is coated with a layer of gold through physical vapor deposition, and finally the electrodes are fabricated by laser etching technique. At the same time, stop screws are installed on the electrode plate to prevent the direct contact between the TM and the electrode plate. The TM is made of titanium alloy material that is machined and then is plated with gold on each surface. The noise of inertial sensor has a lot to do with the machining and assembly errors of TM and electrode housing. The performance of inertial sensor will be affected by the gap between TM and electrode housing as well as electrode area. Finally, the relevant design indexes of electrode housing and TMs are shown in Table 1.

Because the influence of the Earth's gravity needs to be overcome through suspension, the method of side suspension is selected considering the stability and difficulty level of this suspension. The electrode plate in the middle of the electrode housing is removed in order to hang the TM above the ground and adjust its position and attitude. At the same time, two threaded holes are tapped on the TM surface in the

Table 1. Relevant design indexes of electrode housing and TM.

	Parameter	Design index
1	Dimension of TM	40 mm×40 mm×10 mm
2	Parallelism and perpendicularity of six surfaces	≤1 μm
3	Parallelism and perpendicularity between the electrode faces of middle frame and between its electrode faces and end faces	≤2 μm
4	Average electrode distance in X-axis	60 μm
5	Average active clearance in X-axis	±12 μm
6	Average electrode distance in Y-axis and Z-axis	75 μm
7	Average active clearance in Y-axis and Z-axis	±20 μm
8	Electrode area in Y-axis	200 mm^2
9	Electrode areas in Z_1 and Z_2 axes	100 mm^2
10	Electrode area in X_1-axis	500 mm^2
11	Electrode areas in X_2 and X_1 axes	250 mm^2
12	Electrode symmetry	≤1%

positive direction of Z-axis to serve as the interface of the hanging wire. Because only the electrodes in X-axis are left, the performance of inertial sensor in torsion pendulum test is evaluated by measuring the displacement and acceleration of suspended TM in X-axis direction and the electrostatic drive force. Finally, the structure used to evaluate the inertial sensor is shown in Fig. 4. In order to ensure that the distance between the upper and lower electrode plates remains unchanged after removing the intermediate electrode plate, a spacer block is placed between the upper and lower electrode plates and fixed to them. The material of the spacer block is the same as that of the electrode plates, with the thickness error within 1 μm.

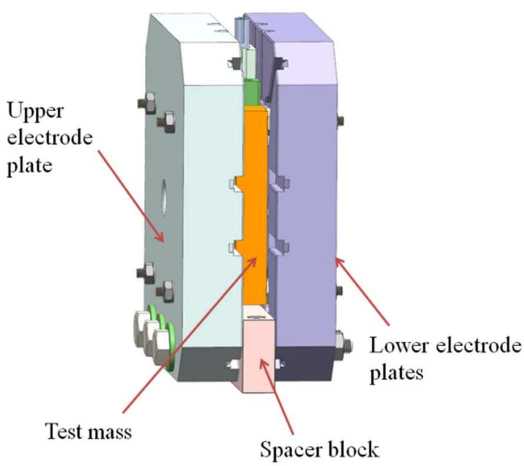

Fig. 4. Structure used to evaluate the inertial sensor.

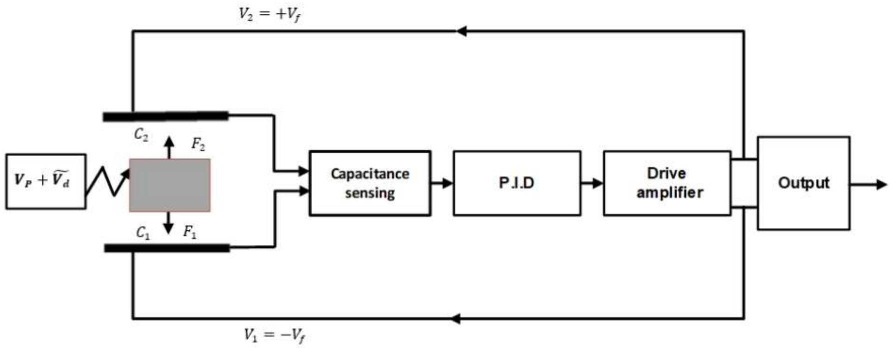

Fig. 5. Basic principle of FEE.

2.2. *Design of front-end electronics*

The main function of FEE of inertial sensor is to detect the capacitance signal generated by the sensitive structure, and apply an appropriate voltage on electrode plates to drive the TM against external nonconservative force. The FEE mainly includes a capacitance sensing circuit and an electrostatic driving circuit. Its basic principle is shown in Fig. 5.

The capacitance sensing circuit can convert the weak capacitance between TM and electrode plate into a voltage signal by applying a 100 kHz sine wave modulation signal to TM to form a resonant capacitance (RC) bridge. This circuit mainly includes a front-end differential bridge circuit, a preamplifier circuit, a band-pass filter circuit, a demodulation filter circuit, an analog-to-digital acquisition circuit, and an excitation source circuit. Its working principle is shown in Fig. 6 in detail.

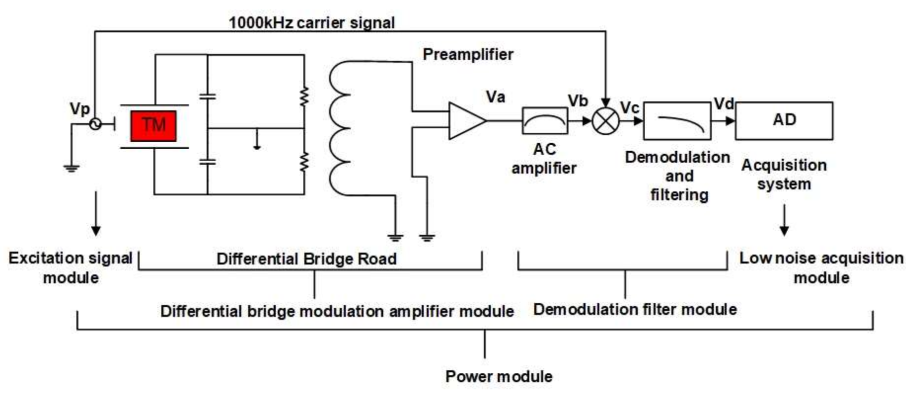

Fig. 6. Capacitance sensing circuit.

The measured voltage signal is transmitted to the PID controller, where the feedback voltage is generated and applied to the electrodes through the electrostatic driving circuit. In order to effectively utilize the area of electrode plate, the capacitance sensing circuit and the electrostatic driving circuit share a common electrode and adopt the technique of frequency division multiplexing, that is, high-frequency excitation voltage is used for capacitance detection, and low-frequency feedback voltage is used for driving. This method can effectively avoid the crosstalk between the driving electrode and the detection electrode. Therefore, before the electrostatic driving voltage enters the electrode, low-pass filtering is needed.

2.3. *Design of torsion pendulum system*

The torsion pendulum system is mainly composed of a suspension mechanism, regulating devices, a vacuum maintenance system and a supporting structure. The composition of torsion pendulum system is shown in Fig. 7. The vacuum maintenance system includes a vacuum chamber and external pumps such as mechanical pump and ion pump. The sensitive structure and the wire for hanging the TM are in the vacuum chamber. The pump vacuumizes the vacuum chamber to high vacuum, and keeps it unchanged to reduce the influence of gas noise in the measurement process.

The suspension mechanism includes not only the wire for hanging the TM, but also the mechanism for adjusting its position and attitude. The suspension wire should have low tension in the torsion direction to better realize the displacement measurement and drive control in the horizontal direction. Finally, a tungsten wire with a diameter of

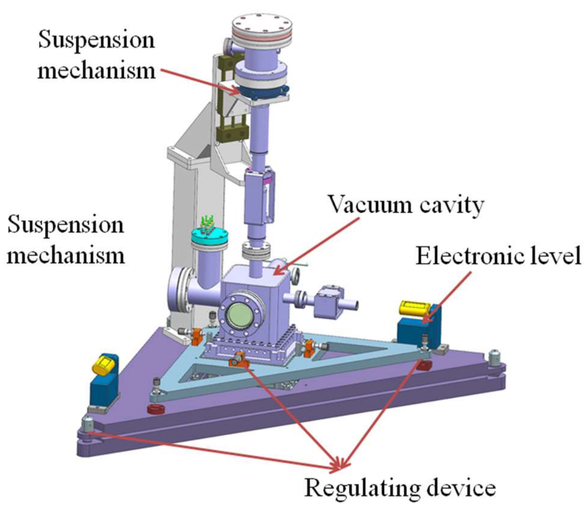

Fig. 7. Torsion pendulum system.

50 μm was selected to hang the TM. The adjustment mechanism in the suspension system mainly consists of a one-dimensional displacement table for adjusting the displacement of TM in the Z-axis direction and a turntable for adjusting the attitude of TM in the rotation around Z-axis. The suspension mechanism is only used for preliminary rough adjustment of the position of TM relative to the upper and lower electrode plates.

The electrode plate is fixed on an adjustable platform which is connected through a flexible hinge. The position of TM relative to the electrode plate can be adjusted more accurately by using the adjusting devices to adjusting the position and attitude of the platform by. The regulating (or adjustment) devices mainly include tilt adjustment device and two-dimensional translation adjustment device. The tilt adjustment device is divided into a coarse adjustment device and a fine adjustment device, both relying on high-precision adjusting screws to realize three-point adjustment. The resolution of fine adjustment device can reach 5 μrad. The two-dimensional adjusting device mainly depends on two groups of adjusting screws to adjust the position of the platform. Each group of adjusting screws are arranged oppositely, with the adjustment resolution of less than 1 μm.

3. Ground Test and Evaluation

The performance of inertial sensor was tested and evaluated on the ground with the torsion pendulum system designed in Sec. 2.3. As shown in Fig. 8, the torsion pendulum system is placed on the air-floating marble platform with active vibration isolation to reduce the influence of ground vibration. The sensitive structure and the FEE are connected by coaxial cable. The FEE and the torsion pendulum must also be grounded. In the laboratory, the environmental cleanliness can reach the level 1000, and the temperature fluctuates within ± 0.02°C. The laboratory is located in Changchun City, Jilin Province, China.

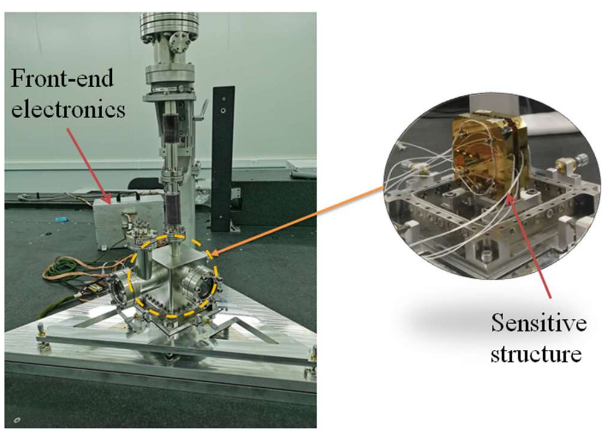

Fig. 8. Torsion pendulum system on marble platform.

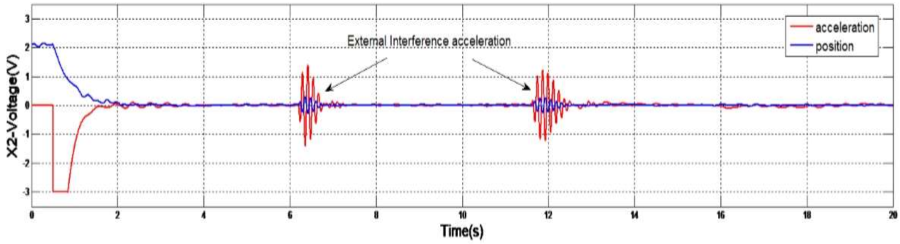

Fig. 9. Curve for closed-loop control stability under test.

3.1. *Test of closed-loop control stability*

The electrode plate is fixed on the translation table of torsion pendulum system, and the TM is hung. The platform is adjusted to horizontal state, and the position of TM relative to the electrode plate is adjusted. When the opposite capacitances are basically equal, the measurement control can begin. The detection curve in the experimental process is shown in Fig. 9. First, the open-loop position detection mode of inertial sensor is turned on. At this time, the driving voltage is 0, and the TM is not at the midway point (i.e. zero position) between the electrode plates. It can be seen that when switching to the closed-loop correction mode, the applied driving voltage rapidly pushes the TM to zero position. When the disturbance is applied artificially, the TM will return to zero position quickly under the control of inertial sensor to resist the influence of external disturbance. The experimental results show that the inertial sensor can stably control the TM.

In order to verify the long-time stability of inertial sensor, the closed-loop acceleration mode was kept active for a long time, and then the test curve in Fig. 10 was obtained. The results show that after 4442 s, the TM was stably controlled in the middle of the electrode plate without drifting.

3.2. *Acceleration resolution*

The acceleration resolution of inertial sensor based on ground evaluation is obtained from the comprehensive analysis of the noise affecting the measurement. The main noise

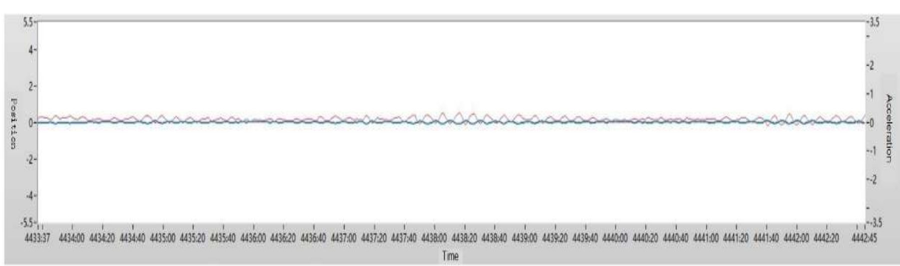

Fig. 10. Stability curve of tested inertial sensor.

sources in the ground evaluation and test of inertial sensor are: seismic noise, temperature fluctuation noise, sensitive-structure machining error, and circuit noise. Then the total noise of the system can be calculated by the following formula:

$$a_{\text{total}} = \sqrt{a_1^2 + a_2^2 + a_3^2 + \cdots}\,, \tag{4}$$

where $a_1 a_1$, $a_2 a_2$ and $a_2 a_3$ are the contribution values of various noises.

The temperature fluctuation noise mainly comes from the fluctuation error caused by the collision between the residual gas molecules and the TM in the vacuum chamber. Its calculation formula is as follows:

$$a_T = \frac{PS}{2m} \cdot \frac{\Delta T}{T} \approx 3 \times 10^{-11} \mathrm{m/s^2/\sqrt{Hz}}\ , \tag{5}$$

where $P = 4 \times 10^{-5}\mathrm{Pa}$ P is the vacuum degree in the vacuum chamber, $P = 4 \times 10^{-5}\mathrm{Pa}$; S is the total area of electrode plate in x-axis direction, $S = 947.4\ \mathrm{mm^2}$.

Because of the machining error of the sensitive structure, the areas of opposite electrode plates may not be exactly the same. Therefore, the noise introduced is as follows:

$$a_{\text{area}} = \frac{\varepsilon_0 \varepsilon_r S \cdot \Delta S}{m d^2}\left(V_p \cdot \Delta V_p + V_d \cdot \Delta V_d\right) \approx 3.6 \times 10^{-12} \mathrm{m/s^2/\sqrt{Hz}}\,, \tag{6}$$

where $\Delta V_p = 1 \times 10^{-5} \mathrm{V/\sqrt{Hz}} \Delta V_p$ is the fluctuation of bias voltage, $\Delta V_p = 1 \times 10^{-5} \mathrm{V/\sqrt{Hz}}$;$\Delta V_d = 2 \times 10^{-5} \mathrm{V/\sqrt{Hz}} \Delta V_d$ is the amplitude fluctuation of excitation signal, $\Delta V_d = 2 \times 10^{-5} \mathrm{V/\sqrt{Hz}}$.

The circuit noise mainly comes from displacement detection noise and electrostatic driving noise. By measuring the power spectral density of the relevant noise interference and calculating with the spring oscillator model of inertial sensor,[9] the displacement detection noise and the driving noise can be obtained as follows:

$$a_{\text{dis}} = 6.8 \times 10^{-10} \mathrm{m/s^2/\sqrt{Hz}}\,, \tag{7}$$

$$a_{\text{act}} = 3.9 \times 10^{-9} \mathrm{m/s^2/\sqrt{Hz}}\,. \tag{8}$$

By querying the relevant parameters of local ground motion and combining them with the vibration isolation conditions in laboratory,[10] the noise is estimated to be

$$a_{\text{ear}} = 1 \times 10^{-6} \mathrm{m/s^2/\sqrt{Hz}}\ . \tag{9}$$

Finally, the total acceleration noise acting on the TM is listed in Table 2.

The "acceleration resolution - power spectral density" curve of inertial sensor in the closed-loop acceleration measurement mode is obtained, as shown in Fig. 11. The results

Table 2. Acceleration noise acting on TM (unit: m/s^2/Hz^1/2).

Thermal gradient	Electrode area asymmetry	Detection noise	Driving noise	Earth pulsation	Total
3×10^{-11}	3.6×10^{-12}	6.8×10^{-10}	3.9×10^{-9}	1×10^{-6}	1×10^{-6}

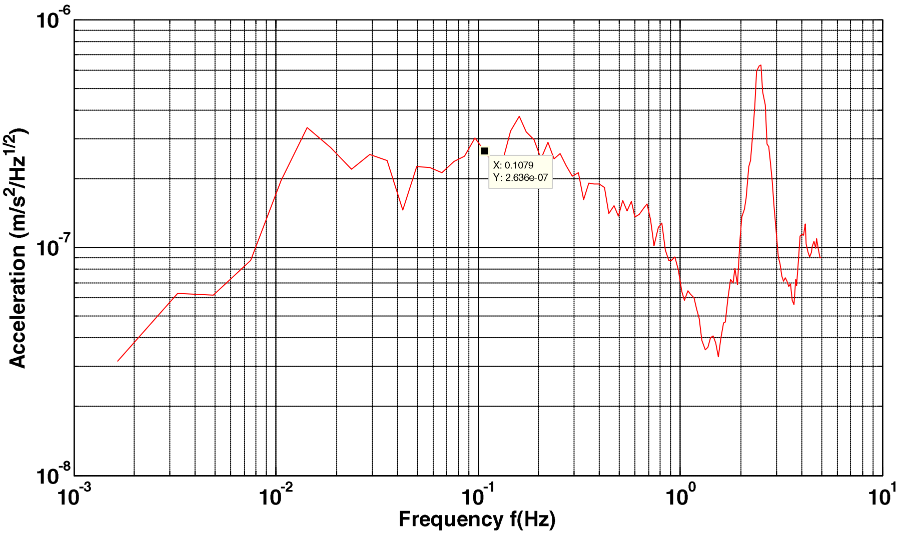

Fig. 11. Acceleration resolution versus power spectral density.

show that at the sensitive frequency of about 0.1 Hz, the acceleration resolution of inertial sensor can reach $2.6\times10^{-7}\mathrm{m/s^2/\sqrt{Hz}}$, thus meeting the Taiji-1 requirement of $9.5\times10^{-7}\mathrm{m/s^2/\sqrt{Hz}}$ in the current environment.

In conclusion, the detection and evaluation of inertial sensor in the ground environment is mainly affected by seismic noise. If this influence can be eliminated by underground laboratory, the acceleration resolution can reach $3.96\times10^{-9}\mathrm{m/s^2/\sqrt{Hz}}$, thus meeting the requirement of $3\times10^{-8}\mathrm{m/s^2/\sqrt{Hz}}$ in Taiji-1 evaluation$3\times10^{-8}\mathrm{m/s^2/\sqrt{Hz}}$.

4. Conclusion

This paper introduces the basic principle and composition of inertial sensor, a key load of Taiji-1 satellite. In addition, a torsion pendulum system is constructed for the ground measurement and evaluation of this inertial sensor system. The closed-loop control stability and acceleration resolution of inertial sensor in the *X*-axis direction are measured and evaluated under laboratory conditions. The experimental results show that in the closed-loop control mode, the inertial sensor exposed to external interference can stably keep the TM in the central position. The analysis of acceleration noise of inertial sensor shows that the ground measurement based on torsion pendulum is mainly affected by ground vibration. In the future, the torsion pendulum system can be optimized to reduce the impact of ground vibration so as to better serve the ground test and evaluation of inertial sensor.

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