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ORIGINAL RESEARCH

Fuzzy model based fault estimation and fault tolerant control for flexible spacecraft with unmeasurable vibration modes

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Abstract

This paper investigates the problem of anti-disturbance fault estimation and attitude tracking fault tolerant control for a flexible spacecraft subject to actuator faults, external disturbances, input saturation, and configuration misalignment. Different form the traditional methods, both the nonlinear dynamics subsystem and attitude control system are reconstructed into Takagi–Sugeno fuzzy models, where partial nonlinear term is remained to assist the design of fuzzy controller. A fuzzy fault estimation observer with proportional–integral adaptive laws is first designed. Without measuring any dynamic information of flexible appendages, estimated values of unknown actuator faults and vibration disturbances of flexible appendages can be obtained simultaneously. This is the most important advantage of the proposed fault estimation scheme over the previous methods. Then, a fuzzy sliding mode fault tolerant control scheme, based on fewer fuzzy rules and less computational burden, is developed under the framework of feedback linearisation and feedforward compensation. The actuator faults and vibration disturbances are counteracted in real time. Regional pole placement and virtual controller with input saturation are adopted to tune the control input and avoid the radical control torque scheduling. Finally, numerical simulations are given to illustrate the validity of the proposed fault estimation and fault tolerant control strategies.

1 | INTRODUCTION

In recent decades, the attitude stabilisation of flexible spacecraft, which is usually composed of a rigid central body and several flexible appendages, has received considerable attention due to its advantages of saving cost and achieving complex space missions. The existence of actuator faults could lead to mission failure and catastrophic consequences for the severe performance degradation or even instability of attitude control system. Reviewing the existing study [1–3], fault estimation and fault tolerant control techniques are effective methods to cope with the unknown actuator faults and enhances the system reliability and maintainability. Different from rigid spacecraft, the intricate and strong coupling characteristics between the rigid and flexible modes will increase the complexity and difficulty in the achievements of accurate fault reconstruction and stable attitude control system [4–6]. However, to the best of the authors' knowledge, none of the existing traditional methods considers the estimation of actuator faults and vibration modes simultaneously in the flexible spacecraft attitude system.

One of the key challenges of fault estimation design in the flexible spacecraft with actuator faults comes from their strong nonlinearity and high uncertainty. The fundamental of observer-based fault estimation is to reconstruct the size and shape of faults by residual information that arise from the mismatch between real plant and observer model [3]. So far, various types of fault estimation approaches for spacecraft have been developed, such as adaptive observer [7], adaptive supertwisting observer [4], intermediate observer [8], and iterative learning observer [9, 10]. Considering actuator faults and external disturbances, an iterative learning observer is proposed to estimate actuator faults of a rigid satellite [9]. In aerospace engineering, apart from the external disturbances, there is always configuration misalignment in spacecraft actuators. It should be pointed out that, even a small degree of misalignment may have a great effect on the fault estimation results by a large adaptive gain or learning rate [10, 11]. In order to estimate actuator faults for a rigid spacecraft with model parameter uncertainties, an adaptive non-linear fault estimation observer proposed in ref. [7], and a fault estimation observer with a stochastically intermediate variable is designed in ref. [8]. However, the spacecraft attitude models considered above have been simplified. The attitude angles of spacecraft are assumed to vary in a small range in ref. [7], and the inertia matrix is assumed to be a diagonal matrix in ref. [8]. From refs. [11–13], actuator faults and disturbances are combined into a lumped disturbance, and the disturbance observer is investigated to address the unknown lumped disturbances. In ref. [14], an adaptive augmented observer is used to simultaneously obtain estimated values of actuator and sensor faults by introducing a new state variable. All the results mentioned above are based on the rigid spacecraft. The problem of sensor fault estimation of flexible spacecraft has been studied in literature [4, 15]. Ref. [16] deals with the actuator fault estimation problem for flexible spacecraft based on an iterative learning observer, while the flexible vibration disturbance caused by flexible appendages is only treated as a lumped disturbance together with the external disturbance. For the actuator complete failure, an adaptive estimation technique is introduced to achieve the estimation of uncertainty of flexibility [17]. However, the actuator fault estimation method applied to flexible spacecraft under the general situation (such as strong flexible vibrations, time varying faults, and large attitude angle working scope) is less considered. Therefore, how to further carry out the estimation of actuator fault and flexible vibration simultaneously for flexible spacecraft under large angle manoeuver is an interesting issue and motivates our study.

For the observer design of complex nonlinear system, Takagi–Sugeno (T–S) fuzzy model has provided a new solution, which consists of a set of locally linearised dynamics connected by fuzzy membership functions. On account of its ability to approximate the complicated nonlinear behaviours of dynamics with any specified accuracy, it has verified to be a very powerful and flexible tool for dealing with non-linear systems [18–20]. In ref. [5], T–S fuzzy modelling method is applied to express the nonlinear behaviours of flexible spacecraft, and a fuzzy robust dissipative control with saturated time-delay input is given. The design of T–S fuzzy model applied to flexible spacecraft mentioned above can be similarly found in refs. [21, 22]. However, for the nonlinear system with complex nonlinearities, the reconstructed T–S fuzzy model may contain a large number of fuzzy local models. It is often difficult for the T–S fuzzy model to achieve stability analysis and control synthesis [23]. In ref. [23, 24], a class of continuous/discrete non-linear systems are represented by the T–S fuzzy models with continuous/discrete nonlinear local models, and the fuzzy models have fewer fuzzy rules than conventional T–S fuzzy models comprised of local linear models. Furthermore, the proposed control design methods with fewer fuzzy rules leads to less computational burden and more relaxed results. Inspired by the above work, the observer-based fault estimation and fault-tolerant control problem for the discrete-time nonlinear systems with unknown perturbation has been studied in ref. [25], and the robust fuzzy model predictive control for the discrete nonlinear systems with parametric uncertainties has been investigated in ref. [26]. Furthermore, a class of complex discrete-time nonlinear system with time-varying delay and actuator faults are considered in ref. [27], and a T–S fuzzy modelling method with nonlinear functions satisfying some sector-bounded conditions is applied. In ref. [28], a fuzzy control scheme for nonlinear impulsive switched systems described by nonlinear T–S fuzzy models is developed, where only some of the nonlinearities are used as premise variables.

Recently, a great deal of results on the stable attitude control for flexible spacecraft has been given. Sliding mode control has been widely applied to the design of fault tolerant control due to its insensitive to external disturbances and model uncertainties [9, 29]. Based on the backstepping technique and adaptive sliding mode control, the attitude tracking fault tolerant control approach has been studied in refs. [7, 30, 31]. In addition to external disturbances and configuration misalignment, the effects of input saturation is also the key factor to be considered in the real attitude control system, especially in the presence of actuator faults. Based on the above consideration, an integral sliding mode control is proposed to generate control torque with desired performance [10]. A nonlinear integral sliding mode control with dual-layer gain adaption scheme is studied to accommodate fault [32]. By introducing the fast nonsingular terminal sliding mode surface, a fault tolerant control method is designed to avert the singularity problem [33]. In addition, the unwinding problem and actuator faults are simultaneously considered in refs. [11, 34]. However, in most of the previous studies concerning fault tolerant control, it is based on the rigid spacecraft and no flexible vibrations are considered. Considering the actuator faults and coupling effect of flexible modes, ref. [35] develops an attitude controller with a fixed-time nonsingular terminal sliding mode, which provides fast fixed-time attitude manoeuver with high precision, singularity avoidance, and chattering free. A fault tolerant control with time variable sliding surface is investigated to deal with the compensation problem caused by actuator faults [36]. Moreover, for the flexible spacecraft with time varying inertia uncertainties and actuator faults, the L2-gain is employed to obtain an adaptive controller with robust performance [37]. Ref. [22] addresses a robust finite-time non-fragile sampled-data control problem via an observer-based control approach. The T–S fuzzy model approach and actuator faults with Bernoulli distribution are considered, while spacecraft attitude model considered is simplified by setting body frame as principal-axis frame. The same assumption constraint can also be found in ref. [17]. Ref. [38] provides a model-free adaptive fault tolerant controller for flexible spacecraft with inertia uncertainties and actuator faults. However, attitude angles of spacecraft are assumed to vary in a small range and inertia matrix is assumed diagonal. The fault tolerant attitude tracking control and vibration suppression for flexible spacecraft without attitude angular velocity measurement are studied in ref. [4]. Based on T–S fuzzy model, an adaptive integral sliding mode control strategy for the flexible spacecraft with configuration misalignment and unknown actuator dead-zone is investigated [21]. Nevertheless, the considered T–S fuzzy model applied to the flexible spacecraft is complex owing to six premise variables.

Motivated by the aforementioned discussion, based on the T–S fuzzy modelling approach, this article is dedicated to address the fault estimation and fault tolerant control problem for flexible spacecraft in the presence of actuator faults, input saturation, external disturbances, and configuration misalignment. The proposed T–S fuzzy model for flexible spacecraft can be applied to manoeuvering at any angle and a large range of angular velocity. Then, a fuzzy adaptive observer is designed for the flexible spacecraft attitude system, which combines the augmentation technique with adaptive laws to address the estimation problem of actuator fault and flexible vibration disturbance. It should be pointed that, reviewing existing methods, the actuator fault estimation method applied to the general flexible spacecraft attitude system (considering model uncertainties, segregating the effect of flexible vibration, and no manoeuver angle constraints) is less considered. Moreover, based on feedback compensation provided by fuzzy observer, a fuzzy adaptive sliding mode fault tolerant control strategy is developed. Based on Lyapunov theory and H_∞ optimisation, all estimation errors in the observer are proved to be ultimately uniformly bounded, and the asymptotically stability and reachability for the closed-loop control system is guaranteed. Compared with existing works, the main contributions of this study are summarised as follows:

1) Compared with the T–S fuzzy model with six premise variables for flexible spacecraft in refs. [5, 21, 22], the proposed method can approximate the flexible spacecraft with only three premise variables. In this study, fewer premise variables that meet the design rationality imply fewer fuzzy rules and less computational burden, which are conducive to the design of fuzzy model. Thus, owing to comprehensive and concise fuzzy rules, the T–S fuzzy model possess higher approximation precision. It is worth mentioning that the T–S fuzzy models of dynamic subsystem (for the observer design) and attitude control system (for the controller design) share the same set of fuzzy rules.
2) The fuzzy adaptive observer proposed can simultaneously obtain the estimation of the system state, actuator fault, and flexible vibration, while real time measurements of the dynamic behaviour of flexible appendages are not required.
3) Based on the proposed fuzzy observer and sliding mode control theory, the proposed fuzzy adaptive fault tolerant controller provides higher control precision and better energy-saving performance with insensitive to external disturbances and flexible vibrations, fault tolerant to actuator faults.

This paper is organised as follows. The dynamic model for flexible spacecraft attitude system with actuator faults is established in Section 2. Section 3 shows a T–S fuzzy fault estimation strategy to achieve online estimation of actuator faults and flexible vibrations. A fuzzy adaptive fault tolerant control strategy is design in Section 4. Section 5 provides the simulation results, including comparisons with some methods from the literature. In Section 6, conclusions of this paper is given.

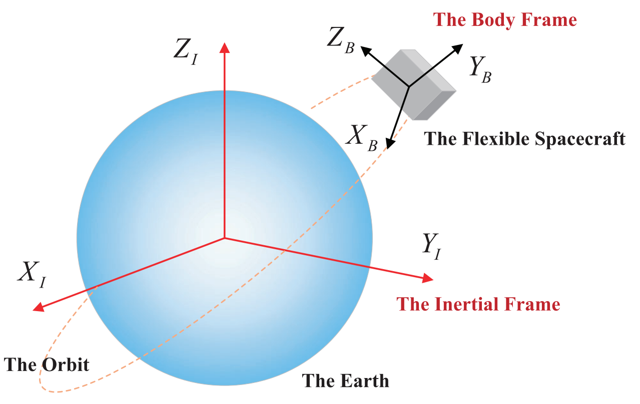

FIGURE 1 The flexible spacecraft in orbit

2 | PROBLEM FORMULATION

The following notations are used. For a matrix A, A^T denotes its transpose form; $\langle A\rangle_S$ denotes the form $A+A^T$; A^+ is the Moore–Penrose inverse matrix; For a square matrix A, A^- is the inverse matrix. $*$ is used for blocks induced by symmetry; diag$(\cdot)$ denotes the block diagonal matrix; I denotes the identity matrix with appropriate dimension; $\|\cdot\|_p$ represents the p-norm in the Euclidean space; $\lambda_{\min}(\cdot)$ represents the minimum eigenvalue of a matrix. For conciseness, the form x is equivalent to the form $x(t)$. $w^\times$ is the cross product operator for $w^T=[w_1 \quad w_2 \quad w_3]\in\mathbb{R}^{1\times 3}$ described by

$$w^\times=\begin{bmatrix}0 & -w_3 & w_2\\ w_3 & 0 & -w_1\\ -w_2 & w_1 & 0\end{bmatrix} \tag{1}$$

2.1 | Flexible spacecraft attitude error dynamics

Figure 1 shows the flexible spacecraft in orbit. The origin of the body frame O_B is defined on the spacecraft centroid. The origin of the inertial frame O_I is located in the centre of the earth.

Considering the actuator configuration misalignment, the flexible spacecraft attitude system can be described as follows [5, 36]:

$$\begin{cases} J\dot{\omega}=-\omega^\times\left(J\omega+\delta^T\dot{\eta}\right)-\delta^T\ddot{\eta}+(\Gamma+\Delta\Gamma)u+u_d,\\ \ddot{\eta}=-C_\eta\dot{\eta}-K_\eta\eta-\delta\dot{\omega}, \end{cases} \tag{2}$$

$$\begin{cases} \dot{q}_v = \frac{1}{2}\left(q_0 I + q_v^{\times}\right)\omega, \\ \dot{q}_0 = -\frac{1}{2} q_v^T \omega, \end{cases} \tag{3}$$

where Equation (2) represents the dynamics subsystem that describes the motion of the flexible spacecraft rigid body, the vibration of the flexible appendages, and the coupling between them. The kinematics subsystem (3) is described by the unit quaternion $q = [q_0 \quad q_v^T] \in \mathbb{R}^{4\times1}$, where q satisfies the constraint $\|q\|_2 = 1$ and represents the attitude orientation of the spacecraft in the body frame O_B with respect to the inertial frame O_I. $J \in \mathbb{R}^{3\times3}$ is the inertia matrix of spacecraft. $\omega^T = [\omega_x \quad \omega_y \quad \omega_z] \in \mathbb{R}^{1\times3}$ is the angular velocity of spacecraft, expressed in O_B relative to O_I. $\Gamma \in \mathbb{R}^{3\times n_u}$ denotes the distribution matrix of actuator and $\Delta\Gamma \in \mathbb{R}^{3\times n_u}$ denotes the unknown perturbation matrix caused by actuator configuration misalignment, where n_u is the number of actuators. $u \in \mathbb{R}^{n_u\times1}$ denotes the expected control input. $u_d \in \mathbb{R}^{3\times1}$ denotes the external disturbance, such as the air drag, solar radiation pressure, and gravity gradient moment. η denotes the generalised flexible displacements that represent the vibration modes. δ denotes the coupling matrix between rigid body and flexible appendages. $C_\eta = \mathrm{diag}\{2\xi_i\omega_{ni}\}$ and $K_\eta = \mathrm{diag}\{\omega_{ni}^2\}(i = 1, 2, \dots, n_\eta)$ are the damping and stiffness matrices, respectively. Where ξ_i and ω_{ni} represent the ith damping ratio and modal natural frequency, respectively; n_η is the number of modal frequencies to be taken into account.

In order to address control issues of attitude tracking and holding for a flexible spacecraft, the attitude tracking error $q_e = (q_{e0}, q_{ev}^T) \in \mathbb{R}^{4\times1}$ is defined to describe the relative orientation between O_B and the desired frame O_D. Giving the desired spacecraft attitude $q_d = (q_{d0}, q_{dv}^T) \in \mathbb{R}^{4\times1}$, q_e can be computed by

$$\begin{bmatrix} q_{e0} \\ q_{ev} \end{bmatrix} = \begin{bmatrix} q_{dv}^T q_v + q_{d0} q_0 \\ q_{d0} q_v - q_{dv}^{\times} q_v - q_{dv} q_0 \end{bmatrix}, \tag{4}$$

where $\|q_e\|_2 = 1$ and $\|q_d\|_2 = 1$.

The angular velocity tracking error $\omega_e \in \mathbb{R}^{3\times1}$ is defined as:

$$\begin{cases} \omega_e = \omega - \Theta_q \omega_d, \\ \Theta_q = \left(q_{e0}^2 - q_{ev}^T q_{ev}\right) I_3 + 2 q_{ev} q_{ev}^T - 2 q_{e0} q_{ev}^{\times}, \end{cases} \tag{5}$$

where ω_d denotes desired attitude angular velocity, Θ_q denotes the rotation matrix with $\dot{\Theta}_q = -\omega_e^{\times}\Theta_q$.

On the basis of above tracking errors, some controllers can be designed to achieve the attitude pointing stability and tracking control, such as classical proportional–integral-derivative control, sliding mode control [4, 6, 21], and adaptive control [37].

Apart from the above, two important problems on flexible spacecraft deserving more attention are actuator faults $f(t) \in \mathbb{R}^{n_u\times1}$ and input saturation that the control input is constrained in a compact set, i.e. $0 \le \|u\|_1 < u_{\max}$. Then, the dynamics subsystem (2) can be rewritten as:

$$\begin{cases} \dot{\omega} = -\left(J - \delta^T\delta\right)^{-1}\omega^{\times}J\omega + \left(J - \delta^T\delta\right)^{-1}\Gamma\left(u + f\right) \\ \quad + \left(J - \delta^T\delta\right)^{-1} d(t), \\ d = \Delta\Gamma\left(u + f\right) + u_d + \delta^T\left(C_\eta\dot{\eta} + K_\eta\eta\right) - \omega^{\times}\delta^T\dot{\eta}, \end{cases} \tag{6}$$

$$\begin{bmatrix} \dot{\eta} \\ \ddot{\eta} + \delta\dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -K_\eta & -C_\eta \end{bmatrix}\begin{bmatrix} \eta \\ \dot{\eta} + \delta\omega \end{bmatrix} + \begin{bmatrix} -\delta \\ C_\eta\delta \end{bmatrix}\omega. \tag{7}$$

The lumped disturbance $d(t)$ comprises two components: 1) $d_1(t) = \Delta\Gamma(u + f) + u_d$ caused by external disturbances and actuator configuration misalignment; 2) Vibration disturbance $T_d(t) = \delta^T(C_\eta\dot{\eta} + K_\eta\eta) - \omega^{\times}\delta^T\dot{\eta}$ caused by flexible appendages.

Remark 1. The actuator fault $f(t)$ and lumped disturbance $d(t)$ share the same control input matrix $(J - \delta^T\delta)^{-1}$ in Equation (6). Obviously, the large disturbance will degenerate the accuracy of fault estimation. Thus, how to estimate the actuator fault with high performance under strong disturbance is a difficulty and challenge, which is also one of the main works of this paper. Note that flexible appendages of spacecraft are made of advanced materials and the damping and stiffness matrices are known. The external disturbance torque undergone by the spacecraft is a relatively small quantity for a long time [21, 38]. Benefiting from modern mature installation technology and high-precision measurement tools, the configuration misalignment is a small quantity. Based on Equations (6) and (7), there is an effective solution to overcome this problem by estimating vibration disturbances and actuator faults at the same time.

2.2 | T–S fuzzy model transformation

The dynamics subsystem (6) and (7) can be reconstructed into a T–S fuzzy system by the following IF-THEN rules.

Model rule i: IF $z_1(t)$ is M_1^i and … and $z_p(t)$ is M_p^i, THEN

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i\Gamma\left(u(t) + f(t)\right) + B_i d_1 + D_{2i} d_2, \\ y(t) = C_i x(t), \\ d_2(t) = A_{d1i}\varpi(t) + B_{d1i} u_2(t), \\ \dot{\varpi}(t) = A_{d2i}\varpi(t) + B_{d2i} u_2(t), \quad i = 1, 2, \dots, r, \end{cases} \tag{8}$$

where

$$A_i = -\left(J - \delta^T\delta\right)^{-1}\omega^{\times}J, B_i = \left(J - \delta^T\delta\right)^{-1}, C_i = I,$$

$$D_{2i} = \left(J - \delta^T\delta\right)^{-1}\left[\delta^T K_\eta \quad \delta^T C_\eta - \omega^{\times}\delta^T\right],$$

$$A_{d1i} = I, u_2(t) = \omega(t), d_2 = \begin{bmatrix} \eta(t) \\ \dot{\eta}(t) \end{bmatrix}, \varpi(t) = \begin{bmatrix} \eta \\ \dot{\eta} + \delta\omega \end{bmatrix},$$

$$A_{d2i} = \begin{bmatrix} 0 & I \\ -K_\eta & -C_\eta \end{bmatrix}, B_{d1i} = \begin{bmatrix} 0 \\ -\delta \end{bmatrix}, B_{d2i} = \begin{bmatrix} -\delta \\ C_\eta \delta \end{bmatrix}. \tag{9}$$

$z_j(t) (j = 1, 2, \ldots, p)$ are measurable premise variables and p is its quantity; M_j^i is the fuzzy set and r is the number of model rules; $x(t)$ is the system state with $x(t) = \omega(t)$; $y(t)$ is the measurement output. In this paper, the state vector $x(t)$ is set as the premise variable $z(t)$. Then, there are common matrices $B_i = B$, $C_i = C, A_{d1i} = A_{d1}, A_{d2i} = A_{d2}, B_{d1i} = B_{d1}$, and $B_{d2i} = B_{d2}$.

The defuzzification process of the model (8) can be represented as:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z)\{A_i x(t) + B\Gamma\big(u(t) + f(t)\big) \\ \qquad \times B d_1(t) + D_{2i} d_2(t)\}, \\ y(t) = Cx(t), \\ d_2(t) = A_{d1}\varpi(t) + B_{d1}u_2(t), \\ \dot{\varpi}(t) = A_{d2}\varpi(t) + B_{d2}u_2(t), \end{cases} \tag{10}$$

where $h_i(z(t)) = \frac{\prod_{j=1}^{p} M_j^i(z_j(t))}{\sum_{i=1}^{r}\prod_{j=1}^{p} M_j^i(z_j(t))}$ with $0 \le M_j^i(z_j(t)) \le 1$ and $\sum_{i=1}^{r} h_i(z(t)) = 1$.

Remark 2. In practice, the actuator faults existing in the dynamic subsystem can be transmitted to the kinematic subsystem through the closed-loop spacecraft attitude control system. However, according to observer design theory, we here only need the dynamic subsystem to realise the estimation of actuator fault. Thus, only attitude angular velocity $\omega(t)$ is the premise variable of T–S fuzzy model proposed, which greatly reduces the number of model rules and the complexity of the T–S fuzzy system.

Based on the above model, some rational assumptions are given to assist the main results.

Assumption 1. The external disturbance $u_d(t)$ and actuator fault $f(t)$ are bounded with properties that $\|u_d(t)\| \le \kappa_d$ and $\|f(t)\| \le \kappa_f$, where both the upper bound $\kappa_d \ge 0$ and $\kappa_f \ge 0$ exist but are unknown.

Assumption 2. The flexible vibration mode η and its derivatives $\dot{\eta}$ and $\ddot{\eta}$, are bounded with properties that $\|\eta\| \le \kappa_{\eta 0}$, $\|\dot{\eta}\| \le \kappa_{\eta 1}$, $\|\ddot{\eta}\| \le \kappa_{\eta 2}$, where $\kappa_{\eta i}, i = 0, 1, 2$ are unknown.

Assumption 3. Both the attitude $q(t)$ and the angular velocity $\omega(t)$ own bounded first-order derivatives.

Assumption 1 is a general constraint and has been widely considered in literature [7, 16, 31, 32, 35], which is reasonable in engineering application owing to physical limitations. Assumption 2 is feasible in practical situation due to the damping in flexible structures [5, 16, 35]. Assumption 3 is natural and realistic because signals $q(t)$, $\dot{q}(t)$, $\omega(t)$, and $\dot{\omega}(t)$ are energy-bounded in practical systems [16, 31, 32].

Before the main results, some necessary lemmas are introduced as follows.

Lemma 1 [15]. *For system $G : (A_i, B_i, C_i, D_i)$, the H_∞ performance with an attenuation level $\gamma > 0$ will be guaranteed if there exist a symmetric positive definite matrix P such that:*

$$\begin{bmatrix} \langle PA_i \rangle_S & PB_i & C_i^T \\ * & -\gamma^2 I & D_i^T \\ * & * & -I \end{bmatrix} < 0. \tag{11}$$

Lemma 2 [39]. *Given symmetric positive definite matrix P, then*

$$\begin{bmatrix} \xi \\ \psi \end{bmatrix}^T \begin{bmatrix} \langle PA \rangle_S & PB \\ * & 0 \end{bmatrix} \begin{bmatrix} \xi \\ \psi \end{bmatrix} < 0 \tag{12}$$

holds for any $\xi \neq 0$ and ψ satisfying $\psi^T\psi \le \xi^T C^T C \xi$ if and only if there exists a scalar $\sigma \ge 0$, such that

$$\begin{bmatrix} \langle PA \rangle_S + \sigma C^T C & PB \\ * & -\sigma I \end{bmatrix} < 0. \tag{13}$$

Lemma 3 [40]. *The eigenvalues of a given matrix A belong to the circular region $O(\phi, \varepsilon)$ with centre $\phi + j0$ and radius ε if and only if there exists a symmetric positive definite matrix P such that the following condition holds*

$$\begin{bmatrix} -P & P(A - \phi I) \\ * & -\varepsilon^2 P \end{bmatrix} < 0. \tag{14}$$

3 | FAULT ESTIMATION STRATEGY DESIGN

This section mainly introduces the design of the proposed fuzzy adaptive fault estimation observer (FAFEO), including the sufficient existence condition given in linear matrix inequality (LMI) and feasibility proof.

3.1 | Fuzzy adaptive observer

Firstly, the following error vectors are defined as:

$$e_Z(t) = Z(t) - \hat{Z}(t), \quad Z = x, y, f, d_2, \varpi, \tag{15}$$

where $\hat{Z}(t)$ is the online estimation of vector $Z(t)$. $e_x(t)$, $e_y(t)$, $e_f(t)$, and $e_d(t)$ denote the state estimation error, residual error, fault estimation error, and disturbance estimation error, respectively. $e_\varpi(t)$ denotes the estimation error of $\varpi(t)$.

For the T–S fuzzy system (10), the FAFEO is given as:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(z)\{A_i\hat{x}(t) + B\Gamma\big(u(t) + \hat{f}(t)\big) \\ \qquad +D_{2i}\hat{d}_2(t) + K_i\big(y(t) - \hat{y}(t)\big)\}, \\ \hat{y}(t) = C\hat{x}(t), \end{cases} \tag{16}$$

where the fuzzy adaptive laws are designed as:

$$\begin{cases} \dot{\hat{f}}_i(t) = \mathrm{Proj}\big[\Psi_i(y,\hat{y})\big],\ i = 1, 2, \ldots, n_u \\ \qquad = \begin{cases} 0, & \text{if } \hat{f}_i(t) \ge f_{\max},\quad \Psi_i(y,\hat{y}) > 0, \\ 0, & \text{if } \hat{f}_i(t) \le f_{\min},\ \Psi_i(y,\hat{y}) < 0, \\ \Psi_i(y,\hat{y}), & \text{otherwise}, \end{cases} \\ \Psi(y,\hat{y}) = \sum_{i=1}^{r} h_i(z)\{F_{1i}(y-\hat{y}) + \theta_1 F_2(\dot{y} - \dot{\hat{y}})\}, \\ \hat{d}_2(t) = A_{d1}\hat{\varpi}(t) + B_{d1}u_2(t),\ \big\|\hat{d}_2(t)\big\|_1 \le \hat{d}_{\max}, \\ \dot{\hat{\varpi}}(t) = \sum_{i=1}^{r} h_i(z)\{A_{d2}\hat{\varpi}(t) + B_{d2}u_2(t) \\ \qquad + G_{1i}(y-\hat{y}) + \theta_2 G_2(\dot{y} - \dot{\hat{y}})\}, \end{cases} \tag{17}$$

where K_i, F_{1i}, and G_{1i} are the gain matrices with appropriate dimensions to be designed. The gain matrix F_2, G_2 and adjustable scalars θ_1, θ_2 are chosen by the designer. $\hat{d}_{\max}$ denotes the allowable maximum value of $\hat{d}_2(t)$. $f_{\max}$ and $f_{\min}$ denote the maximum and minimum allowable values of $\hat{f}_i(t)$, respectively.

Subtracting system (16) from Equation (10), the dynamic error equation is obtained by

$$\begin{cases} \dot{e}_x(t) = \sum_{i=1}^{r} h_i(z)\left\{(A_i - K_iC)\, e_x(t) + B\Gamma e_f(t) \right. \\ \qquad \left. +D_{2i}e_d(t) + Bd_1(t)\right\}, \\ \dot{e}_y(t) = Ce_x(t), \\ \dot{e}_f(t) = \sum_{i=1}^{r} h_i(z)\left\{\dot{f}(t) - F_{1i}Ce_x(t) - \theta_1 F_2 C\dot{e}_x(t)\right\}, \\ e_d(t) = A_{d1}e_{\varpi}(t), \\ \dot{e}_{\varpi}(t) = \sum_{i=1}^{r} h_i(z)\left\{A_{d2}e_{\varpi}(t) - G_{1i}Ce_x(t) \right. \\ \qquad \left. -\theta_2 G_2 C\dot{e}_x(t)\right\}, \end{cases} \tag{18}$$

With the augmented variable $\bar{e}^T(t) = [e_x^T \quad e_f^T \quad e_{\varpi}^T]$, the fuzzy system (18) can be rewritten as:

$$\dot{\bar{e}}(t) = \sum_{i=1}^{r} h_i(z)\{\bar{M}^{-1}\big(\bar{A}_i - \bar{K}_i\bar{C}\big)\bar{e}(t) + \bar{M}^{-1}\bar{B}_d\bar{d}_o(t)\}, \tag{19}$$

where

$$\bar{M} = \begin{bmatrix} I & 0 & 0 \\ \theta_1 F_2 C & I & 0 \\ \theta_2 G_2 C & 0 & I \end{bmatrix}, \bar{A}_i = \begin{bmatrix} A_i & B\Gamma & D_{2i} \\ 0 & 0 & 0 \\ 0 & 0 & A_{d2} \end{bmatrix},$$

$$\bar{K}_i = \begin{bmatrix} K_i \\ F_{1i} \\ G_{1i} \end{bmatrix}, \bar{C} = \begin{bmatrix} C & 0 & 0 \end{bmatrix},$$

$$\bar{B}_d = \begin{bmatrix} B & 0 \\ 0 & I \\ 0 & 0 \end{bmatrix}, \bar{d}_o(t) = \begin{bmatrix} d_1(t) \\ \dot{f}(t) \end{bmatrix}. \tag{20}$$

Note that $\bar{M}$ is nonsingular irrespective of the terms $\theta_1 F_2 C$ and $\theta_2 G_2 C$.

Performing Laplace transformation on the augmented dynamic error system (19), the estimation error can be derived from

$$\begin{aligned} &\bar{e}(s) = G_{\bar{e}\bar{d}}(s)\bar{d}_o(s), \\ &G_{\bar{e}\bar{d}}(s) = \sum_{i=1}^{r} h_i(z)\left\{ \left[sI - \bar{M}^{-1}(\bar{A} - \bar{K}_i\bar{C})\right]^{-1}\bar{M}^{-1}\bar{B}_d \right\}, \end{aligned} \tag{21}$$

where the transfer function $G_{\bar{e}\bar{d}}(s)$ is a multivariable function of $\{\bar{K}_i, \bar{M}\}$. Clearly, $\bar{e}(t)$ is not only related to $\dot{f}(t)$ and $u_d(t)$, but also related to $u(t)$ and $f(t)$ caused by actuator configuration misalignment. In order to attenuate the influence of above disturbances on estimation, the H_∞ optimisation is introduced into the design of FAFEO.

Remark 3. From Equation (17), the online fault estimator can be derived as:

$$\begin{aligned} \hat{f}(t) = \sum_{i=1}^{r} h_i(z)\Bigg\{ & F_{1i}\int_{t_0}^{t} Ce_x(\tau)d\tau + \theta_1 F_2 Ce_x(t) \\ & - \theta_1 F_2 Ce_x(t_0)\Bigg\}, \end{aligned} \tag{22}$$

where t_0 denotes a certain moment before the fault occurrence. It is obvious that the presented fault estimator $\hat{f}(t)$ is a proportional–integral (PI) function of $e_x(t)$, where only the current measurement output are required. Similarly, we can obtain the same conclusion on the online disturbance estimator.

3.2 | Observer-based fault estimation design

Theorem 1. *For systems (10) and (16), let an H_∞ performance index $\gamma_1 > 0$, and a circular pole constraint region $O(\phi_1, \varepsilon_1)$ for $\bar{M}^{-1}(\bar{A}_i - \bar{K}_i\bar{C})$ be given. For prescribed scalars θ_1, θ_2, and matrices F_2, G_2, if there exist matrices Y_{1i}, $Q > 0$ satisfying*

$$\begin{bmatrix} \langle \bar{M}^T Q\bar{A}_i - \bar{M}^T Y_{1i}\bar{C}\rangle_S + I & \bar{M}^T Q\bar{B}_d \\ * & -\gamma_1^2 I \end{bmatrix} < 0, \tag{23}$$

$$\begin{bmatrix} -Q & -\phi_1 Q\bar{M} + Q\bar{A}_i - Y_{1i}\bar{C} \\ * & -\varepsilon_1^2 \bar{M}^T Q\bar{M} \end{bmatrix} < 0, \tag{24}$$

$$i = 1, 2, \ldots, r,$$

then the augmented system (19) is asymptotically stable and satisfies $\|G_{\bar{e}\bar{d}}(s)\|_\infty < \gamma_1$. The gain matrices of the fuzzy adaptive observer are computed by $\bar{K}_i = Q^{-1}Y_{1i}$.

Proof. Define the Lyapunov candidate as:

$$V_0(t) = \bar{e}^T(t)\bar{M}^T Q\bar{M}\bar{e}(t). \tag{25}$$

For an symmetric positive definite matrix Q, the matrix $\bar{M}^T Q\bar{M}$ is also symmetric positive definite. Clearly, $V_0(t)$ is positive definite.

Firstly, the stability of the system (19) with $\bar{d}_o(t) = 0$ is proved. The time derivative of Equation (25) is

$$\begin{aligned} \dot{V}_0(t) &= \dot{\bar{e}}^T(t)\bar{M}^T Q\bar{M}\bar{e}(t) + \bar{e}^T(t)\bar{M}^T Q\bar{M}\dot{\bar{e}}(t) \\ &= \sum_{i=1}^r h_i(z)\bar{e}^T(t)\left(\bar{A}_i - \bar{K}_i\bar{C}\right)^T \bar{M}^{-T}\bar{M}^T Q\bar{M}\bar{e}(t) \\ &\quad + \sum_{i=1}^r h_i(z)\bar{e}^T(t)\bar{M}^T Q\bar{M}\bar{M}^{-1}\left(\bar{A}_i - \bar{K}_i\bar{C}\right)\bar{e}(t) \\ &= \sum_{i=1}^r h_i(z)\left\{\bar{e}^T(t)\left\langle \bar{M}^T Q\left(\bar{A}_i - \bar{K}_i\bar{C}\right)\right\rangle_S \bar{e}(t)\right\} \end{aligned} \tag{26}$$

It can be readily found that $\dot{V}_0(t) < 0$ is equivalent to

$$\left\langle \bar{M}^T Q\left(\bar{A}_i - \bar{K}_i\bar{C}\right)\right\rangle_S < 0. \tag{27}$$

Based on Lyapunov stability theorem, the system (19) is asymptotically stable as the inequality (27) is satisfied.

Then, when $\bar{d}_o(t) \neq 0$, it is proved that Equation (19) has H_∞ performance index [40, 41]

$$\|G_{\bar{e}\bar{d}}(s)\|_\infty < \gamma_1. \tag{28}$$

According to Lemma 1, the system (19) possess H_∞ performance with an attenuation level γ_1 if the following condition holds:

$$\begin{bmatrix} \left\langle \bar{M}^T Q\left(\bar{A}_i - \bar{K}_i\bar{C}\right)\right\rangle_S + I & \bar{M}^T Q\bar{B}_d \\ * & -\gamma_1^2 I \end{bmatrix} < 0. \tag{29}$$

Obviously, Equation (29) holds, implying that inequality (27) also holds. Finally, we have the condition (23) results from Equation (29) by defining $\bar{Y}_i = P\bar{K}_i$ and Schur complement.

Now, the convergence of $\bar{e}(t)$ is proved. According to Lemma 2 and assuming that $\sigma = 1$, for Equation (29), when

$$\bar{e}^T(t)\bar{e}(t) \geq \gamma_1^2 \bar{d}_o^T(t)\bar{d}_o(t), \tag{30}$$

it can guarantee that

$$\dot{V}_0(t) = \sum_{i=1}^r h_i(z)\begin{bmatrix} \bar{e} \\ \bar{d}_o \end{bmatrix}^T \begin{bmatrix} \left\langle \bar{M}^T Q\left(\bar{A}_i - \bar{K}_i\bar{C}\right)\right\rangle_S & \bar{M}^T Q\bar{B}_d \\ * & 0 \end{bmatrix}\begin{bmatrix} \bar{e} \\ \bar{d}_o \end{bmatrix} < 0, \tag{31}$$

and there is $T > 0$, for any $t > t_0 + T$ such that

$$\|\bar{e}(t)\|_2 \leq \gamma_1\|\bar{d}_o(t)\|_2, \tag{32}$$

which indicates the dynamic error system (19) is asymptotically stable and estimate error $\bar{e}(t)$ is ultimately uniformly bounded.

For the condition (24), by Lemma 2, there is a symmetric positive definite matrix $\bar{M}^T Q\bar{M}$ such that

$$\zeta = \begin{bmatrix} -\bar{M}^T Q\bar{M} & -\phi_1 \bar{M}^T Q\bar{M} + \bar{M}^T Q(\bar{A} - \bar{L}\bar{C}) \\ * & -\varepsilon_1^2 \bar{M}^T Q\bar{M} \end{bmatrix} < 0, \tag{33}$$

from which it follows that the eigenvalues of $\bar{M}^{-1}(\bar{A}_i - \bar{K}_i\bar{C})$ belong to the given circular region $O(\phi_1, \varepsilon_1)$. Then we can obtain Equation (24) via $Y_{1i} = Q\bar{K}_i$ and $\tilde{M}\zeta\tilde{M}^T$ with $\tilde{M} = diag(\bar{M}^{-1}, I)$. This completes the proof. □

Remark 4. Reviewing the existing approaches [40, 41], H_∞ optimisation and regional pole placement are effective methods in coping with disturbance and tuning the transient response, respectively. Here, H_∞ design and regional pole placement are combined to assist in the design of adaptive observer and improve the fault estimation performance, which are also applied to the controller design.

4 | FAULT TOLERANT CONTROL STRATEGY DESIGN

In this section, based on the fuzzy adaptive observer, a fuzzy adaptive sliding mode control strategy is developed. The control algorithm is divided into two steps: 1) The T–S fuzzy model of attitude control system is reconstructed; 2) A fuzzy adaptive sliding mode controller is developed to deal with the problem of attitude tracking and holding under actuator faults and input saturation.

4.1 | Attitude control system

From Equations (2)–(5), the attitude error dynamics system of flexible spacecraft can be derived as:

$$\begin{cases} \dot{q}_{ev} = \dfrac{1}{2}\left(q_{e0}I + q_{ev}^{\times}\right)\omega_e, \\ \dot{q}_{e0} = -\dfrac{1}{2}q_{ev}^T\omega_e, \end{cases} \tag{34}$$

$$\begin{aligned} \dot{\omega}_e = &-\left(J-\delta^T\delta\right)^{-1}\omega^{\times}J\omega_e + \left(J-\delta^T\delta\right)^{-1}\left(\Gamma\left(u+f\right)+d\right) \\ &- \left[\left(J-\delta^T\delta\right)^{-1}\omega^{\times}J - \omega_e^{\times}\right]\Theta_q\omega_d - \Theta_q\dot{\omega}_d, \end{aligned} \tag{35}$$

Define a new control error variable $s = \omega_e + \beta q_{ev}$, $\beta > 0$. Then, the new attitude control system can be obtain as follows:

$$\begin{aligned} \dot{s} = &-\left(J-\delta^T\delta\right)^{-1}\omega^{\times}Js + \left(J-\delta^T\delta\right)^{-1}\Gamma\left(u+f\right) \\ &+ \left(J-\delta^T\delta\right)^{-1}d + g(s), \end{aligned} \tag{36}$$

where $g(s) = \frac{1}{2}\beta(q_{e0}I + q_{ev}^{\times})\omega_e + (J-\delta^T\delta)^{-1}\omega^{\times}J\beta q_{ev} - [(J-\delta^T\delta)^{-1}\omega^{\times}J - \omega_e^{\times}]\Theta_q\omega_d - \Theta_q\dot{\omega}_d$.

Based on T–S fuzzy modelling approach, the system (36) can be reconstructed as

$$\begin{aligned} \dot{s}(t) = \sum_{i=1}^{r} h_i(z)\big\{&A_i s(t) + g(s) + B\Gamma\left(u(t) + f(t)\right) \\ &+ Bd_1(t) + D_{2i}d_2(t)\big\}. \end{aligned} \tag{37}$$

The same fuzzy rules as in the Section 2.2 are used, and the matrices A_i, B, D_{2i} is consistent with that in Equation (10). The complex nonlinear term $g(s)$ is introduced into the fuzzy model (37). It retains some nonlinear characteristics of the nonlinear model (36), which increases the accuracy of T–S model proposed. On the other hand, it can reduce the number of fuzzy rules, which results in less amount of calculation. It should be noted that, the term $g(s)$ can be obtain in real time from known information, which contributes to the achievement of feedback linearisation.

Remark 5. The nonlinear term $g(s)$ is bounded.

$$\begin{aligned} \|g(s)\| = &\left\|\frac{1}{2}\beta\left(q_{e0}I + q_{ev}^{\times}\right)\omega_e + \left(J-\delta^T\delta\right)^{-1}\omega^{\times}J\beta q_{ev}\right. \\ &\left.-\left[\left(J-\delta^T\delta\right)^{-1}\omega^{\times}J - \omega_e^{\times}\right]\Theta_q\omega_d - \Theta_q\dot{\omega}_d\right\| \\ \leq &\left\|\frac{1}{2}\beta\left(q_{e0}I + q_{ev}^{\times}\right)\omega_e\right\| + \left\|\left(J-\delta^T\delta\right)^{-1}\omega^{\times}J\beta q_{ev}\right\| \\ &+\left\|\left[\left(J-\delta^T\delta\right)^{-1}\omega^{\times}J - \omega_e^{\times}\right]\Theta_q\omega_d\right\| + \left\|\Theta_q\dot{\omega}_d\right\| \\ \leq &\frac{1}{2}\beta\|\omega_e\| + c_1\beta\|q_{ev}\| + c_2\|\omega_d\| + \|\dot{\omega}_d\|, \end{aligned} \tag{38}$$

where $\|q_{e0}I + q_{ev}^{\times}\| = 1$, $c_1 \geq 0$ and $c_2 \geq 0$ are bounded constants. All variables including $\omega_e, q_{ev}, \omega_d, \dot{\omega}_d$, and β are bounded, the term $g(s)$ is bounded consequently.

Remark 6. The comparison of T–S fuzzy model for flexible spacecraft attitude system. In refs. [5, 21, 22], the attitude and angular velocity are selected as premise variables, and consequently, the design of corresponding fuzzy rules is complex. It is worth mentioning that, in this paper, only three premise variables are needed to approximate the nonlinear model. Moreover, the T–S fuzzy model proposed has higher fitting accuracy and larger continuous definition domain owing to multiple rules are fired at the same time.

4.2 | Fuzzy adaptive sliding mode controller design

A linear sliding surface, also recognised as error variable above mentioned, is defined as follows:

$$s = \omega_e + \beta q_{ev}. \tag{39}$$

For the sake of ensuring the reachability of the assigned sliding surface, a T–S fuzzy adaptive sliding mode controller is introduced as follows:

$$\begin{cases} u(t) = u_{eq}(t) + u_{vs}(t), \\ u_{eq}(t) = \displaystyle\sum_{i=1}^{r} h_i(z)\left\{v - \hat{f} - \Gamma^{+}B^{-}D_{2i}\hat{d}_2 - \Gamma^{+}B^{-}g(s)\right\}, \\ v(t) = \displaystyle\sum_{j=1}^{r} h_j(z)K_c^i s, \\ u_{vs}(t) = -\Gamma^{+}\sigma_1 s - \Gamma^{+}\sigma_2\dfrac{s}{\|s\|_2}, \end{cases} \tag{40}$$

where $\sigma_2 > \frac{\|B\|}{\lambda_{\min}(B)}\sum_{i=1}^{r} h_i(z)\|\Gamma e_f(t) + d_1(t) + B^{-}D_{2i}e_d(t)\|$, $\sigma_1 > 0$. According to Assumptions 1 and 2, and adaptive laws (17), the gain $\sigma_2 > 0$ has a lower bound which can be calculated from the bounded signals $d_1(t)$, $e_f(t)$, and $e_d(t)$. $u_{eq}(t)$ denotes the equivalent control component and is chosen to ensure that $\dot{s} = 0$ for all time. $u_{vs}(t)$ denotes the variable structure component, which is to be chosen to ensure that the sliding surface $s = 0$ is attractive and can be reached in finite time. The virtual controller $v(t)$ is designed by state feedback control and K_c^i is the controller gain to be designed.

Theorem 2. *For the flexible spacecraft attitude control T–S fuzzy system (37) and T–S fuzzy adaptive sliding mode controller (40), let an H_∞ performance index $\gamma_2 > 0$, the initial state $s(0)$ upper bound $\|s(0)\|_2 < \mu$ and a circular pole constraint region $\mathrm{O}(\phi_2, \varepsilon_2)$ for $A_i + B\Gamma K_c^i$ be given. For prescribed scalar $\varsigma > 0$, if there exist matrices Y_{2i}, $X > 0$*

satisfying

$$\begin{bmatrix} \langle A_i X + B\Gamma Y_{2i} \rangle_S & \bar{B}_{ci} & X \\ * & -\gamma_2^2 I & 0 \\ * & * & -I \end{bmatrix} < 0, \quad (41)$$

$$\begin{bmatrix} -X & A_i X + B\Gamma Y_{2i} - \phi_2 X \\ * & -\epsilon_2^2 X \end{bmatrix} < 0, \quad (42)$$

$$-X + \mu^2 I < 0, \quad (43)$$

$$\begin{bmatrix} -X & -Y_{2i}^T \\ * & -\varsigma^2 I \end{bmatrix} < 0, \ i = 1, 2, \dots, r, \quad (44)$$

where $\bar{B}_{ci} = [B\Gamma \quad D_{2i} \quad B]$. *Then the fuzzy system (37) is global asymptotically stable, which implies* $\lim_{t\to\infty} s = 0$, $\lim_{t\to\infty} \omega_e = 0$, $\lim_{t\to\infty} q_{ev} = 0$. *The virtual controller* $v(t)$ *with the* H_∞ *performance index* γ_2 *satisfies* $\|v(t)\|_2 < \varsigma$. *The gain matrices of the virtual controller are computed by* $K_c^i = X^{-1} Y_{2i}$.

Proof. Firstly, the stability of sliding mode is proved. According to the sliding mode theory, once the state trajectory $s(t)$ reaching and sliding conditions are satisfied, the system is finally driven to stay in the sliding mode:

$$s = \omega_e + \beta q_{ev} = 0. \quad (45)$$

Select the Lyapunov function as:

$$V_1 = q_{ev}^T q_{ev} + (1 - q_{e0})^2 = 2(1 - q_{e0}). \quad (46)$$

Clearly, the time derivative of the Lyapunov function V_1 is

$$\dot{V}_1 = -2\dot{q}_{e0} = q_{ev}^T \omega_e = -\beta q_{ev}^T q_{ev} \le 0. \quad (47)$$

For $\dot{V} = 0$ only if $q_{ev} = 0$, the equilibrium state $q_{ev} = 0$ is global asymptotic stability. According to Equation (45), we have $\lim_{t\to\infty} q_{ev} = 0$, $\lim_{t\to\infty} \omega_e = 0$. Consequently, based on the sliding surface (39), the stability of the system in the sliding surface is guaranteed.

Then, the reaching of the sliding vector $s(t)$ is proved. The time derivative of the sliding surface (39) is

$$\begin{aligned} \dot{s} &= \sum_{i=1}^r h_i(z)\{A_i s + g(s) + B\Gamma(u(t) + f(t)) \\ &\quad + Bd_1 + D_{2i} d_2\} \\ &= \sum_{i=1}^r h_i(z)\Bigg\{A_i s + g(s) + B\Gamma\Bigg(\sum_{j=1}^r h_j(z)\Bigg\{v - \hat{f} \\ &\quad - \Gamma^+ B^- D_{2j}\hat{d}_2 - \Gamma^+ B^- g(s) - \Gamma^+\sigma_1 s - \Gamma^+\sigma_2 \frac{s}{\|s\|_2}\Bigg\} \\ &\quad + f\Bigg) + Bd_1 + D_{2i} d_2\Bigg\} \\ &= \sum_{i=1}^r h_i(z)\Bigg\{A_i s + B\Bigg(\Gamma v - \sigma_1 s - \sigma_2 \frac{s}{\|s\|_2} + \Gamma e_f(t)\Bigg) \\ &\quad + Bd_1(t) + D_{2i} e_d(t)\Bigg\}. \end{aligned} \quad (48)$$

By substituting Equation (48) into the sliding condition, we have

$$\begin{aligned} s^T \dot{s} &= \sum_{i=1}^r h_i(z) s^T \Bigg\{A_i s + B\Bigg(\Gamma v - \sigma_1 s - \sigma_2 \frac{s}{\|s\|_2} + \Gamma e_f(t)\Bigg) \\ &\quad + Bd_1 + D_{2i} e_d(t)\Bigg\} \quad (49) \\ &= \sum_{i=1}^r h_i(z)\Bigg\{s^T(A_i + B\Gamma K_c^i)s - s^T B\sigma_1 s - s^T B\sigma_2 \frac{s}{\|s\|_2} \\ &\quad + s^T B\big(\Gamma e_f(t) + d_1 + B^- D_{2i} e_d(t)\big)\Bigg\} \\ &\le \sum_{i=1}^r h_i(z)\{-s^T B\sigma_1 s - \|s^T\|_2 \lambda_{\min}(B)\sigma_2 \\ &\quad + \|s^T\|_2 \|B\| \|\Gamma e_f(t) + d_1(t) + B^- D_{2i} e_d(t)\|\} \\ &\le -\sum_{i=1}^r h_i(z) s^T B\sigma_1 s. \end{aligned} \quad (49)$$

where K_c^i in controller $v(t)$ is designed such that the matrix $A_i + B\Gamma K_c^i$ is Hurwitz. Thus, $s^T(A_i + B\Gamma K_c^i)s(t) \le \lambda_{\max}(A_i + B\Gamma K_c^i)\|s(t)\|_2^2 \le 0$ can be guaranteed.

Then, the design of K_c^i is given. From Equation (48), we have

$$\dot{s} = \sum_{i=1}^r h_i(z)\{(A_i + B\Gamma K_c^i)s + \bar{B}_{ci}\bar{d}_c(t)\}, \quad (50)$$

where $\bar{d}_c(t) = [\begin{matrix} e_f(t) \\ e_d(t) \\ -\sigma_1 s - \sigma_2 s/\|s\|_2 + d_1(t) \end{matrix}]$.

Select a Lyapunov function as:

$$V_2(t) = s^T(t) P s(t). \quad (51)$$

The time derivative of the Lyapunov function (51) is

$$\begin{aligned} \dot{V}_2(t) &= \dot{s}^T(t) P s(t) + s^T(t) P \dot{s}(t) \\ &= \sum_{i=1}^r h_i(z)\Big\{s^T \big\langle P(A_i + B\Gamma K_c^i)\big\rangle_S s + 2\bar{d}_c^T \bar{B}_{ci}^T P s\Big\}, \end{aligned} \quad (52)$$

Clearly, the condition that the matrix $A_i + B\Gamma K_c^i$ is Hurwitz is equivalent to

$$\big\langle P(A_i + B\Gamma K_c^i)\big\rangle_S < 0. \quad (53)$$

According to Lemma 2, the system (40) possess H_∞ performance with an attenuation level γ_2 if the following condition holds:

$$\begin{bmatrix} \left\langle P\left(A_i + B\Gamma K_c^i\right)\right\rangle_S & P\bar{B}_{ci} & I \\ * & -\gamma_2^2 I & 0 \\ * & * & -I \end{bmatrix} < 0. \tag{54}$$

The eigenvalues of $A_i + B\Gamma K_c^i$ belong to the given circular region $\mathrm{O}(\phi_2, \varepsilon_2)$ if the following condition holds:

$$\begin{bmatrix} -P & P\left(A_i + B\Gamma K_c^i - \phi_2 I\right) \\ * & -\varepsilon_2^2 P \end{bmatrix} < 0. \tag{55}$$

Define $X = P^{-1} > 0$. Pre- and post-multiplying Equation (54) by $\mathrm{diag}(X, I, I)$ and its transpose matrix, and Pre- and post-multiplying Equation (55) by $\mathrm{diag}(X, X)$ and its transpose matrix, we have

$$\begin{bmatrix} \left\langle \left(A_i + B\Gamma K_c^i\right)X\right\rangle_S & \bar{B}_{ci} & X \\ * & -\gamma_2^2 I & 0 \\ * & * & -I \end{bmatrix} < 0, \tag{56}$$

$$\begin{bmatrix} -X & \left(A_i + B\Gamma K_c^i - \phi_2 I\right)X \\ * & -\varepsilon_2^2 X \end{bmatrix} < 0. \tag{57}$$

The conditions (41) and (42) are obtained by $Y_{2i} = K_c^i X$.

Next, the constraint $\|v(t)\|_2 < \varsigma$ is proved. Assuming that $s^T(0)Ps(0) < 1$, by Schur complement, we have

$$\begin{bmatrix} 1 & s^T(0) \\ s(0) & P^{-1} \end{bmatrix} > 0. \tag{58}$$

According to $\|s(0)\|_2 < \mu$ and Equation (58), we assume that

$$\mu^2 I < X. \tag{59}$$

From the stability analysis above, for any $t > 0$ such that

$$s^T(t)Ps(t) \le s^T(0)Ps(0) < 1. \tag{60}$$

The condition $\|v(t)\|_2 < \varsigma$ leads to

$$\begin{aligned} v^T v &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \left(K_c^i s\right)^T K_c^j s \\ &\le \frac{1}{2}\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \left\{\left(K_c^i s\right)^T K_c^i s + \left(K_c^j s\right)^T K_c^j s\right\} \\ &= \sum_{i=1}^{r} h_i s^T \left(K_c^i\right)^T K_c^i s < \varsigma^2. \end{aligned} \tag{61}$$

Clearly, the inequality (61) holds, if

$$\sum_{i=1}^{r} h_i s^T \left(K_c^i\right)^T K_c^i s - \varsigma^2 s^T P s \le 0, \tag{62}$$

which is equivalent to $\left(K_c^i\right)^T K_c^i - \varsigma^2 P < 0$. Finally, the condition (44) can be obtained by invoking Schur complement and $Y_{2i} = K_c^i X$ on the following inequality:

$$X\left\{\left(K_c^i\right)^T K_c^i - \varsigma^2 P\right\}X = X\left(K_c^i\right)^T K_c^i X - X\varsigma^2 < 0. \tag{63}$$

This completes the proof. □

Remark 7. In this paper, we combine H_∞ design with regional pole placement to assist the state feedback control $v(t)$ and improve the control performance. In addition, adaptive observer and feedforward compensation design are used to compensate strong disturbances and actuator faults. An H_∞ method is adopted to attenuate the influence of disturbances and faults which are not fully compensated.

In engineering application, the chattering phenomenon usually exists in the sliding mode control due to the sign function $\mathrm{sgn}(s) = s/\|s\|$ in the variable structure controller. According to refs. [7, 21, 31], the sign function can be replaced by a saturation function or saturation-like function so to attenuate the chattering of the controller. Thus, using the continuous saturation-like function $s/(\|s\| + \sigma_2')$, the variable structure controller $u_{vs}(t)$ in Equation (40) is modified as follows:

$$u_{vs}(t) = -\Gamma^+\sigma_1 s - \Gamma^+\sigma_2 \frac{s}{\|s\|_2 + \sigma_2'}, \tag{64}$$

where $\sigma_2' > 0$ is a small parameter to be designed.

5 | NUMERICAL SIMULATION

In this section, simulation results on a flexible spacecraft are provided to show the feasibility and benefits of the proposed strategy. The numerical simulation including three parts: 1) T–S fuzzy model; 2) actuator fault estimation; 3) fault tolerant control. The T–S fuzzy model of flexible spacecraft attitude system proposed in refs. [5, 21, 22], the fault estimation observer proposed in ref. [10], and the fault tolerant control strategy proposed in refs. [21, 35] are considered for comparison. Moreover, the energy function $E = \int_0^T \|u(t)\|_2^2 \mathrm{d}t$ is introduced to quantitatively characterise the energy-cost of controller [10–12].

Considering that the vibrations of flexible appendages are mainly determined by low-frequency modes, the first four-order nature frequencies are adopted in the simulation. The nature frequencies of the flexible appendages are set as $\omega_{n1} = 1.5362$ rad/s, $\omega_{n2} = 2.2076$ rad/s, $\omega_{n3} = 3.7466$ rad/s, $\omega_{n4} = 3.7466$ rad/s. The corresponding damping ratios are $\zeta_{n1} = 0.0056$, $\zeta_{n2} = 0.0086$, $\zeta_{n3} = 0.013$, $\zeta_{n4} = 0.013$.

In this paper, the actuator distribution matrix is set as $\Gamma = I_3$. The inertia matrix, coupling matrix, external disturbance, and perturbation matrix are given as follows [21, 31]:

$$J_0 = \begin{bmatrix} 350 & 3 & 4 \\ 3 & 270 & 10 \\ 4 & 10 & 190 \end{bmatrix} \mathrm{kg}\cdot\mathrm{m}^2,$$

$$\delta = \begin{bmatrix} 6.45637 & 1.27814 & 2.15629 \\ -1.25819 & 0.91756 & -1.67264 \\ 1.11687 & 2.48901 & 0.83674 \\ 1.23637 & -2.65810 & -1.12530 \end{bmatrix} \mathrm{kg}^{1/2}\cdot\mathrm{m/s}^2,$$

$$u_d(t) = \begin{bmatrix} -3 + 4\cos(0.2\pi t) - \cos(0.4\pi t) \\ 4 + 3\sin(0.2\pi t) - 2\cos(0.4\pi t) \\ -3 + 4\sin(0.2\pi t) - 3\sin(0.4\pi t) \end{bmatrix} 10^{-4}\mathrm{N}\cdot\mathrm{m},$$

$$\Delta\Gamma = \begin{bmatrix} 0 & \Delta a_2\cos\Delta b_2 & \Delta a_3\cos\Delta b_3 \\ \Delta a_1\cos\Delta b_1 & 0 & \Delta a_3\cos\Delta b_3 \\ \Delta a_1\cos\Delta b_1 & \Delta a_2\cos\Delta b_2 & 0 \end{bmatrix},$$

$$\Delta a = (0.2, 0.1, 0.1)\,\mathrm{deg}, \Delta b = (0.15, 0.1, 0.1)\,\mathrm{deg}.$$

Generally, three scenarios of actuator faults are considered: fault-free (healthy), intermittent fault, and time varying fault. Here are three channels defined as:

$$f_1(t) = \begin{cases} 0.5(t-40)^{0.2}\sin(0.2(t-40))\mathrm{N}\cdot\mathrm{m}, & 40\ \mathrm{s} < t, \\ 0\mathrm{N}\cdot\mathrm{m}, & \text{others}, \end{cases}$$

$$f_2(t) = \begin{cases} 1\mathrm{N}\cdot\mathrm{m}, & 40\ \mathrm{s} < t \le 70\ \mathrm{s}, \\ 1.5\mathrm{N}\cdot\mathrm{m}, & 100\ \mathrm{s} < t, \\ 0\mathrm{N}\cdot\mathrm{m}, & \text{others}, \end{cases} \qquad (66)$$

$$f_3(t) = 0\mathrm{N}\cdot\mathrm{m}.$$

In this example, according to the general working situation of the flexible spacecraft, the operating regions of premise variables are set as $\omega_j(t) \in [-0.7 \quad 0.7]\mathrm{rad/s}$, $j = 1, 2, 3$. Based on eight operating points [42], which are $(0.7, 0.7, 0.7)$, $(0.7, 0.7, -0.7)$, $(0.7, -0.7, 0.7)$, $(0.7, -0.7, -0.7)$, $(-0.7, 0.7, 0.7)$, $(-0.7, 0.7, -0.7)$, $(-0.7, -0.7, 0.7)$, $(-0.7, -0.7, -0.7)$, the membership functions of T–S fuzzy sets $M_j^i (i = 1, 2, \ldots, 8, j = 1, 2, 3)$ are illustrated in Figure 2. The T–S fuzzy model of the flexible spacecraft is expressed as follows:

Model rule 1: IF $\omega(t)$ is $\begin{bmatrix} 0.7 & 0.7 & 0.7 \end{bmatrix}$, THEN

$$\dot{x}(t) = A_1 x(t) + B\big(\Gamma(u + f) + d_1\big) + D_{21} d_2,$$

$$\dot{s}(t) = A_1 s(t) + g(s) + B\big(\Gamma(u + f) + d_1\big) + D_{21} d_2.$$

$\vdots$

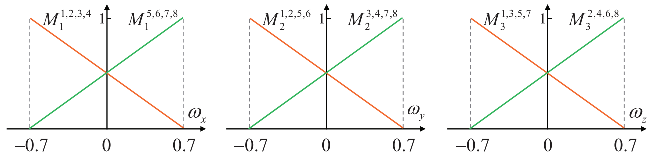

FIGURE 2 Membership functions

Model rule 8: IF $\omega(t)$ is $\begin{bmatrix} -0.7 & -0.7 & -0.7 \end{bmatrix}$, THEN

$$\dot{x}(t) = A_8 x(t) + B\big(\Gamma(u + f) + d_1\big) + D_{28} d_2,$$

$$\dot{s}(t) = A_8 s(t) + g(s) + B\big(\Gamma(u + f) + d_1\big) + D_{28} d_2.$$

Remark 8. In this paper, T–S fuzzy models (10) and (37) adopt the same set of fuzzy rules, and share the state matrix and control input matrix. In this numerical simulation, the design of fault estimation observer and fault tolerant controller are based on fuzzy models, and the flexible spacecraft attitude system expresses in nonlinear system.

5.1 | T–S fuzzy model

The effectiveness of the proposed T–S fuzzy model (37) is demonstrated on a feedback control loop (see Equation (69)). The T–S fuzzy model in ref. [5], which is also adopted in refs. [21, 22], is considered as comparison. Considering a general situation, the initial flexible spacecraft attitude states are chosen by $q_v^T(0) = (0.1, -0.1, 0.1)$, $\omega^T(0) = (0.02, -0.03, 0.01)$, $\eta(0) = 0_{4\times1}$, $\dot{\eta}(0) = 0_{4\times1}$. The desired states are given by $q_{dv}^T = (0.3, 0.2, -0.4)$, $\omega_d = 0_{3\times1}$. As shown in Figure 3, the proposed T–S fuzzy model (37) has good approximation performance on the actual nonlinear model (36). However, the T–S fuzzy model proposed in ref. [5] deviates from the actual model (36), where the number of fuzzy rules fired to approximate the actual model is less.

5.2 | Actuator fault estimation

The gain matrices F_2, G_2, and adjustable parameters θ_1, θ_2 are selected as:

$$F_2 = \begin{bmatrix} 2 & 0 & 0 \\ * & 1 & 0 \\ * & * & 1 \end{bmatrix}, G_2^T = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix},$$

$$\theta_1 = 20, \theta_2 = 20. \qquad (68)$$

We set regional pole constraint as $\mathrm{O}(-10, 10)$, the feasibility radius of LMIs in Theorem 1 as 1×10^7, and $\gamma_1^2 \le 0.1$, which are conducive to avert the singularity of gain matrix $\bar{K}_i$ and enhance the numerical stability. The designed parameters for proposed observer (16) are provided in Appendix (1). The simulation analysis is performed for two manoeuver process, which is shown in Table 1. The initial flexible spacecraft attitude states are chosen by $q_v^T(0) = (0.3, 0.2, -0.4)$,

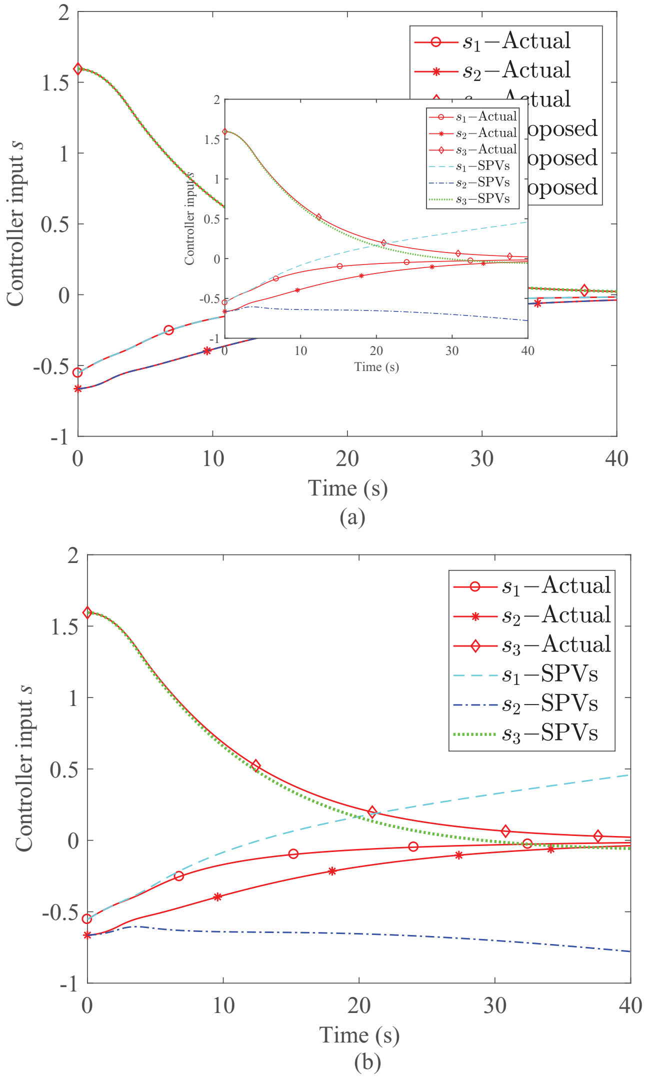

FIGURE 3 (a) Comparison between the T–S fuzzy model proposed and actual model; (b) comparison between the T–S fuzzy model in ref. [5] and actual model

TABLE 1 Manoeuver sequence of flexible spacecraft

Time (s)	Fault events	Manoeuver events
0–40	No	Yes
40–80	Yes	No
80–200	Yes	Yes

$\omega^T(0) = (0.02, -0.03, 0.01)$, $\eta(0) = 0_{4\times1}$, $\dot{\eta}(0) = 0_{4\times1}$. The desired states are set as $\omega_d = 0_{3\times1}$, $q_{dv}^T = (0.3, 0.2, -0.4)$ at 0–80 s, and $\omega_d = 0_{3\times1}$, $q_{dv}^T = (0.2, -0.1, -0.1)$ at 80–200 s. Moreover, the actuator faults with different amplitude are applied to analyse the performance of the method proposed. The weak faults are defined as $0.2f_i(t), i = 1, 2, 3$ and the condition is only used here.

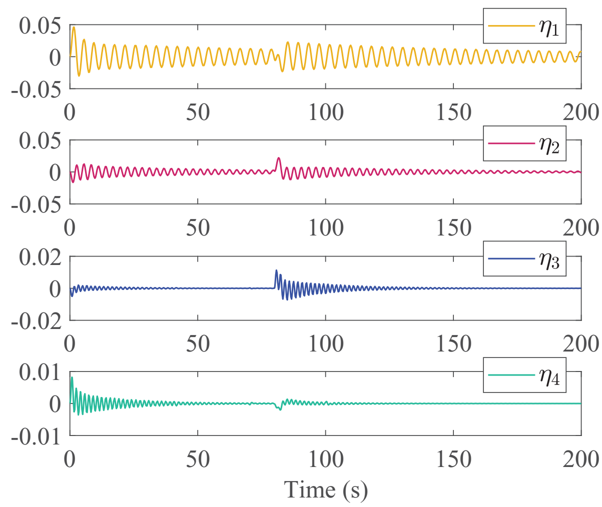

FIGURE 4 The vibration mode of flexible appendages

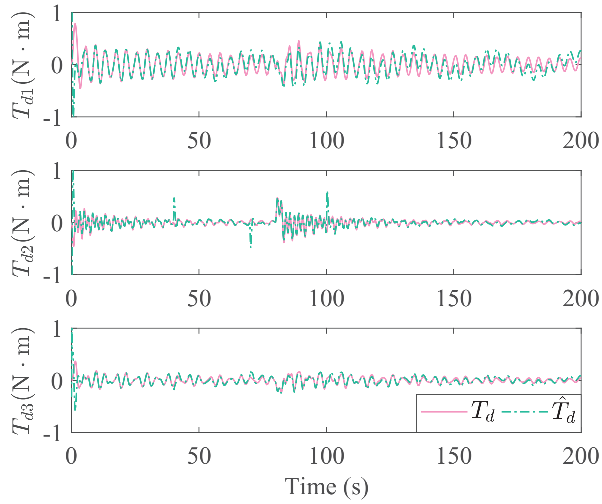

FIGURE 5 The estimation of vibration disturbances

Figure 4 illustrates the vibration mode of the flexible appendages under the proposed control strategy. As shown in Figure 5, the proposed observer is able to estimate the vibration disturbances caused by flexible appendages during spacecraft manoeuver. Clearly, the estimation of vibration disturbances is robust to actuator faults and attitude manoeuver. The estimation and estimated error of angular velocity are drawn in Figure 6. It should be pointed that all the estimated values also show the fluctuation with fault characteristics as actuator faults occur, especially for the time-varying fault. The estimation error convergence bound is $\|e_x(t)\|_2 \leq 4 \times 10^{-7}$ rad/s in 15 s, and increases to $\|e_x(t)\|_2 \leq 3 \times 10^{-5}$ rad/s as the actuator faults occurs. Moreover, Figure 7 depicts the comparison of simulation results under the intermittent fault and time-varying fault, respectively. The week faults are also considered in Figure 8. Obviously, the proposed observer can estimate both the system states and actuator faults simultaneously, and all estimation errors asymptotically converge to a small bounded interval.

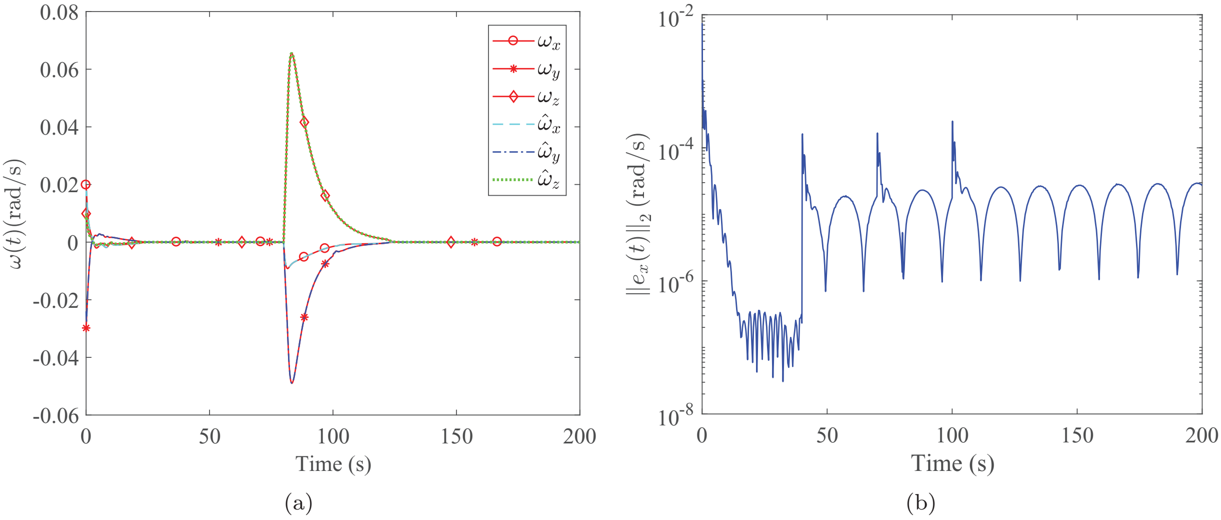

FIGURE 6 (a) The estimation of angular velocity; (b) the estimation error of angular velocity

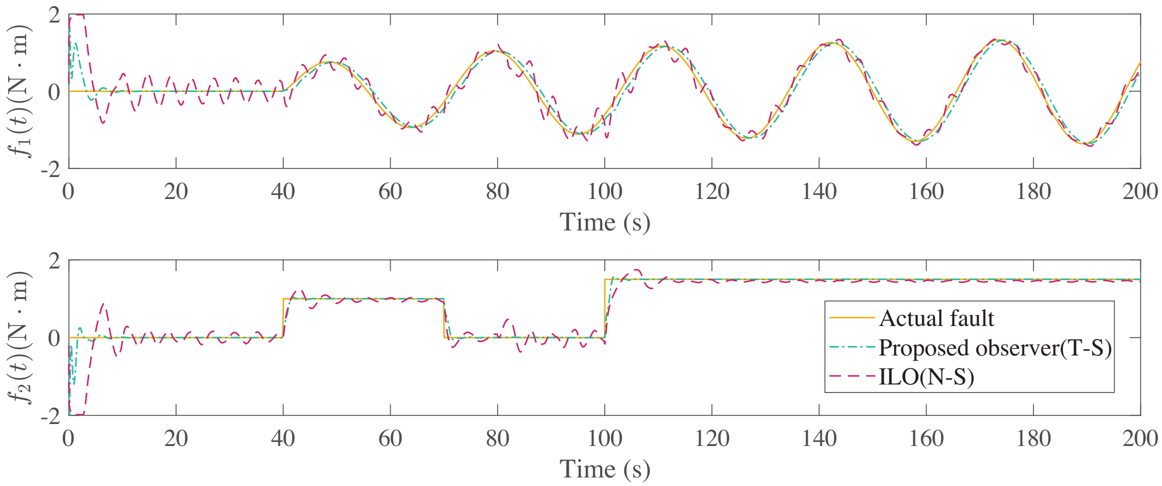

FIGURE 7 Comparison of fault estimation

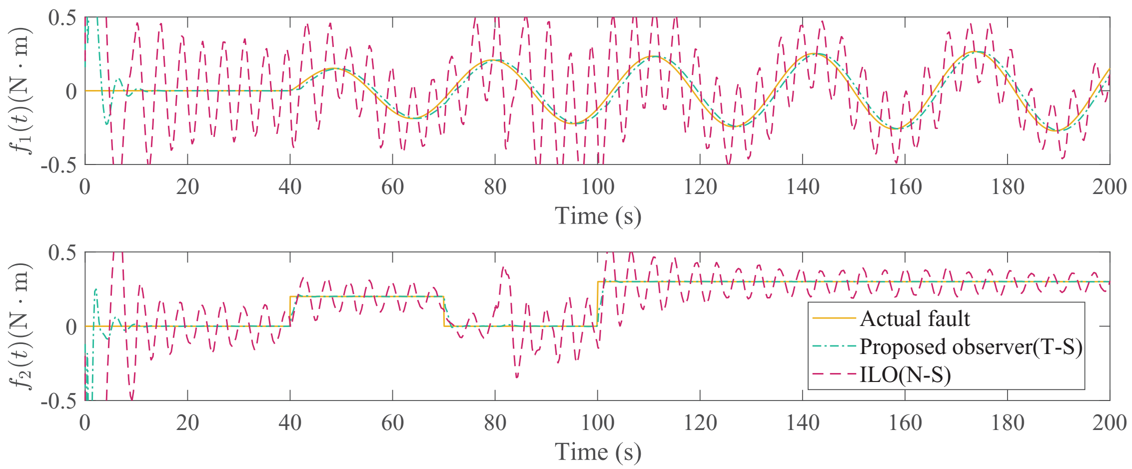

FIGURE 8 Comparison of fault estimation (under weak faults)

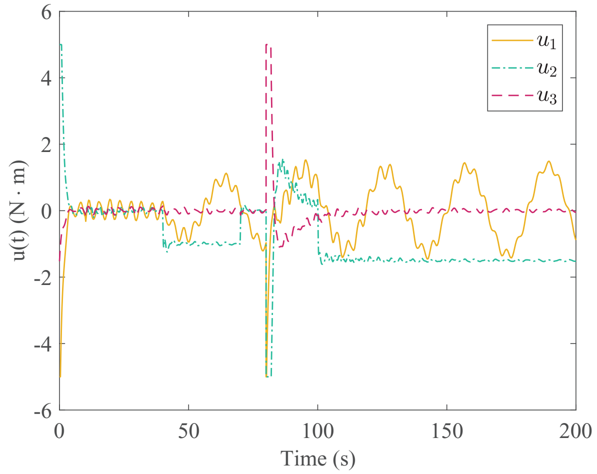

FIGURE 9 Time trajectory of control input torque

Compared with the iterative learning observer (ILO) based on nonlinear system (N-S) in ref. [10], the proposed method have lower fault estimation errors and higher estimation accuracy, especially for weak faults.

5.3 | Fault tolerant control

Three fault tolerant control methods are considered to draw comparisons. The control input saturation is set as $\|u\|_1 < u_{\max} = 5\text{N} \cdot \text{m}$. Note that the variable s, as mentioned in formula (36) earlier, is defined as $s = \omega_e + \beta q_{ev}$.

(a) Based on the feedforward compensation including nonlinear term $g(s)$ and estimations $\hat{f}(t)$, $\hat{d}_2(t)$, the classical proportional-derivative (PD) control can be designed as:

$$u(t) = -k_p q_{ev} - k_d \omega_e - \hat{f}(t) - B^+ D_{2i} \hat{d}_2(t) - B^+ g(s), \tag{69}$$

where $k_p = 20$ and $k_d = 60$ are proportional term and derivative term, respectively.

(b) According to ref. [21], the adaptive integral sliding mode control (AISMC) can be designed as:

$$u(t) = -\sum_{i=1}^{r} h_i(z)\Gamma^+ \left\{ \left\| K_c^i s(t) \right\|_2 + \sigma + \hat{d}_m(t) \right\} \frac{S}{\|S\|_2 + \sigma'} - \Gamma^+ B^+ g(s), \tag{70}$$

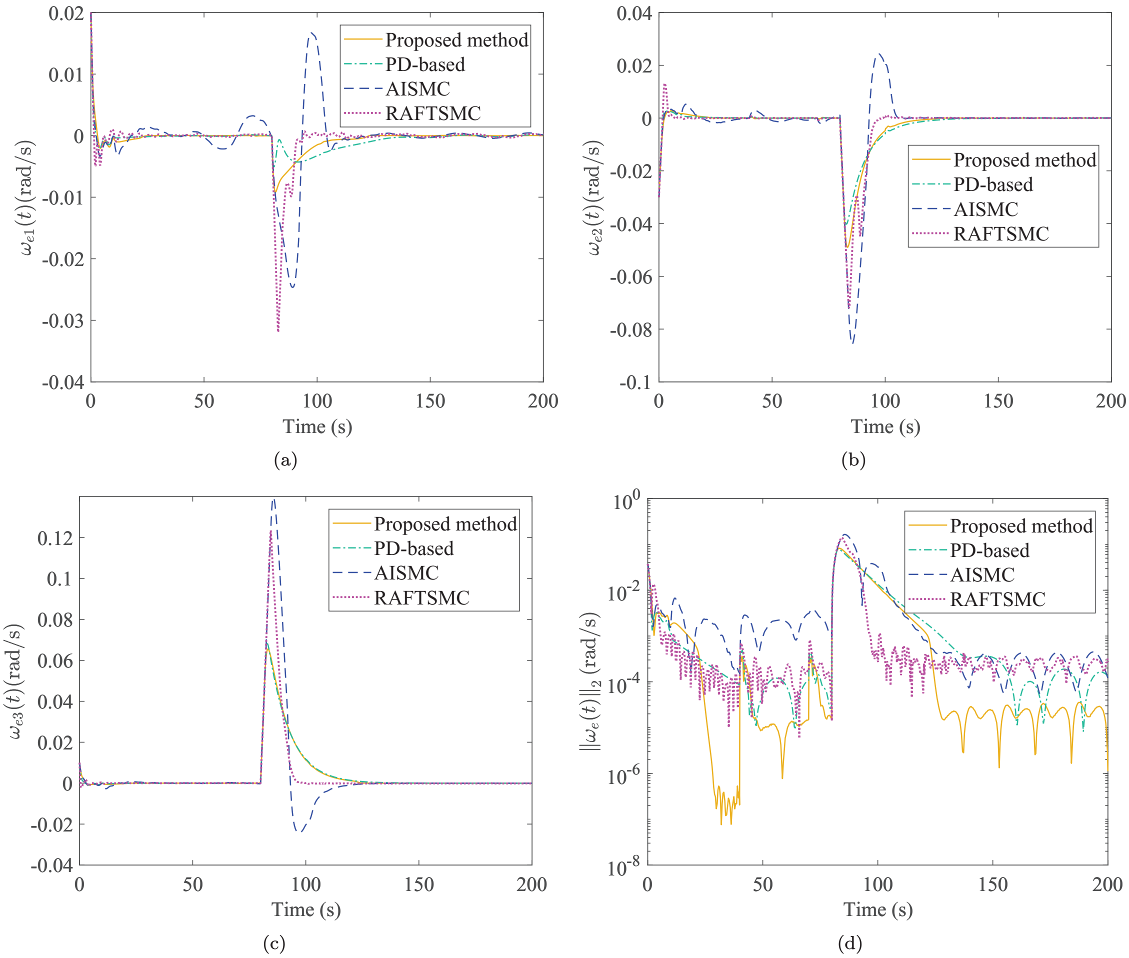

FIGURE 10 Comparison of angular velocity tracking errors: (a) channel with time varying fault; (b) channel with intermittent fault; (c) channel without fault; (d) norm of angular velocity tracking errors

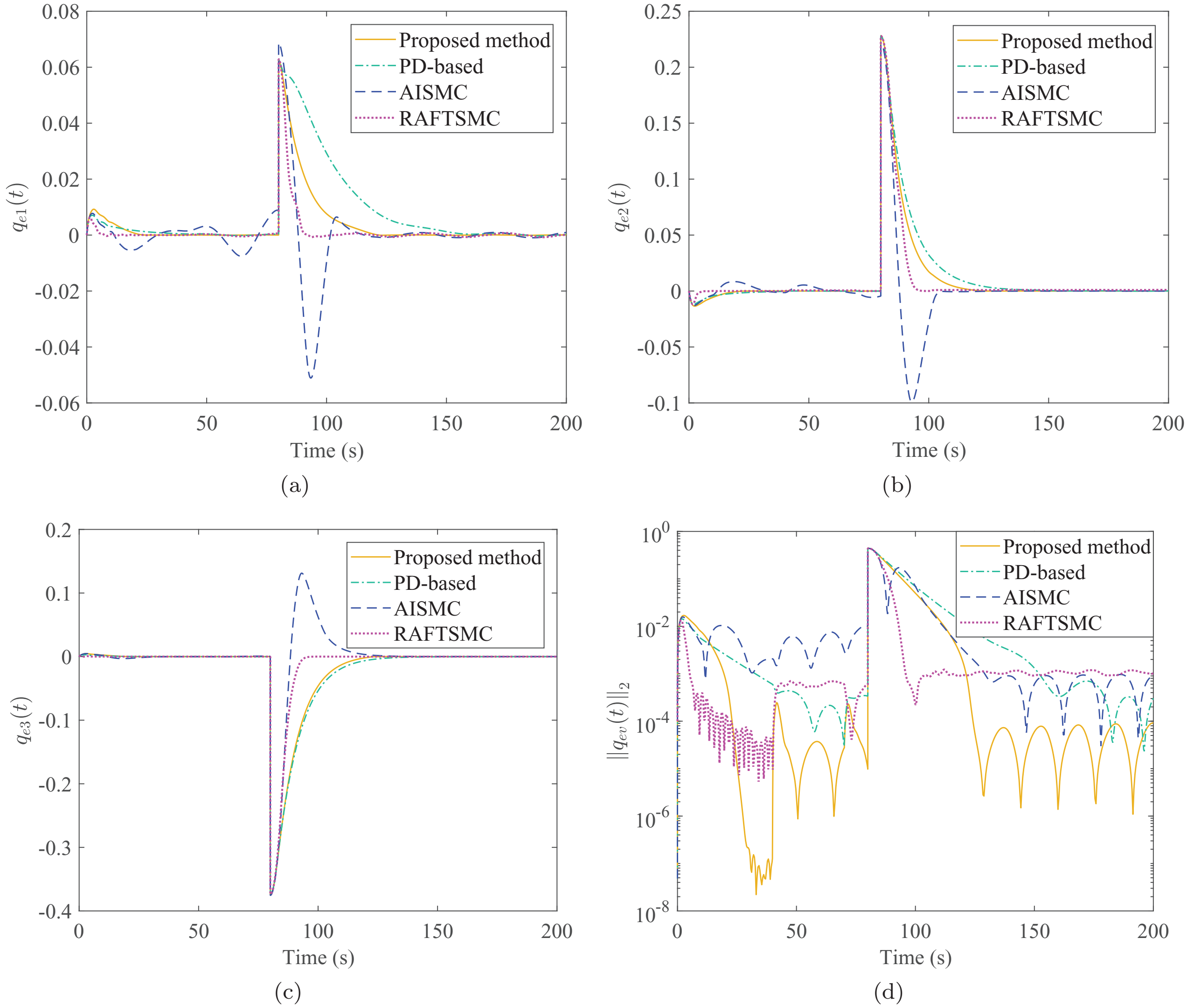

FIGURE 11 Comparison of attitude tracking errors: (a) channel with time varying fault; (b) channel with intermittent fault; (c) channel without fault; (d) norm of attitude tracking errors

$$S(t) = s - \int_0^t \sum_{i=1}^{r} h_i(z)\left(A_i + BK_c^i\right)sdt, \hat{d}_m(t) = \|S\|_2, \tag{71}$$

where Equation (71) denotes the integral sliding surface; $\beta = 2$, $\sigma = 0.002$, and $\sigma' = 0.01$. $\hat{d}_m(t)$ denotes the estimation of the upper bound $d_m \geq \|d(t)\|_2$ of lumped disturbance $d(t)$. K_c^i is computed by Theorem 1 in ref. [21]. It is noted that, as applied in literature [11–13, 16], actuator faults and disturbances are considered as a lumped disturbance.

(c) In ref. [35], a nonsingular fast fixed-time terminal sliding surface and a robust adaptive fixed-time sliding mode control (RAFTSMC) scheme are considered. The designed parameters of the controller in ref. [35] are provided as follows: $k_1 = k_2 = 0.2$, $l_1 = 8.8$, $l_2 = -252$, $\phi = 0.01$, $r_1 = 1.2$, $r_2 = 0.6$, $r_3 = 1.5$, $r_4 = 0.6$, $\kappa_1 = 0.4$, $\kappa_2 = 20$, $\kappa_3 = 0.5$, $\kappa_4 = 8$, $\gamma_1 = 1$, $\gamma_2 = 10$, $\gamma_3 = 10$, $\varepsilon = 0.01$, $g_1 = g_2 = 1$, $p_1 = p_2 = 0.1$.

Based on above simulation situation in fault estimation, we obtain the simulation results of attitude tracking control and comparisons. The designed parameters for proposed controller (40) are provided in Appendix (2). Figure 9 depicts the time trajectory of control input torque, where the signal is always constrained within the allowable maximum magnitude. Figures 10 and 11 show the comparison of simulation results under the intermittent fault and time varying fault, including attitude tracking errors and angular velocity tracking errors. It can be seen, all control methods can achieve attitude stability under actuator faults, and the effect of time varying fault on steady-state error is relatively greater. As displayed in Figures 10(d) and 11(d), the tracking errors of angular velocity and attitude quaternion obtained by all controllers converge to a small neighbourhood of zero in the end. Differently,

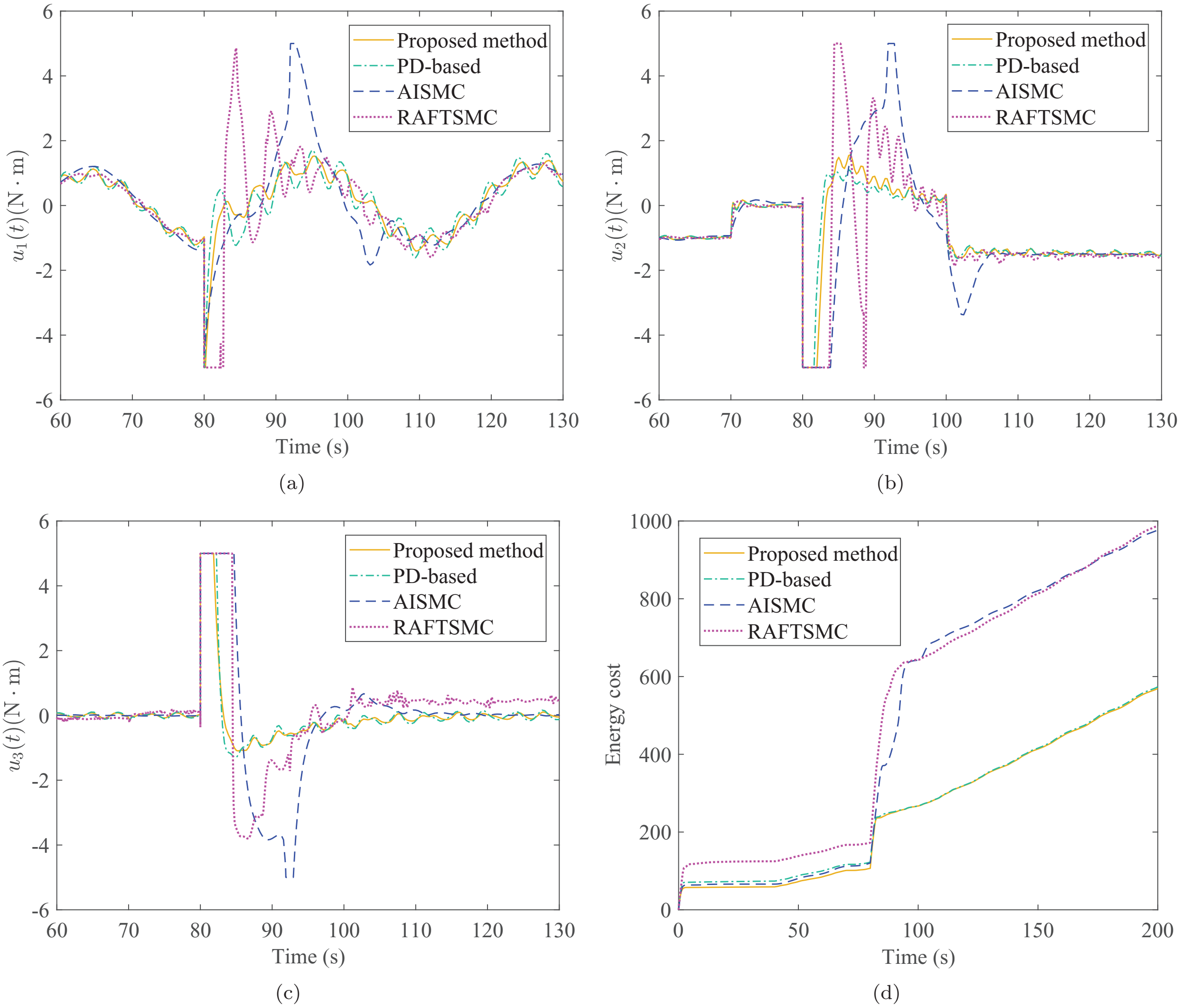

FIGURE 12 Comparison of control input torque: (a) channel with time varying fault; (b) channel with intermittent fault; (c) channel without fault; (d) controller energy cost

compared with PD-based method and AISMC scheme, the fault tolerant control algorithm proposed has better performance in terms of convergence rate and steady-state error. The RAFTSMC has the fastest convergence performance and the proposed control scheme gets superior steady-state errors. According to energy function mentioned above, the comparison of energy utilisation is drawn in Figure 12. It is obvious that the energy-cost by AISMC and RAFTSMC is relatively high, and the energy consumption gap comes from the initial stage of spacecraft manoeuver. Due to regional pole placement and saturation virtual controller $v(t)$, the control input given by proposed control strategy is not too radical when the control error signal is large. Therefore, the spacecraft obtain less fluctuation of control input torque which results in better energy-saving performance. It is worth noting that although PD-based method can achieve low energy consumption, it leads to larger steady-state error and slower convergence rate.

Overall, the above analysis results of three parts have sufficiently verified the effectiveness and advantages of the developed strategy.

6 | CONCLUSION

In this paper, based on T–S fuzzy models, we presented an observer-based anti-disturbance adaptive sliding mode control scheme to solve fault estimation and fault tolerant control problem of a flexible spacecraft subject to unknown actuator faults, external disturbances, configuration misalignment, and input saturation. The T–S fuzzy model was taken to express the nonlinear dynamics of flexible spacecraft attitude system. The estimation of actuator fault and flexible vibration is completed by a fuzzy adaptive observer proposed. Moreover, based on the adaptive observer and sliding mode control theory, a fuzzy adaptive fault tolerant controller was developed to ensure the reachability of the sliding surface and the stabilisation of the system. The sufficient stability conditions were given in the terms of linear matrix inequality, which can be easily calculated and has high numerical stability. Furthermore, the H_∞ optimisation was adopted to attenuate the effect of disturbances. Regional pole placement was applied to tune the system transient response. The simulation results with comparisons has verified the effectiveness and superiority of this paper. As future work, the proposed strategy can be extended to address the issue of anti-disturbance fault estimation of flexible spacecraft with more actuators, such as six thrusters.

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CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

Yanbo Li: Conceptualization, data curation, formal analysis, investigation, methodology, software, validation, visualization, writing - original draft, writing - review and editing. Wei Xu: Conceptualization, formal analysis, methodology, project administration, resources, supervision, writing - original draft, writing - review and editing. Lin Chang: Conceptualization, formal analysis, funding acquisition, investigation, resources, writing - original draft.

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APPENDIX A

(1) The designed parameters of proposed observer (16) are provided as follows:

$$\gamma_1 = 1.6563,\ f_{\max} = 2,\ f_{\min} = -2,\ \hat{d}_{\max} = 0.2,$$

$$K_1 = \begin{bmatrix} 11.0797 & 1.7060 & 0.1698 \\ 2.0634 & 15.3316 & -3.7670 \\ 1.1124 & -3.2298 & 17.4597 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 11.2338 & 1.8061 & 1.0575 \\ 0.3014 & 16.8966 & -1.4905 \\ 2.8869 & -2.5642 & 17.1767 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} 11.3012 & 1.3731 & -0.3320 \\ 3.1385 & 14.9069 & -3.4138 \\ 2.0857 & -4.0953 & 14.7673 \end{bmatrix},$$

$$K_4 = \begin{bmatrix} 13.5644 & 0.6913 & 1.1004 \\ 2.6231 & 15.8636 & -2.9494 \\ 1.6844 & -4.8170 & 15.5415 \end{bmatrix},$$

$$K_5 = \begin{bmatrix} 11.3442 & 2.2118 & 1.2684 \\ 0.9174 & 16.4726 & -4.7243 \\ 6.0193 & -2.6507 & 18.1318 \end{bmatrix},$$

$$K_6 = \begin{bmatrix} 10.6319 & 2.1572 & 0.2341 \\ 0.0696 & 17.1557 & -2.4063 \\ 1.1002 & -0.5380 & 16.0206 \end{bmatrix},$$

$$K_7 = \begin{bmatrix} 11.9018 & 0.7661 & 0.0607 \\ 2.0015 & 15.4469 & -3.9478 \\ 3.2444 & -2.6290 & 13.9143 \end{bmatrix},$$

$$K_8 = \begin{bmatrix} 11.9489 & 0.8640 & 0.1272 \\ 0.4371 & 16.8437 & -2.8419 \\ 1.2606 & -2.6794 & 14.0976 \end{bmatrix};$$

$$F_{11} = \begin{bmatrix} 9.7237\times 10^3 & 326.8199 & 1.8290\times 10^3 \\ -6.3344 & 1.0247\times 10^4 & -3.2907\times 10^3 \\ -746.9617 & -1.2268\times 10^3 & 8.3579\times 10^3 \end{bmatrix},$$

$$F_{12}=\begin{bmatrix}9.0309\times10^{3} & 2.7778\times10^{3} & 2.1771\times10^{3}\\ -4.6158\times10^{3} & 1.1244\times10^{4} & -2.5613\times10^{3}\\ 66.1973 & -570.1799 & 8.2265\times10^{3}\end{bmatrix},$$

$$F_{13}=\begin{bmatrix}9.6426\times10^{3} & 398.4586 & -212.2283\\ 34.0224 & 1.0012\times10^{4} & -2.9185\times10^{3}\\ 2.9817\times10^{3} & -1.7920\times10^{3} & 7.6271\times10^{3}\end{bmatrix},$$

$$F_{14}=\begin{bmatrix}1.0171\times10^{4} & 2.4550\times10^{3} & -210.9471\\ -3.5754\times10^{3} & 1.0542\times10^{4} & -3.0415\times10^{3}\\ 2.6386\times10^{3} & -1.5275\times10^{3} & 7.6178\times10^{3}\end{bmatrix},$$

$$F_{15}=\begin{bmatrix}9.1963\times10^{3} & 453.3374 & 3.3619\times10^{3}\\ -918.4914 & 1.1172\times10^{4} & -2.2130\times10^{3}\\ 1.3119\times10^{3} & -3.5679\times10^{3} & 9.4382\times10^{3}\end{bmatrix},$$

$$F_{16}=\begin{bmatrix}8.6973\times10^{3} & 2.8241\times10^{3} & 1.6734\times10^{3}\\ -3.8500\times10^{3} & 1.1534\times10^{4} & -1.2398\times10^{3}\\ -539.3831 & -1.9341\times10^{3} & 8.0846\times10^{3}\end{bmatrix},$$

$$F_{17}=\begin{bmatrix}1.0057\times10^{4} & -389.7244 & 96.0473\\ 42.9327 & 1.0556\times10^{4} & -1.2508\times10^{3}\\ 4.2230\times10^{3} & -3.8485\times10^{3} & 7.5450\times10^{3}\end{bmatrix},$$

$$F_{18}=\begin{bmatrix}9.3632\times10^{3} & 1.9728\times10^{3} & -305.1111\\ -4.2603\times10^{3} & 1.1604\times10^{4} & -1.1329\times10^{3}\\ 3.5869\times10^{3} & -3.4873\times10^{3} & 7.5922\times10^{3}\end{bmatrix};$$

$$G_{11}=\begin{bmatrix}320.6129 & -5.1408 & 159.0961\\ -142.4824 & 290.5667 & -348.9390\\ 202.5739 & 167.4318 & -91.7588\\ 173.1203 & -149.7373 & -83.8280\\ 431.2368 & 86.5778 & 142.8238\\ -898.6241 & 455.6093 & -1157.4772\\ 541.0819 & 324.0231 & 180.4034\\ 684.5043 & -139.6698 & 143.7356\end{bmatrix},$$

$$G_{12}=\begin{bmatrix}305.5511 & 83.2036 & 168.8351\\ -275.0154 & 293.5678 & -325.2914\\ 131.8068 & 214.6869 & -77.3230\\ 210.9485 & -145.2681 & -88.5007\\ 360.4500 & 234.6058 & 170.6782\\ -1114.33017 & 135.7276 & -1144.0781\\ 231.3908 & 558.8776 & 261.2598\\ 756.9340 & -51.7888 & 129.6552\end{bmatrix},$$

$$G_{13}=\begin{bmatrix}361.7443 & -10.4094 & 82.7031\\ -213.7139 & 300.2449 & -309.3160\\ 164.6629 & 170.8180 & -117.1624\\ 99.9721 & -134.5523 & -98.5022\\ 482.6662 & 82.1496 & 34.3687\\ -1331.2870 & 498.5755 & -810.7609\\ 443.0646 & 331.6670 & 64.3321\\ 551.2211 & -116.0938 & 33.4766\end{bmatrix},$$

$$G_{14}=\begin{bmatrix}375.0407 & 59.3479 & 83.4535\\ -292.5768 & 297.4400 & -322.4400\\ 140.9596 & 209.5630 & -119.4195\\ 160.0084 & -123.8689 & -98.5515\\ 476.3326 & 195.0656 & 29.8964\\ -1496.0033 & 243.5520 & -786.4493\\ 341.6687 & 509.1828 & 72.8951\\ 706.7613 & -8.0327 & 30.5024\end{bmatrix},$$

$$G_{15}=\begin{bmatrix}329.4036 & -27.5213 & 219.7104\\ -214.5960 & 362.0529 & -350.0487\\ 158.7602 & 204.1559 & -71.9300\\ 133.8327 & -120.2928 & -104.5593\\ 421.0692 & 70.1396 & 252.8766\\ -1141.8168 & 762.1123 & -1426.4282\\ 388.1852 & 446.0579 & 336.5068\\ 621.8390 & -89.9025 & 161.9484\end{bmatrix},$$

$$G_{16}=\begin{bmatrix}287.8983 & 68.5705 & 150.4751\\ -242.2390 & 328.5386 & -281.4231\\ 142.58864 & 231.3742 & -71.8559\\ 208.7085 & -125.3695 & -109.8762\\ 341.6599 & 221.3584 & 158.6230\\ -948.2652 & 309.6604 & -1014.3476\\ 276.6781 & 618.2566 & 290.4692\\ 753.0822 & -17.0462 & 46.5165\end{bmatrix},$$

$$G_{17}=\begin{bmatrix}391.6389 & -60.4696 & 92.2539\\ -245.0695 & 356.4513 & -265.0377\\ 161.0947 & 182.5473 & -94.7841\\ 87.2154 & -114.6821 & -115.8267\\ 518.6878 & 17.8405 & 68.4526\\ -1542.7317 & 881.1959 & -761.7621\\ 432.2399 & 364.3024 & 180.1956\\ 563.2590 & -122.3085 & -13.6815\end{bmatrix},$$

$$G_{18} = \begin{bmatrix} 357.6202 & 22.5913 & 80.2989 \\ -335.4756 & 362.5720 & -262.0622 \\ 108.6588 & 233.2093 & -99.0336 \\ 151.2859 & -109.6418 & -122.4343 \\ 431.5542 & 153.8080 & 47.5774 \\ -1548.3732 & 605.1020 & -713.4597 \\ 194.7192 & 595.3380 & 147.4134 \\ 690.0607 & -21.4134 & -52.0813 \end{bmatrix}. \quad \text{(A.1)}$$

(2) The designed parameters of proposed controller (40) are provided as follows:

$$\gamma_2 = 1.7919, \mu = 0.1, (\phi_2, \varepsilon_2) = (20, 20), \beta = 2,$$

$$\varsigma = 5\sqrt{3}, \sigma_1 = 0.2, \sigma_2 = 0.15, \sigma_2' = 1 \times 10^{-4},$$

$$K_c^1 = \begin{bmatrix} -34.7440 & 1.1063 & 1.0937 \\ 1.4111 & -33.8341 & -0.8464 \\ 0.3764 & -0.5589 & -20.8476 \end{bmatrix},$$

$$K_c^2 = \begin{bmatrix} -35.4191 & 0.5616 & 0.7990 \\ 1.4483 & -33.8696 & -1.6043 \\ 0.1094 & -1.6136 & -20.9780 \end{bmatrix},$$

$$K_c^3 = \begin{bmatrix} -31.9910 & 0.9290 & 0.8885 \\ 1.5486 & -33.8368 & -1.0351 \\ 1.9635 & -0.6481 & -19.9350 \end{bmatrix},$$

$$K_c^4 = \begin{bmatrix} -32.5344 & 0.5510 & 0.5865 \\ 1.2052 & -33.8601 & -1.4684 \\ 1.6329 & -1.2981 & -20.0662 \end{bmatrix},$$

$$K_c^5 = \begin{bmatrix} -35.0347 & -0.6622 & 0.8585 \\ -0.0102 & -34.0008 & -0.8408 \\ 0.0632 & -0.9848 & -21.1602 \end{bmatrix},$$

$$K_c^6 = \begin{bmatrix} -35.2320 & -1.2476 & 1.2101 \\ 0.0100 & -33.8693 & -1.6387 \\ 0.8286 & -2.0212 & -21.1286 \end{bmatrix},$$

$$K_c^7 = \begin{bmatrix} -32.1281 & -0.5289 & 0.6218 \\ 0.2065 & -34.0080 & -1.0504 \\ 1.5953 & -1.1829 & -20.1715 \end{bmatrix},$$

$$K_c^8 = \begin{bmatrix} -32.2891 & -0.9208 & 0.9916 \\ -0.1776 & -33.8574 & -1.4950 \\ 2.2599 & -1.8550 & -20.1519 \end{bmatrix}. \quad \text{(A.2)}$$