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Property of Many-Body Localization in Heisenberg Ising Chain Under Periodic Driving

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Abstract
We study the property of the many-body localization in Heisenberg Ising Chain model with periodic driving by using the method of matrix exact diagonalization. We consider a driving protocol in which the system's Hamiltonian is periodically switched between two operators. The first segment is a disordered Ising system, and acts for time T_0; It is worth noting that the Hamiltonian of the second part is an operator dependent on time, and acts for time T_1, so the driving period is $T = T_0 + T_1$. We choose excited state fidelity to observe the phase transition between the localized phase and the ergodic phase of the system, which reflects the property of many-body localization in Heisenberg Ising Chains under periodic driving. Through the study, we find that when the disorder strength h is small, the system is in the ergodic phase, periodic driving can cause the occurrence of a transition from the ergodic phase to the localized phase, while the system is in the localized phase with a large disorder strength h, the transition from the localized phase to the ergodic phase will occur under the periodic driving. For these two cases, they all show that there is a critical driving period T_c, when the driving period is greater than T_c, the system will have a phase transition, meanwhile, T_c decreases with the increase of driving strength. Furthermore, we also get that the system size and disorder strength also effect the critical point of the driving period. The critical point decreases as the strength of disorder increases and decreases with the increase of the system size.

Keywords Many-body localization · Quantum phase transition · Thermodynamics · Disordered spin chain models

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1 Introduction

A static disorder potential can lead to a complete absence of diffusion in a closed quantum system which has received extensive attention. This is the Anderson localization [1, 2] mentioned in Anderson's previous articles more than half a century ago. It shows that a static disordered potential may lead to a complete absence of diffusion in an closed quantum system and has reached complete conclusion that non-interacting systems in one and two dimensions will be localized for arbitrary disorder [3, 4], even for very small disorder. Until more recently, Basko et al. [5] gave new conclusions to receive this idea of many-body localization. Many features of many-body localization (MBL) have been explored [6–18]. It has been displayed that bipartite entanglement entropy between two sectors of the system shows a characteristic logarithmic growth in the many-body localized phase. Many-body localization at nonzero temperature is a quantum transition, which is very important for many-body quantum physics and statistical mechanics: it is a quantum "glass transition" [19].

Strong disorder has many functions. First of all, the tendency of infinite absorption of energy from time-periodic driving filed in closed quantum many-body system is an obstacle to find new non-equilibrium phase. In addition, the interaction system with strong disorder may lead to the appearance of the MBL phase, which does not allow the transport of energy and particles [20]. Then for the ground state of correlated system, disorder is also of great importance and may inhibit conductance. Periodically driven systems can exhibit nontrivial steady state even in the absence of interaction limits. In a periodically driven system, the topological many-body states can have very long lifetimes, although it is usually metal stable [21]. Periodically driven a quantum system in time can change its long-term dynamics and trigger topological order. Whether locally driven or globally driven, the ergodic system is always heated to infinite temperature in the energy space [22]. Periodically driven systems retain the memory of their initial conditions for any length of time in the MBL phase [23]. In a disordered interacting system, suppressing jump amplitude can increase the relative strength of disorder and interactions, potentially driving the transition from a static delocalized system to a localized system [24]. Driving interacting systems are different from non-interacting systems: the long-term behavior of non-interacting systems is described by generalized Gibbs ensemble [25, 26]. The influence of periodic driving on ergodic system and localized system is very different: when the ergodic system is heated to infinite temperature, its Floquet eigenstate is delocalized in energy space, while localized system only absorbs energy locally [20]. While a complete classification of non-ergodic systems remains an open problem, it has recently established that many-body localization provides a robust mechanism of ergodicity breaking in systems with quenched disorder [23–26]. The response of disordered system to periodic driving provides a natural experimental probe for solid and cold atomic systems [27–32]. The dynamics of an isolated quantum many-body system with Hamiltonian switching periodically between disordered and non-disordered operators is the focus of our study.

For the periodic driving, its Hamiltonian is a time-dependent periodic function, $H(t+T) = H(t)$, which properties are determined by the unitary Floquet operator . If the evolution operator over one period, the Floquet operator has the form [26],

$$\widehat{F} = \mathcal{T}\exp\left\{-i\int_0^T H(t)dt\right\} \tag{1}$$

here $\mathcal{T}\exp$ is a time-ordered exponential. In the eigenstate basis $|\Psi_\alpha\rangle$ and the quasienergies θ_α of $\widehat{F}$, it can be written as $\widehat{F} = \sum_{n=1}^{D} e^{-i\theta_\alpha}|\Psi_\alpha\rangle\langle\Psi_\alpha|$, where D is the dimension of the system. An effective Floquet Hamiltonian H_F can be introduced, as $\widehat{F} = e^{-iH_F T}$, with same eigenstates $|\Psi_\alpha\rangle$,

Through studying the properties of the Floquet eigenstates, one can identify two phase: the localized phase [6–12], in which almost all eigenstates have area-law entanglement entropy, and the eigenstate thermalization hypothesis is violated; the delocalized phase, in which eigenstates have volume-law entanglement and obey the ETH [13–18]. In this paper, we mainly research the property of the many-body localization in disordered Heisenberg Ising Chain with periodic driving. The model we choose is a one-dimensional istropic Heisenberg spin -1/2 chain, and the phase transition between the ergodic phase and the localized phase is observed by excited state fidelity. We consider a driving protocol in which the system's Hamiltonian is periodically switched between two operators. The first segment is a disordered system, and acts for time T_0, it is worth noting that the second Hamiltonian is an operator related to the trigonometric function of time. Through observing the phase transition between the localized phase and the ergodic phase, we can see the effect of periodic driving on the property of the many-body localization in Heisenberg Ising model and study the behavior of this disordered system under global driving.

2 Numerical Model

We study a one-dimensional isotropic Heisenberg spin -1/2 Chain with many-body nearest interactions, L bits, and open boundary conditions. We consider a driving procotol in which the system's Hamiltonian is periodically switched between two operators, H_0 and H_1, the disordered Hamiltonian H_0 acts for time T_0 and has the form

$$H_0 = \sum_i h_i S_i^z + S_i^z S_{i+1}^z \tag{2}$$

Where random filed h_i is uniformly distributed in the interval [-h,h], h is disorder strength; and the delocalizing Hamiltonian H_1 we choose:

$$H_1(t) = \sum_i (\mathbf{S}_i \cdot \mathbf{S}_{i+1} + S_i^z) * V_o * \cos(\omega t - \pi/4) \tag{3}$$

which acts for time T_1, hence, the driving period is $T = T_0 + T_1$. Then the Floquet operator in our model is

$$\widehat{F} = \mathcal{T} \exp\left\{-i \int_0^{T1} H_1(t)dt\right\} e^{-iH_0 T_0}. \tag{4}$$

It is worth noting that Hamiltonian H_1 is time-dependent. Then we want to discuss the influence of this type periodically driving on the disordered system. We tune the strength of the kick T_1 to observe the transition between the localized phase and the ergodic phase.

3 Results and Discussion

In order to observe the phase transition between the localized phase and the ergodic phase, we select the excited fidelity. The ground-state fidelity [33–36] per lattice site is defined as the overlap of the first ground-state with parameter λ and λ+δλ, that is,

$$F_0(\lambda, \lambda + \delta\lambda) = |\langle \psi_0(\lambda) \mid \psi_0(\lambda + \delta\lambda)\rangle| \tag{5}$$

Analogously, we get the definition of the fidelity of the n-th excited state $\psi_n(T, h)$ for this periodically driven system, as the overlap between $|\psi_n(T, h)\rangle$ and $|\psi_n(T, h + \delta h)\rangle$. It is special that, here the fidelity is not defined with respect to the same parameter T [37–40], but with respect to the disorder strength, while δh is a small shift, with the following forms: $\delta h = \epsilon h$, let $\epsilon = 10^{-3}$.

$$F_n(T, h + \delta h) = |\langle \psi_n(T, h) \mid \psi_n(T, h + \delta h)\rangle| \tag{6}$$

It has been shown that [34] not only the ground state fidelity but also the excited state fidelity plays an essential role. Then for each disordered implementation, we select the many-body eigenstate $|\psi_n\rangle$ in the excited state in the middle segment of the energy ordered list of all data. Because the excited states represent high-energy states, thus we look only at excited state and avoid states that represent low-temperature. We next calculate the fidelity F_n for each eigenstate $|\psi_n\rangle$. The averaged $\mathsf{E}[F]$ was obtained by averaging over all chosen excited states and disordered realizations. The standard libraries for exact matrix diagonalization are adopted for numerical analyses. For each disorder amplitude h, We used 10000 disorder realizations for N=6, 1000 disorder realizations for N=8 and N=10, 100 disorder realizations for N=12 to yield the data illustrated in this article. Then we plot the averaged excited state fidelity $E[F]$ as a function of the driving period T_1, while we let $T_0 = 1$.

From Figs. 1, 2, 3, 4 and 5, the disordered Heisenberg Ising Chain system is in the localized phase with large disorder strength. We study the phase transition from the localized phase to the ergodic phase. According to Figs. 1 and 2, $h = 3.5$ and $h = 10$ respectively, it shows that the phase transition from the localized phase to the ergodic phase does occur. It indicates that both the driving strength V_0 and the disorder strength h affect the critical point of the phase transition. According to Fig. 1, one also can get the critical point T_c: for $V_0 = 10, T_c \to 0.7$; $V_0 = 20, T_c \to 0.5$; $V_0 = 30, T_c \to 0.4$. According to Fig. 2, for $V_0 = 5, T_c \to 1.5$, $V_0 = 10, T_c \to 0.8$, $V_0 = 30, T_c \to 0.5$, $V_0 = 50, T_c \to 0.3$. These datas show that the critical point T_c decreases as driving strength V_0 increases, and the larger the disorder strength, the larger the critical driving period is. In Fig. 3, we select $n = 8$ and

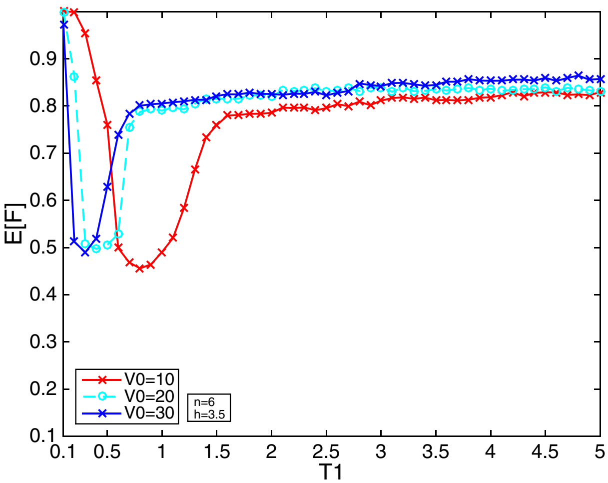

Fig. 1 Averaged excited state fidelity as a function of driving period T_1 for driving strength from 10 to 20. The disorder strength $h = 3.5$, the system size is $n = 6$. $E[F]$ decays substantially under the periodic driving until T approaches to the critical point T_c. The critical point T_c decreases as driving strength V_0 increases

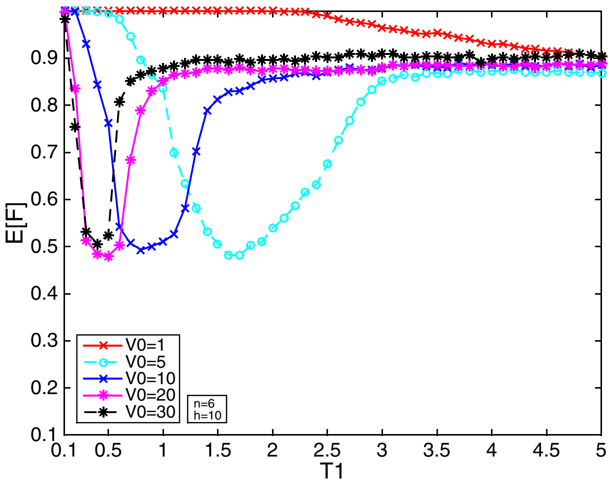

Fig. 2 Averaged excited state fidelity as a function of T_1 for different values of driving strength from 1 to 50. The disorder strength is $h = 10$, the system size is $n = 6$. $E[F]$ decays substantially under the periodic driving until T approaches to the critical point T_c. The point T_c decreases as driving strength V_0 increases

$h = 10$ to further explore the effect of system size on the phase transition. It also shows that the critical point T_c decreases with the driving strength increases as the previous figure. Particularly, We can see that the system size also has influence on the phase transition. In order to illustrate this, in Figs. 4 and 5, we further select $n = 6, n = 8$ and $n = 10$ to observe the behavior of the system with same disorder strength and under same driving strength V_0. Both Figs. 4 and 5 all indicate that the system size does effect the critical point, the larger the system size, the smaller the critical driving period T_c is.

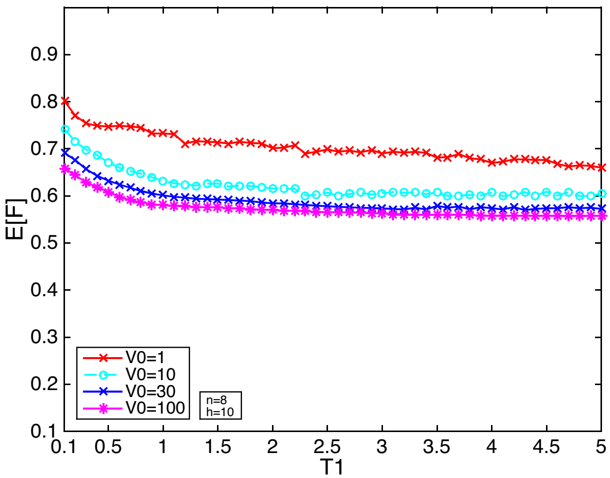

Fig. 3 Averaged excited state fidelity as a function of T_1 for different values of driving strength from small to large. The disorder strength is $h = 10$ and the system size is $n = 8$. $E[F]$ decays under the periodic driving until T approaches to the critical point T_c, then $E[F]$ changes slowly and tents to a straight line. One can get that $E[F]$ decreases with V_0 increases

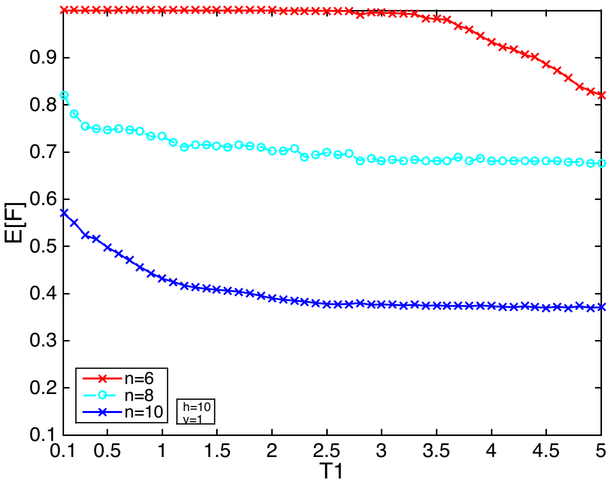

Fig. 4 Averaged excited state fidelity as a function of driving period T_1 for representative system size. The disorder strength $h = 10$ and the driving strength $V_0 = 1$. The system size n are indicated in the legend. $E[F]$ decreases with driving period T increases until T approaches to the critical point T_c, then $E[F]$ changes slowly and tends to a straight line. The larger the system size, the smaller the critical point T_c is

From Figs. 6 to 8, the disordered Heisenberg Ising Chain system is in the ergodic phase with $h = 0.5$. We study the phase transition from the ergodic phase to the localized phase. They all show that for this disordered system, the phase transition from the ergodic phase to the localized phase also occurs under periodic driving. In Fig. 6, the system size is $n = 6$, $h = 0.5$, we plot the excited state fidelity $E[F]$ as a function of driving period T_1 for representative values of driving strength V_0. We then explore the effects of driving strength and

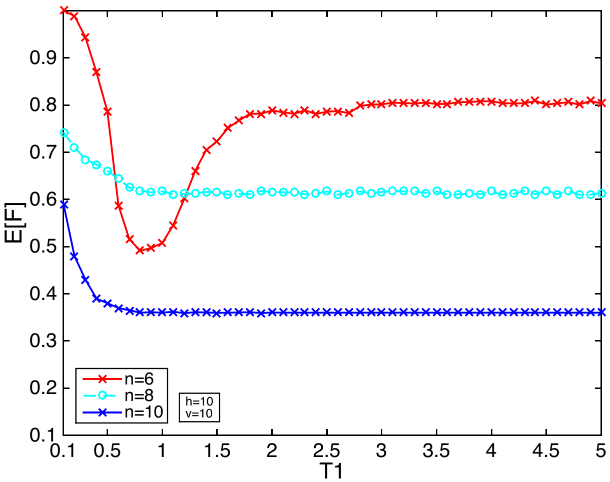

Fig. 5 Averaged excited state fidelity as a function of driving period T_1 for the value of disorder strength $h = 10$, the driving strength $V_0 = 10$. The system size n are indicated in the legend. $E[F]$ decreases with driving period T increases until T approaches to the critical point T_c, then $E[F]$ changes slowly and tents to a straight line. The larger the system size, the smaller the critical point T_c is

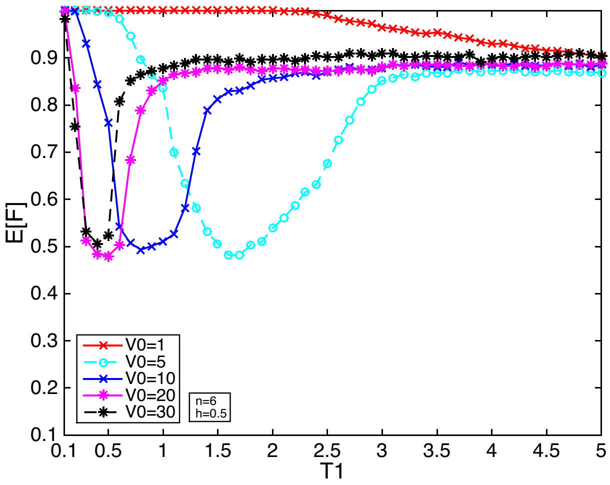

Fig. 6 Averaged excited state fidelity as a function of driving period T_1 for the disorder strength $h = 0.5$, the system size is $n = 6$. The driving strength V_0 are indicated in the legend. $E[F]$ decays substantially under the periodic driving until T approaches to the critical point T_c. The critical point T_c decreases as driving strength V_0 increases

system size on the property of the many-body localizaton in this periodical driving Heisenberg Ising Chain. By comparing the curves, one can get the driving strength also affect the critical point of the this phase transition. The larger the driving strength, the smaller the critical driving period T_c is, and the faster the $E[F]$ decays. in Fig. 6. In Fig. 7, we select $n = 8$, $h = 0.5$, It shows that the critical point decreases as the driving strength increases as the previous figure. By comparing, we can get that the system size has the same effect on

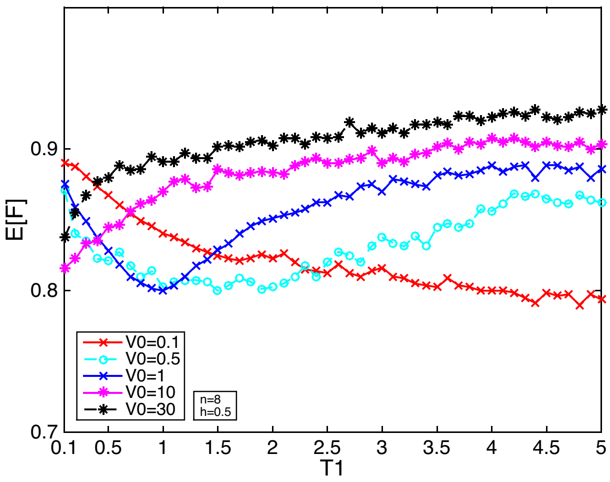

Fig. 7 Averaged excited state fidelity as a function of driving period T_1 for the value of disorder strength $h = 0.5$, the system size is $n = 8$. The driving strength V_0 are indicated in the legend. When V_0 is small, $E[F]$ decays until T approaches to the critical point T_c, then $E[F]$ turns to increase approximately approaching to a stable value; when V_0 is large, $E[F]$ increases with driving strength increases

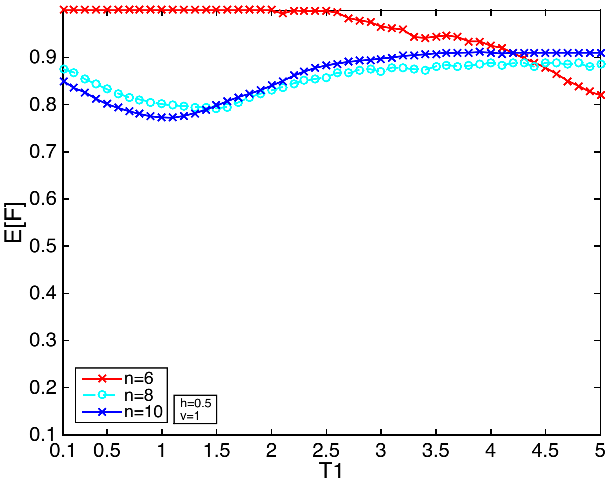

Fig. 8 Averaged excited state fidelity as a function of driving period T_1 for the value of disorder strength $h = 0.5$, the driving strength $V_0 = 1$. The system size n are indicated in the legend. $E[F]$ decreases with driving period T increases until T approaches to the critical point T_c, then $E[F]$ changes slowly and tents to a straight line. The larger the system size, the smaller $E[F]$ is

the critical point. In order to illustrate this, in Fig. 8, we select $n = 6$, $n = 8$ and $n = 10$ and verify the conclusion that the larger the system size, the smaller the critical point is. Then we observe the behavior of the system with same driving strength and verified the previous conclusions.

4 Summary

In this paper, we use the exact matrix diagonalization to explore the property of the many-body localization in disordered Heisenberg Ising Chain with periodically driving. We study a one-dimensional isotropic Heisenberg spin -1/2 Chain with many-body nearest interactions, L bits, and open boundary conditions. The Hamiltonian of the disordered system is composed of piecewise functions, in which the Hamiltonian H_0 is the disorder term and acting for time T_0. It is specially noted that the Hamiltonian is an operator related to the trigonometric function of time, acting for T_1. Through observing the excited state fidelity, when the disorder strength h is small, the system is in the ergodic phase, periodic driving can cause the phase transition from the ergodic phase to the localized phase; while the h is large, the system is in the localized phase, the phase transition from the localized phase to the ergodic phase will occur under the periodical driving, it is proved that the periodic driving will induce the phase transition between the localized phase and the ergodic phase. Furthermore, we also get that the system size, the disorder strength and driving strength all effect the critical point of the driving period. The conclusion is that the larger the disorder strength, the smaller the critical driving period is; the larger the system size, the smaller the critical driving period is; the larger driving strength, the smaller the critical driving period is. We hope that the present work can provide a meaningful tool for gaining a better understanding of the MBL transition and ergodicity breaking in quantum systems, and we will research this interesting phenomenon further in our future work.

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Data Availability The datasets generated during and analyzed during the current study are available from the corresponding author on reasonable request.

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